Online Aappendix to "Digital Assets and the Exorbitant Dollar Privilege"

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1 Model without Stablecoins

For the analysis that follows, it is convenient to define

$$\tilde{d}_t = d_t - B_t.$$

The variable d_t is the agent's demand for domestic bonds and $B_t \geq 0$ is the per-capita stock of domestic debt. Thus, $\tilde{b}_t$ is the agent's demand for domestic bonds in *excess* of the per-capita stock of domestic public debt. We impose that $f_{t+1} \geq 0$, since domestic agents cannot issue foreign bonds. Using the definition of $\tilde{d}_t$ and the government budget $T_t = B_t - B_{t+1}/R_t$, we can rewrite the agent's budget constraint as

$$c_t + p_t k_{t+1} + \frac{\tilde{d}_{t+1}}{R_t} + \frac{(1+\varphi(F_{t+1}))f_{t+1}}{R_t^*} = (z_t + p_t)k_t + \tilde{d}_t + f_t. \tag{1}$$

With this, we can show that agents' decisions are linear in the end-of-period wealth net of the government debt, namely, $a_t = (z_t + p_t)k_t + \tilde{d}_t + f_t$.

Lemma 1.1. *Given a_t and $\{p_t, R_t, R_t^*, \varphi(F_{t+1})\}_{t=1}^{\infty}$, agents' policies are*

$$\begin{aligned} c_t &= (1-\beta)a_t, \\ p_t k_{t+1} &= \phi_t \beta a_t, \\ \frac{\tilde{d}_{t+1}}{R_t} + \frac{(1+\varphi(F_{t+1}))f_{t+1}}{R_t^*} &= (1-\phi_t)\beta a_t, \end{aligned}$$

where ϕ_t satisfies $\mathbb{E}_t \left[\frac{\max\left\{R_t\,,\, R_t^*/(1+\varphi(F_{t+1}))\right\}}{\phi_t\left(\frac{z_{t+1}+p_{t+1}}{p_t}\right)+(1-\phi_t)\cdot\max\left\{R_t\,,\, R_t^*/(1+\varphi(F_{t+1}))\right\}} \right] = 1.$

Proof. A domestic agent maximizes expected lifetime welfare subject to its budget constraint eq. (1) and the non-negativity constraint $f_{t+1} \geq 0$ choosing k_{t+1}, f_{t+1}, and $\tilde{d}_{t+1}$. The first order conditions are

$$k_{t+1}: \qquad \frac{p_t}{c_t} = \beta \mathbb{E}_t \left(\frac{z_{t+1} + p_{t+1}}{c_{t+1}} \right). \tag{2}$$

$$f_{t+1}: \qquad \frac{1}{c_t} \geq \beta \frac{R_t^*}{1+\varphi(F_{t+1})} \mathbb{E}_t \left(\frac{1}{c_{t+1}} \right). \tag{3}$$

$$\tilde{d}_{t+1}: \qquad \frac{1}{c_t} = \beta R_t \mathbb{E}_t \left(\frac{1}{c_{t+1}} \right). \tag{4}$$

Equation (3) is satisfied with equality if $f_{t+1} > 0$ and with the inequality if $f_{t+1} = 0$. There are three cases, which depend on equilibrium interest rates R_t and R_t^*:

- **Case 1,** $R_t > \frac{R_t^*}{1+\varphi(F_{t+1})}$**:** Eq. (4) implies that eq. (3) holds with inequality. Thus, $f_{t+1} = 0$.
- **Case 2:** $R_t = \frac{R_t^*}{1+\varphi(F_{t+1})}$. In this case, eq. (3) holds with equality, so $f_{t+1} > 0$. The agent is indifferent between f_{t+1} and $\tilde{d}_{t+1}$. Individual portfolios cannot be determined (and $f_{t+1} = 0$ is also an individual solution). Only the aggregate value, F_{t+1}, is determined.
- **Case 3:** $R_t < \frac{R_t^*}{1+\varphi(F_{t+1})}$. This case is not possible in equilibrium because it will violate eq. (3). The agent would take an infinitely large negative position in $\tilde{d}_{t+1}$ and an infinitely large position in f_{t+1}. Since all agents would do that, in equilibrium there would be an infinite demand of F_{t+1} that reduces the equilibrium interest rate R_t^* until $R_t \geq \frac{R_t^*}{1+\varphi(F_{t+1})}$.

To solve for the individual portfolio, guess that

$$p_t k_{t+1} = \phi_t \beta a_t$$

$$\frac{\tilde{d}_{t+1}}{R_t} + \frac{f_{t+1}(1+\varphi_t)}{R_t^*} = (1-\phi_t)\beta a_t$$

Together, they imply that $c_t = (1-\beta)a_t$. Replacing these guesses in the FOCs above, under the two possible Cases, namley Case 1 and Case 2, verifies the guesses. The expression for ϕ_t can be recovered from the respective first order condition. To see this, consider Case 1. Since $f_{t+1} = 0$, only eqs. (2) and (4) hold with equality. Multiply eq. (2) by $k_{t+1}p_{t+1}$ and eq. (4) by $\tilde{d}_{t+1}\frac{1}{R_t}$ and add the two expressions to verify that the guess is satisfied. Then use the guess in eq. (4) to solve for ϕ_t, with $\max\left\{R_t\,,\, R_t^*/(1+\varphi(F_{t+1}))\right\} = R_t$. We can use a similar procedure for the proof under Case 2. □

In the paper we claimed that a steady state equilibrium without Stablecoins has the following properties:

- The US interest rate is lower than $1/\beta - 1$ and lower than in a closed economy.
- The RoW interest rate is lower than the US, $R^{RoW} = R^{US}/(1+\varphi(F^{RoW}))$.
- RoW holds US bonds, $F^{RoW} > 0$, but the US does not hold RoW bonds, $F^{US} = 0$.

We derive these properties assuming that the demand for safe assets (government bonds) increases with the volatility of the shock z. This is a standard result in models with uninsurable idiosyncratic risks. Let's first derive the conditions that determine a steady state equilibrium. We denote expectations with $\mathbb{E}$ and $\mathbb{E}^*$ to emphasize that the z shocks follow different stochastic processes.

Proposition 1.1. *Given steady state domestic and foreign debt levels $\{B, B^*\}$, prices and aggregate allocations are determined by the following conditions:*

$$\begin{aligned}
\phi &= \mathbb{E}\left[\frac{z+p}{z+p+\tilde{D}+F}\right], && (5)\\
p &= \frac{\beta\phi(Z+\tilde{D}+F)}{(1-\beta\phi)}, && (6)\\
R &= \frac{(1-\beta\phi)(\tilde{D}+F)}{\beta(1-\phi)(Z+\tilde{D}+F)}, && (7)\\
C &= Z+\tilde{D}+F-\frac{\tilde{D}+F}{R}, && (8)\\
\phi^* &= \mathbb{E}^*\left[\frac{z^*+p^*}{z^*+p^*+\tilde{D}^*+F^*}\right], && (9)\\
p^* &= \frac{\beta\phi^*(Z^*+\tilde{D}^*+F^*)}{(1-\beta\phi^*)}, && (10)\\
R^* &= \frac{(1-\beta\phi^*)(\tilde{D}^*+F^*)}{\beta(1-\phi^*)(Z^*+\tilde{D}^*+F^*)}, && (11)\\
C^* &= Z^*+\tilde{D}^*+F^*-\frac{\tilde{D}^*+F^*}{R^*}, && (12)\\
&\tilde{D}+F^*=0, && (13)\\
&\tilde{D}^*+F=0, && (14)\\
&F\cdot\left[R-R^*/(1+\varphi(F))\right]=0 && (15)\\
&F^*\cdot\left[R/(1+\varphi(F^*))-R^*\right]=0 && (16)
\end{aligned}$$

where $Z=\int z\mu_z$, $Z^*=\int z\mu_z^*$, $\tilde{D}=\int\tilde{d}$, $\tilde{D}^*=\int\tilde{d}^*$, $F=\int f$, $F^*=\int f^*$, $C=\int c$, $C^*=\int c^*$.

Proof. Aggregating individual portfolios we obtain $p\int k=\phi\beta\int a$. We can then use the fact that in equilibrium $k=1$ to derive eq. (6), where Z denotes the long-run average of idiosyncratic shock z. Adding up the portfolio decisions we obtain

$$\int\frac{\tilde{d}}{R}+\int\frac{f(1+\varphi)}{R^*}=\phi\beta\int a,$$

which we can rewrite more compactly as

$$\frac{\tilde{D}}{R} + \frac{F(1+\varphi)}{R^*} = \phi\beta[Z + \tilde{D} + F + p].$$

Finally, using the price

$$\frac{\tilde{D}}{R} + \frac{F(1+\varphi)}{R^*} = \frac{(1-\phi)\beta}{1-\beta\phi}[Z + \tilde{D} + F].$$

If $R \geq \frac{R^*}{1+\varphi(F)}$, this delivers eq. (7) (because either the condition holds with equality, in which case both assets are in demand, or $F = 0$ when it holds with inequality). Replacing eq. (7) into the steady state expression for ϕ derived in Lemma I.1 delivers eq. (5). Aggregate consumption eq. (12) is obtained by aggregating the consumer budget constraint and imposing a steady state. Expressions (9) to (12) are solved for in a similar fashion for the other country. Equations (13) and (14) are the market clearing conditions for domestic and foreign bonds in steady state. Finally, eqs. (15) and (16) correspond to the aggregate steady state complementary slackness conditions corresponding to the domestic and foreign countries. Note that for the foreign country, the slackness condition is reversed. □

In a closed economy $F^{US} = F^{RoW} = 0$ (agents cannot hold foreign bonds) and $\tilde{D}^{US} = \tilde{D}^{RoW} = 0$ (aggregate holdings of bonds must be equal to the domestic supplies). This implies that $\phi = 1$, that is, aggregate savings must be fully allocated to land. The interest that clears the market then satisfies

$$1 = R\mathbb{E}\left(\frac{p}{z+p}\right), \tag{17}$$

with $p = \frac{\beta Z}{1-\beta}$. Since $[p/(z+p)]$ is a convex function of z, under some regularity conditions the expectation $\mathbb{E}[p/(z+p)]$ increases in the volatility of z[1]. Condition (17) then implies that the interest rate R decreases with the volatility of the shock. Since the US faces lower idiosyncratic volatility than the RoW, in a closed economy we must have that $R^{US} > R^{RoW}$.

To show that the two interest rates are smaller than $1/\beta$, consider condition (4). In a steady state $c_t = \mathbb{E}c_{t+1}$. However, $1/\mathbb{E}c_{t+1} < \mathbb{E}(1/c_{t+1})$. Then, eq. (4) implies that $\beta R < 1$.

Consider now the open economy version of the model. Before opening up we have that $R^{US} > R^{RoW}$. Therefore, as soon as the RoW is allowed to acquire US assets, it will reduce the holdings of domestic bonds D^{RoW} and increase the holding of US bonds F^{RoW}. But this will decrease the US interest rate and increase the interest rate in RoW. This will continue until $R^{US}/(1+\varphi(F^{RoW}) = R^{RoW}$. Thus, $F^{RoW} > 0$ and $F^{US} = 0$.

2 Model with Stablecoins

The US-consumer budget constraint is

[1]It is easy to see that this would hold if z was iid and uniformly distributed.

$$c_t + p_t k_{t+1} + \frac{\tilde{d}_{t+1} + s_{t+1}}{R_t^{US}} + \frac{(1+\varphi(F_{t+1}))f_{t+1}}{R_t^{RoW}} = \underbrace{(z_t + p_t)k_t + \tilde{d}_t + s_t + f_t}_{=a_t}.$$

US superscripts are omitted for readability from all variables except interest rates. Here we are using that $R^S = R^{US}$. Note that if US agents hold both $\tilde{d}_{t+1}$ and s_{t+1}, they must be indifferent between holding the two assets. We can then define $\hat{d}_t = \tilde{d}_t + s_t$ and rewrite the budget constraint as

$$c_t + p_t k_{t+1} + \frac{\hat{d}_{t+1}}{R_t^{US}} + \frac{(1+\varphi(F_{t+1}))f_{t+1}}{R_t^{RoW}} = \underbrace{(z_t + p_t)k_t + \hat{d}_t + f_t}_{=a_t}. \tag{18}$$

The RoW-consumer budget constraint (now omitting RoW superscripts from all variables except interest rates) is

$$c_t + p_t k_{t+1} + \frac{\tilde{d}_{t+1}}{R_t^{RoW}} + \frac{(1+\varphi(F_{t+1}))f_{t+1} + s_{t+1}}{R_t^{US}} = (z_t + p_t)k_t + \tilde{d}_t + s_t + f_t. \tag{19}$$

RoW agents are not indifferent between holding US bonds and Stablecoins because of the cost φ. We focus on steady state equilibria where aggregate variables are constant. In an economy with Stablecoins, the steady state equilibrium is determined as stated in Proposition 2.1.

Proposition 2.1. *Given steady state domestic and foreign debt levels $\{B, B^*\}$, prices and aggregate allocations are determined by the following conditions:*

$$\begin{aligned}
\phi &= \mathbb{E}\left[\frac{z+p}{z+p+\hat{D}}\right], && (20)\\
p &= \frac{\beta\phi(Z+\hat{D})}{(1-\beta\phi)}, && (21)\\
R &= \frac{(1-\beta\phi)(\hat{D})}{\beta(1-\phi)(Z+\hat{D})}, && (22)\\
C &= Z + \hat{D} - \frac{\hat{D}}{R}, && (23)\\
\phi^* &= \mathbb{E}^*\left[\frac{z^*+p^*}{z^*+p^*+\tilde{D}^*+S^*}\right], && (24)\\
p^* &= \frac{\beta\phi^*(Z^*+\tilde{D}^*+S^*)}{(1-\beta\phi^*)}, && (25)\\
R^* &= \frac{(1-\beta\phi^*)(\tilde{D}^*+S^*)}{\beta(1-\phi^*)(Z^*+\tilde{D}^*+S^*)}, && (26)\\
C^* &= Z^*+\tilde{D}^*+S^* - \frac{\tilde{D}^*+S^*}{R^*}, && (27)\\
&\tilde{D} + \kappa(S+S^*) = 0, && (28)\\
&\tilde{D}^* = F^* = F = 0, && (29)\\
&R = R^* && (30)
\end{aligned}$$

where $Z = \int z\mu_z$, $Z^* = \int z\mu_z^*$, $\hat{D} = \int \hat{d}$, $\tilde{D}^* = \int \tilde{d}^*$, $F^* = \int f^*$, $C = \int c$, $C^* = \int c^*$.

Proof. We follow the same steps as the proofs for the economy without Stablecoins to derive the equilibrium conditions (20)-(23). For the foreign country, there is now an additional first order condition, wrt s^*_{t+1}, since agents are not indifferent between holding US bonds and Stablecoins,

$$k^*_{t+1}: \qquad \frac{p^*_t}{c^*_t} = \beta \mathbb{E}^*_t \left(\frac{z^*_{t+1} + p^*_{t+1}}{c^*_{t+1}} \right). \tag{31}$$

$$f^*_{t+1}: \qquad \frac{1}{c^*_t} \geq \beta \frac{R_t}{1 + \varphi(F^*_{t+1})} \mathbb{E}^*_t \left(\frac{1}{c^*_{t+1}} \right). \tag{32}$$

$$\tilde{d}^*_{t+1}: \qquad \frac{1}{c^*_t} = \beta R^*_t \mathbb{E}^*_t \left(\frac{1}{c^*_{t+1}} \right). \tag{33}$$

$$s^*_{t+1}: \qquad \frac{1}{c^*_t} \geq \beta R_t \mathbb{E}^*_t \left(\frac{1}{c^*_{t+1}} \right). \tag{34}$$

The inequality in eq. (34) appears because of the constraint that $s_{t+1} \geq 0$ (e.g. agents cannot generate Stablecoins themselves, they need to go through the DAO). This equation modifies the foreign country portfolio decision relative to that characterized in Lemma 1.1.

We start by showing that $F^* = 0$ in equilibrium. Suppose not, then $R_t > \frac{R_t}{1+\varphi(F^*_{t+1})}$ for any $F^* > 0$. From eq. (34), this implies eq. (32) holds with inequality, so $f^*_{t+1} = 0 => F^* = 0$.

Given this, there are three cases, which depend on equilibrium interest rates R_t and R^*_t

- **Case 1,** $R^*_t > R_t$**:** Eq. (33) implies that eq. (34) holds with inequality, implying $s^*_{t+1} = 0$. Can this be an equilibrium? No, because if $R^*_t > R_t$, eq. (4) implies that $\frac{1}{c_t} < \beta R^*_t \mathbb{E}^*_t \left(\frac{1}{c^*_{t+1}} \right)$, which would violate eq. (3).

- **Case 2:** $R^*_t = R_t$. The agent is indifferent between s^*_{t+1} and $\tilde{d}^*_{t+1}$.

- **Case 3:** $R^*_t < R_t$. This case is not possible in equilibrium because it will violate eq. (34). The agent would take an infinitely large negative position in $\tilde{d}^*_{t+1}$ and an infinitely large position in s^*_{t+1}. Since all agents would do that, in equilibrium there would be an infinite demand of S^*_{t+1}, and hence for B^*, that would reduced the equilibrium interest rate R^*_t until $R^*_t = R_t$.

We established that $R^*_t = R_t$ (eq. 30). This implies that if $F_{t+1} > 0$, then $R^*_t > \frac{R_t}{1+\varphi(F_{t+1})}$, but under this condition eq. (3) implies $f_{t+1} = 0$. In other words, the US does not hold foreign bonds in equilibrium $F_{t+1} = 0$. As a result, $\tilde{D}^* = 0$. Using a similar argument, we can also show that $F^*_{t+1} = 0$. Using these results, a guess similar to the one in the Lemma, and some algebra allow us to derive eqs. (24) to (27). Finally, eq. (28) is just the market clearing condition for US bonds, $\tilde{D} + \kappa(S^* + S) = 0$, where we took into account that the DAO only requires a proportion κ of US debt to satisfy the world aggregate demand for Stablecoins, $S^* + S$.

□

Proposition 2.2. *When $\kappa = 1$, the US interest rate R^{US} is lower in the steady state with Stablecoins. For a sufficiently small κ, the US interest rate R^{US} is higher with Stablecoins.*

Proof. Without Stablecoins we have established that $R > R^*$, $F = 0$, $F^* > 0$, $\tilde{D} < 0$ and $\tilde{D}^* = 0$. With Stablecoins we have $R = R^*$, $F = F^* = 0$, and $\tilde{D} = -\kappa(S + S^*)$ and $\tilde{D}^* = 0$.

Let's start with the steady state without Stablecoins. Starting from this equilibrium, suppose that Stablecoins can be created. For the moment, however, suppose that the US interest rate R does not change. For the RoW, Stablecoins give a return of R which is greater than the return on US bonds, $R/(1+\varphi(F^*))$ and RoW bonds, R^*. This implies that the RoW interest rate R^* must rise to R. Otherwise, nobody will buy RoW bonds. With this (still partial equilibrium) adjustment, RoW holds Stablecoins, $S^* > 0$, all domestic bonds, $\tilde{D}^* = 0$, but no US bonds, $F^* = 0$. Essentially, the RoW replaces US bonds with Stablecoins. But since the return from Stablecoins is greater than the net return from US bonds, the purchase of Stablecoins S^* is bigger than the US bonds F^* held before the introduction of Stablecoins. Under the assumption that the US interest rate R does not change (for the moment), will the demand for US bonds increase or decrease with the introduction of Stablecoins?

There are two effects. On the one hand, since RoW agents no longer hold US bonds (F^* goes to zero), the demand declines. On the other hand, however, Stablecoins require reserves in US bonds in the quantity κS^*. This increases the demand for US bonds. Therefore, whether the overall demand for US bonds increases or decreases depends on the relative importance of these two effects.

When $\kappa = 1$, the reserve demand increases by S^*, which is greater than F^* (since Stablecoins provide a return R that is greater than $R/(1 + \varphi(F^*))$). This implies that the introduction of Stablecoins generates a net increase in the demand for US bonds. To clear the market, then, the US interest rate has to drop. However, if κ is small, the increase in the demand for reserves, κS^*, will also be small. In the limit with $\kappa = 0$ there will not be any increase in demand for reserves. Thus, for a sufficiently small κ, the demand for reserves κS^* will be smaller than the RoW holding of US bonds before Stablecoins, F^*. In this case there will be an excess supply of US bonds that must be absorbed by US agents. This requires an increase in the US interest rate R.

In general, the higher is κ, the larger is the increase in the demand for bond reserves after the introduction of Stablecoins. This should lead to a negative relationship between the US interest rate R^{US} and κ.

□