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ABSTRACT

We study how cross-country macroeconomic spillovers caused by sovereign default affect equilibrium bailouts. Because of portfolio diversification, the default of one country causes a macroeconomic contraction also in other countries. This generates a self-interest for these other countries to bailout the defaulting country. A novel insight of the paper is that bailouts could be efficient not only ex-post (after the debt has been issued) but also ex-ante (before the issuance of the debt). Although anticipated bailouts create the typical moral hazard problem leading countries to issue more debt, this may correct for the under issuance of public debt that would result from the lack of cross-country policy coordination.

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1 Introduction

During the last 30 years we have witnessed a dramatic increase in the international integration of financial markets. This allowed governments to 'export' their public debt, that is, to borrow from foreign countries. Figure 1 plots the share of public debt for the US and the largest EU countries from 1997 to 2010 and shows that the share has increased substantially during this period.

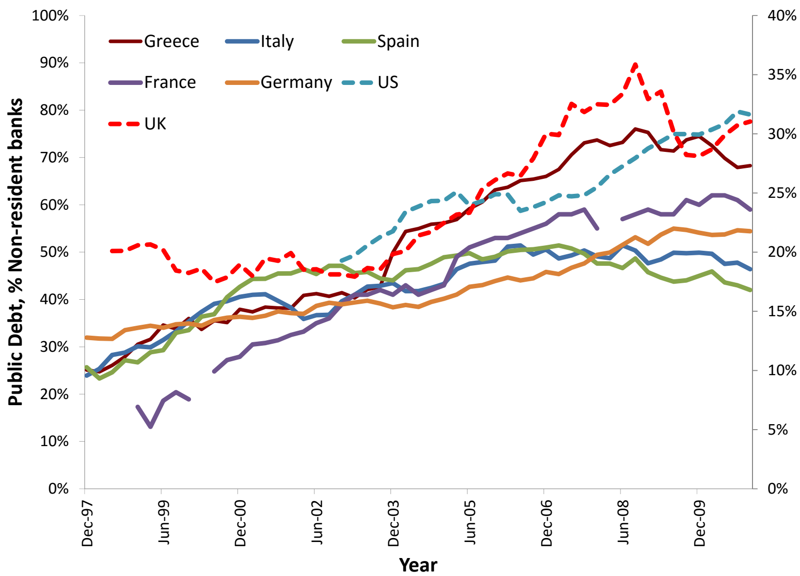

Figure 1: Share of public debt held abroad. Left axis for solid line countries. Right axis for dotted line countries. Source: Merler and Pisani-Ferry (2012).

During the same period, countries have also 'imported' foreign public debt as domestic residents increased their holdings of securities issued in other countries. Figure 2 plots the ownership of foreign debt instruments, including government debt, for several countries since 1980. As can be seen from the figure, the ownership of foreign debt has increased significantly, especially since the mid 1990s.

The two figures illustrate an important trend in global financial markets: the cross-country diversification of financial portfolios. This is a general trend that is not limited to debt instruments but it extends to portfolio investments and FDI. In this paper, however, we focus on debt instruments and, especially, sovereign debt, because of their role in providing liquidity.

Although international diversification may improve investment efficiency and allows for greater risk-sharing, it also increases macroeconomic interdependence across countries. This implies that the macroeconomic impact of a crisis in one country, including crises associated with sovereign default, spills to other countries. In this paper we show that the macroeconomic spillover of sovereign default is central for understanding why non-defaulting countries are willing to bailout defaulting countries.

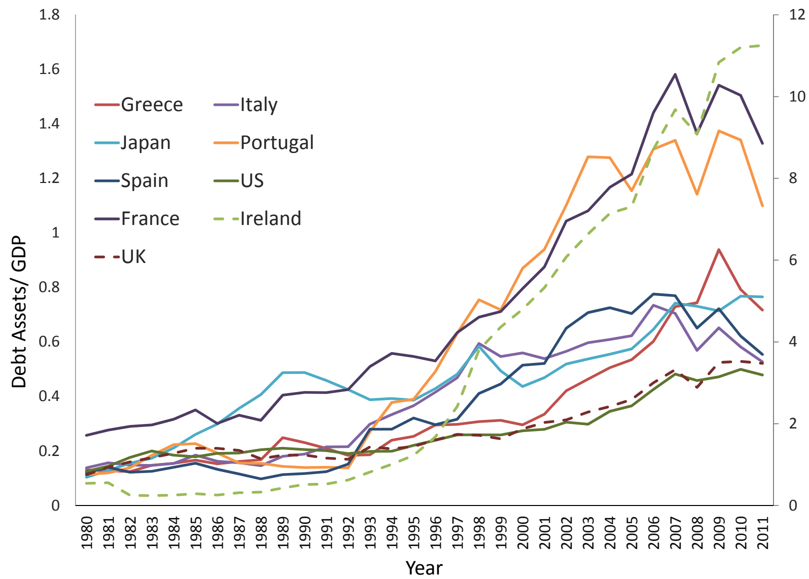

Figure 2: External debt assets as a percentage of GDP (Greece, Portugal, Japan, UK, US, France, Ireland, Italy, Spain). Left axis for solid line countries. Right axis for dotted line countries. Source: Lane and Milesi-Ferreti (2007).

We further show that, although the anticipation of bailouts leads countries to issue more debt (the typical moral hazard problem), still this could be welfare improving. In other words, bailouts could be efficient not only ex-post—that is, after the debt has been issued—but also ex-ante.

We show this result in a two-country model in which public debt can be held to insure production risks and higher supply of public debt improves production efficiency. However, in a world in which financial markets are internationally integrated, part of the debt issued by one country is acquired and held by foreign residents. This implies that the benefits of issuing debt are shared with other countries. Since the benefits are shared, there is less incentive for each individual country to issue debt. In equilibrium, then, the supply of public debt is inefficiently low. This is a consequence of the lack of policy coordination across countries. Anticipated bailouts counterbalance this inefficiency: since the 'expected' repayment cost is lower when a country anticipates a future bailout, each country issues more debt, partially correcting for the inefficiency caused by the lack of cross-country policy coordination.

The macroeconomic spillovers of sovereign default is crucial for this result: in absence of spillovers, there is no incentive to bailout defaulting countries and, of course, there is no anticipation of bailouts.

The focus on macroeconomic spillovers differentiates our paper from the large body of literature on sovereign default. Most of the contributions in this literature are based on the 'small open economy' paradigm. In a small open economy, sovereign default does not generate any 'macroeconomic' cost for the lending countries. The only cost for the lending countries is the capital loss associated with lower repayment. But as the European debt crisis has shown, the possibility that the default of some countries could have sparked a financial and macroeconomic crisis in other countries was a serious concern. This concern was motivated by the international portfolio composition of lending countries, banks in particular: since banks in core countries hold the debt of periphery countries, default could endanger the financial and macroeconomic stability also of the core countries. Although we do not model the banking sector explicitly, our theoretical framework formalizes this idea and studies how the macroeconomic spillovers shape the incentives to bailout defaulting countries.

In addition to highlighting the centrality of cross-country spillovers for bailouts, we also show that portfolio diversification plays a central role in the decision to default. If a larger share of sovereign debt is held by foreigners, the incentive to default for the debtor country increases since it redistributes wealth from foreign residents to domestic residents. In our paper, however, we highlight a different mechanism through which financial diversification affects the incentive to default. We show that international diversification increases the incentive of a country to default because the 'domestic macroeconomic cost' is smaller.

Why is the macroeconomic cost of default smaller when the country is financially diversified? The central mechanism is the disruption of financial markets induced by default. When a government defaults on its debt, the holders of government debt incur capital losses. To the extent that financial wealth held by private agents is important for economic decisions, this has a negative effect on economic activity. When financial markets are integrated (and portfolios diversified), domestic residents hold a smaller share of wealth in domestic assets and a larger share in foreign assets. This implies that when the domestic government defaults, the wealth losses of domestic residents are smaller, causing a smaller macroeconomic contraction. Then, being the macroeconomic cost smaller, the government's incentive to default is higher. Using cross-country data where the propensity to default is captured by interest rate spreads on sovereign debt, we find that the portfolio diversification of a country is correlated with these spreads.

The mechanism described above points out that it is not only the quantity of domestic debt held by foreigners that matters for the choice of a country to default but also the debt issued by foreign countries and held by domestic agents. Of course, the quantity of foreign debt held by domestic agents depends on the external supply of foreign debt. This introduces a channel through which the supply of foreign debt affects the incentive of a country to default: as foreign countries supply more debt, agents in the domestic country will buy more of the foreign debt and become more diversified. Higher diversification then implies that the macroeconomic cost of default in the domestic country is lower, which in turn increases the government incentive to default even if the quantity of domestic debt held by foreigners remains unchanged. This shows that the default of one country could be triggered by the debt issued by other countries (higher international liquidity).

The role played by external factors for the choice of a country to default is another dimension in which our paper differs from most of the literature on sovereign default. The majority of studies focus on the internal factors that lead a country to default. For example, a sequence of negative productivity or fiscal shocks leads the country to borrow more and, if the economic conditions continue to deteriorate, it becomes optimal or necessary for the country to default. In our study we show that some of the factors that cause a country to default may not originate domestically. In particular, the debt issued by *other* countries (higher international liquidity) may also be an important factor.

2 Literature review

This paper builds on a large literature on public debt with incomplete markets. The main role of government debt in our paper is to partially complete the assets market when agents are subject to uninsurable idiosyncratic risk. The mechanism is similar to that studied in Aiyagari and McGrattan (1998), Azzimonti, de Francisco, and Quadrini (2014), Azzimonti and Yared (2018), Bhandari, Evans, Golosov and Sargent (2017), and Floden (2001), although there is no sovereign default in these studies.[1]

Because of the possibility of default, our paper is also related to a growing literature on external sovereign default that builds on Eaton and Gersovitz (1981). Examples include Aguiar and Gopinath (2006), Aguiar and Amador (2016), Arellano (2008), Cuadra, Sanchez, and Sapriza (2010), Pouzo and Presno (2014) and Yue (2010)). Aguiar and Amador (2014) and Tomz and Wright (2013) provide earlier reviews of this literature, whereas the handbook chapters by Aguiar et al (2016) and D'Erasmo, Mendoza, and Zhang (2016) provide a more recent discussion of the literature on sovereign default and sustainable public debt.

Our paper also relates to the political economy literature that emphasizes the redistributive effects of sovereign default. Alesina and Tabellini (1990), Aghion and Bolton (1990), Drazen (1998), D'Erasmo and Mendoza (2016, 2017), Dovis, Golosov, and Shourideh (2016) emphasize the importance of domestic heterogeneity and focus on the redistributive consequences of default. Amador (2003), Aguiar, Amador, Farhi, and Gopinath (2013), Hatchondo, Martinez, and Sapriza (2009), and Mendoza and Yue (2012), instead, focus on the international redistribution of sovereign default.

Our paper is also related to the literature that endogenizes the cost of default by assuming that public debt provides liquidity (see Guembel and Sussman (2009),

[1]Our paper differs from Azzimonti, de Francisco, and Quadrini (2014) in two additional dimensions. First, the stock of public debt affects the labor market and hence the aggregate level of production. In Azzimonti, de Francisco, and Quadrini (2014), instead, aggregate production is fixed, which implies that public debt does not have any macroeconomic effects neither domestically or abroad. As already discussed, the macroeconomic consequences of sovereign default and the resulting international spillovers are central to the analysis of the paper presented here. Second, we also allow for aggregate uncertainty, whereas Azzimonti, de Francisco, and Quadrini (2014) considers only idiosyncratic uncertainty. As we will see, aggregate shocks play an important role in the decision to default.

Broner, Martin, and Ventura (2010), Broner and Ventura (2011), Gennaioli, Martin, and Rossi (2013), Brutti (2011), and Di Casola and Sichlimiris (2014)). Some recent papers study the interaction between sovereign debt and domestic financial institutions (e.g. Boccola (2016), Farhi and Tirole (2017), Perez (2015), and Sosa-Padilla (2018)). As in our paper, the cost of default is endogenous as it disrupts production and causes a recession. The analysis of these studies, however, is based on small open economies and, therefore, there are not international macroeconomic spillovers. Our study, instead, highlights the importance of macroeconomic spillovers associated with sovereign default using a model with 'large' economies. International macroeconomic spillovers, which are negligible in small open economy models, are central for understanding the optimality of bailouts.

Arellano and Bai (2013) also consider an environment in which sovereign default affects other countries. The mechanism is based on the interest rate channel.[2] Our channel of transmission, instead, relies on the macroeconomic consequences of sovereign default (where recessions are exported to other countries through the destruction of foreign portfolios holdings). Our study is also related to contributions that analyze debt restructuring through bargaining, such as Yue (2010) and Bai and Zhang (2009).[3]

There is also a connection to the literature that studies the effects of central government bailouts on sub-national units (see Chari and Kehoe (2007), Cooper et al. (2008), and Dovis and Kirpalani (2017)). A novel insight from our paper is that bailouts could improve welfare not only *ex-post* but also *ex-ante*, since they counterbalance the inefficiencies induced by the lack of cross-country coordination of policies. Our work is also related to the literature studying efficiency of bailouts. Differently from Farhi and Tirole (2012) who find that bailouts generate excessive financial fragility and are hence welfare reducing, we find that bailouts can improve welfare from an ex-ante perspective. Bianchi (2016) also finds that bailouts may be efficient ex-ante, but the bailout studied in this paper is from the domestic government to the domestic private sector. Instead, we study cross-country bailouts. Dovis (2018), Fink and Scholl (2016), and Roch and Uhlig (2018) analyze the optimality of sovereign bailouts provided by international financial institutions (modeled as risk neutral investors), while we discuss renegotiation among two large economies, both of which may have incentives to default.

3 The model

We start presenting our theory with a very stylized model that allows us to provide the key results analytically. The model will then be extended in Section 6.

The economy has two large countries: the 'home' country denoted with superscript H and the 'foreign' country denoted with superscript F. The main difference between

[2]See also Borri and Verdelhan (2009), Park (2013), Lizarazo (2013), and Pouzo and Presno (2016).

[3]See Niepelt (2016) and Mihalache (2016) for an alternative renegotiation protocols.

the two countries is in the degree of commitment to repay debt obligations. We assume that the foreign country always repays in the second period (safe) whereas the home country may choose to default on part or all of its debt (risky). This assumption will be relaxed in Section 7 where both countries could default.

In each country there are two types of agents: a measure 1 of workers and a measure 1 of entrepreneurs. Workers in country $i \in \{H, F\}$ value consumption and leisure with the quasilinear utility

$$\ln(c_1^i) + \beta\varphi(c_2^i, \ell_2^i), \qquad \varphi(c, \ell) = c - \ell^\nu$$

where c_1^i and c_2^i denote consumption in period 1 and 2, respectively, ℓ_2^i is the supply of labor in period 2, and $\nu > 1$. Production takes place only in the second period and, therefore, wages are paid only in period 2.

Workers are excluded from financial markets (hand-to-mouth). Their consumptions in the two periods are

$$\begin{aligned} c_1^i &= e + T_1^i, \\ c_2^i &= e + w_2^i \ell_2^i + T_2^i, \end{aligned}$$

where e is an endowment received in both periods, T_1^i and T_2^i are the government transfers received in period 1 and 2 respectively, and w_2^i is the wage rate earned in period 2.

Notice that endowments are the same in the two countries, which explains why they are not indexed by the country superscript i. The stark assumption that workers cannot borrow simplifies the exposition but it is not essential. Our results would hold in an environment in which workers have access to credit but they are subject to a borrowing limit. Workers maximize their lifetime utility subject to the budget constraints. The solution provides the supply of labor ℓ_2^i.

The lifetime utility of entrepreneurs takes the form

$$d_1^i + \beta d_2^i.$$

where d_1^i and d_2^i denote their consumption in periods 1 and 2, respectively. Entrepreneurs receive an endowment a in period 1 (common in both countries) and produce in the second period with labor using the linear technology

$$y_2^i = z_2^i l_2^i,$$

where l_2^i is the input of labor, $z_2^i \in \{z_L, z_H\}$ is an aggregate productivity shock which is country specific, with $z_L < z_H$. An important assumption is that the draw of aggregate productivity z_2^i takes place at the beginning of period 2 before entrepreneurs choose the input of labor.

A special feature of this model is that the cost of labor depends not only on the wage bill but also on the financial wealth of entrepreneurs. To use a compact notation we denote the labor cost with the function $\varpi(w, l, m)$, where w is the wage rate, l is the input of labor, and m is the financial wealth of the entrepreneur. The function $\varpi(.)$ is strictly increasing in w and l—capturing the direct cost of labor—and strictly decreasing in the entrepreneur's wealth m. The extension in Section 6 provides a micro-foundation for this function which derives from the risk of hiring more labor. As we will see, the idea is as follows: Since entrepreneurs face higher risk when their wealth is low, in order to produce they require a higher premium over the marginal product of labor. For the moment, however, we capture this idea in reduced form by assuming that m has a negative impact on the cost of labor.

Given the value of their financial assets m_2^i, entrepreneurs choose l_2^i in order to maximize second period profits,

$$\pi_2^i = z_2^i l_2^i - \varpi(w_2^i, l_2^i, m_2^i).$$

There is no market for contingent claims and the only assets that entrepreneurs can trade are one-period government bonds. Under a financially integrated economy, entrepreneurs can buy bonds from home and foreign countries. We denote by b_1^{ji} the bonds issued in period 1 by country j and purchased by an entrepreneur residing in country $i \in \{H, F\}$ (i.e. the first superscript indicates the nationality of the government that issued the debt and the second superscript indicates the nationality of the holder of the debt). The equilibrium price for the bond is $1/R_1^j$.

The budget constraints for entrepreneurs in the first and second periods are,

$$\begin{aligned} d_1^i &= a - \frac{b_1^{Fi}}{R_1^F} - \frac{b_1^{Hi}}{R_1^H}, \\ d_2^i &= \pi_2^i + m_2^i, \end{aligned}$$

where a is the endowment received only in period 1. We can think of a as the wealth of entrepreneurs accumulated up to period 1. Wealth held in period 2 is

$$m_2^i \equiv \delta_2 b_1^{Hi} + b_1^{Fi}. \tag{1}$$

Here, $\delta_2 \leq 1$ denotes the proportion of the debt issued by the home country that is repaid in period 2. We allow for partial default which is a common feature of the data.[4] The post-default value of home issued bonds is then $\delta_2 b_1^{Hi}$. Because the foreign country always repays its debt obligations, the value of foreign issued bonds is b_1^{Fi}.

The cross-country asymmetry in debt repayment implies that in equilibrium $R_1^H \neq R_1^F$. Denoting with capital letters aggregate variables, the total debt issued by country $j \in \{H, F\}$ is $B_1^j = B_1^{jH} + B_1^{jF}$.

The revenues raised by governments in period 1 are distributed to workers with lump-sum transfers, that is,

$$T_1^j = \frac{B_1^j}{R_1^j}. \tag{2}$$

[4]See also Arellano, Mateos-Planas and Rios-Rull (2013) for a model with partial default.

Effectively, governments borrow on behalf of workers. Notice that, since B_1^j is not restricted to be positive, governments could choose to lend instead of borrowing. In period 2 the debt will be repaid (partially or in full) and to do so governments raise taxes from workers (or make transfers if $B_1^j < 0$). Thus,

$$T_2^H = -\delta_2 B_1^H, \qquad T_2^F = -B_1^F. \tag{3}$$

Although country F always repays the debt in full, country H could repay only a fraction δ_2. The actual repayment is $\delta_2 B_1^H$.

4 Equilibrium in Period 2

To characterize the equilibrium, we proceed backward. We first characterize the equilibrium in period 2 and then we move back to period 1. The equilibrium in period 2 takes as given the debt issued by the two countries in period 1, B_1^j, and the portfolio composition of entrepreneurs, B_1^{ji}, with $i, j \in \{H, F\}$. Since in this section we focus on period 2, we abstract from time subscripts. Following is the detailed description of the sequence of events within period 2:

1. Aggregate productivity shocks z^i are realized.
2. The H (home) country chooses repayment δ.
3. Entrepreneurs choose the input of labor l^i, workers choose the supply of labor ℓ^i, and the wage w^i clears the labor market in each country $i \in \{H, F\}$.
4. Production and consumption take place.

This timing assumption implies that the default decision of the home country depends on the conditions of the economy, captured by aggregate productivity z^H.

4.1 Equilibrium given policy

We start characterizing the competitive equilibrium given the stocks of public debt $\{B^H, B^F\}$ and the repayment policy chosen by the home government, δ. The problem solved by workers in country $i \in \{H, F\}$ is

$$\begin{aligned} W^i \quad &= \quad \max_{c,\ell} c - \ell^\nu \\ &\quad\quad \textbf{s.t.} \\ &\quad\quad c = e + w^i \ell + T^i, \end{aligned} \tag{4}$$

with $T^H = -\delta B^H$ and $T^F = -B^F$.

From the first order condition we obtain the supply of labor as a function of the wage rate,

$$\ell^i = \left(\frac{w^i}{\nu}\right)^{\frac{1}{\nu-1}}. \tag{5}$$

The assumption of quasi-linearity implies that the supply of labor is independent of transfers (no income effect). The assumption $\nu > 1$ ensures that the supply is increasing in the wage rate w^i.

Entrepreneurs maximize second period consumption,

$$U^i = \max_l \left\{ z^i l - \varpi(w^i, l, m^i) + m^i \right\}, \tag{6}$$

where m^i is defined in eq.(1).

To gain analytical tractability, we make the following assumption:

Assumption 1 *The cost of labor takes the form* $\varpi(w, l, m) = wl\left(\frac{l}{m}\right)$.

With this specification of the cost function, the demand for labor, derived from Problem (6), takes the form

$$l^i = \frac{z^i}{2w^i} m^i, \tag{7}$$

which is linear in the post-default financial wealth of entrepreneurs m^i. As we will see, this property also applies to the micro-foundation of the cost function derived from the general model presented in Section 6.

The labor market equilibrium is derived by equating the aggregate demand of labor to the supply as illustrated in Figure 3 and characterized in Lemma 2.

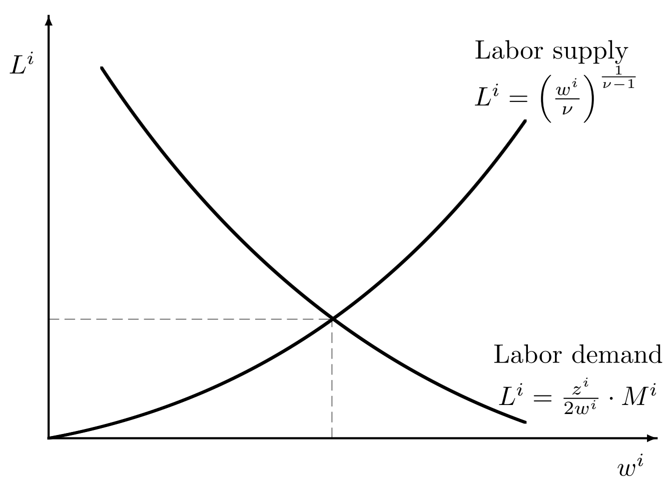

Figure 3: Labor Market Equilibrium.

Lemma 2 *Equilibrium labor and wage rate in country i are equal to*

$$L^i = \left(\frac{z^i M^i}{2\nu}\right)^{\frac{1}{\nu}},$$

$$w^i = \left(\frac{z^i M^i}{2}\right)^{1-\frac{1}{\nu}} \nu^{\frac{1}{\nu}}.$$

Proof. By equating eqs. (5) to (7) and solving for L^i and w^i, we obtain the above expressions. ■

The variable M^i denotes the 'aggregate' financial wealth of entrepreneurs in country i, after the repayment of government debt. It is the sum of the repaid debts issued by home and foreign countries, that is,

$$M^i = \delta B^{Hi} + B^{Fi}. \tag{8}$$

This expression makes clear that a default in the home country, that is, $\delta < 1$, reduces entrepreneurs' wealth which in turn reduces the aggregate demand for labor. In Figure 3, this is captured by a shift to the left of the demand curve. As a result, both wages and labor decline in equilibrium. The decline in labor then implies a decline in output (macroeconomic contraction).

What is important—and this differentiates our paper from most of the literature on sovereign default—is that the endogenous macroeconomic contraction arises not only in the defaulting country (H country), but also in the non-defaulting country (F country). This is because the default of the home country ($\delta < 1$) also reduces M^F, that is, the wealth of foreign entrepreneurs (see equation (8)). As we will see, this feature of the model is crucial for understanding the bailout incentives of creditor countries.

Another important feature of the model is that the size of the induced contraction is larger when productivity is high. We state this formally in the following corollary.

Corollary 3 *The drop in output $Y^i(z^i)$ induced by the default of the home country is larger in booms,*

$$\frac{\partial Y^i(z_H)}{\partial \delta} > \frac{\partial Y^i(z_L)}{\partial \delta}. \tag{9}$$

Proof. Replacing equilibrium conditions into the production function, we obtain $Y^i(z^i) = (z^i)^{1+1/\nu}\left[\frac{\delta B^{Hi}+B^{Fi}}{2\nu}\right]^{1/\nu}$. The result follows from taking the derivative w.r.t. δ and noting that $z_H > z_L$. ■

This property has two implications: (i) the incentive for the home country to default increases when the economy is in recession; (ii) the incentive of the foreign country to bailout the home country increases when the foreign country experiences an economic boom. Thus, the business cycle is important not only for affecting the decision to default (which is a feature of many sovereign default models) but also the decision to bailout a defaulting country (which is a novel feature of our model).

4.2 Optimal default decision without renegotiation

The optimal policy of the home government in period 2 consists of the choice of debt repayment to maximize the weighted sum of the utility of workers and entrepreneurs,

$$V^H = \max_{\delta \leq 1} \quad \Big\{ \Psi \, U^H + (1-\Psi) \, W^H \Big\}, \tag{10}$$

where Ψ is the relative weight assigned to entrepreneurs, and U^H and W^H denote the indirect utilities of entrepreneurs and workers as defined in (4) and (6).[5]

The state variables for the problem solved by the home government are the stock of debt B^H, the portfolio composition of home entrepreneurs, B^{HH} and B^{FH}, and the realization of home productivity z^H.

To characterize the solution, it is convenient to consider first the relaxed problem where the repayment of debt is not subject to the constraint $\delta \leq 1$. The first order condition for the relaxed problem can be written as,

$$\underbrace{\left[-\Psi \, \pi^H - (1-\Psi) \left(w^H + \frac{\partial \varphi}{\partial L^H} \right) \right] \frac{\partial L^H}{\partial M^H} B^{HH}}_{\textit{1. Macroeconomic effect}} \quad +$$

$$\underbrace{\Big[\Psi L^H - (1-\Psi) L^H \Big] \frac{\partial w^H}{\partial M^H} B^{HH}}_{\textit{2. Price effect}} \quad + \quad \underbrace{\Big[-\Psi B^{HH} + (1-\Psi) B^H \Big]}_{\textit{3. Redistribution effect}} \quad = \quad 0$$

It has three main terms, each capturing the welfare effect of default through a particular channel. We can also see the separate impact of each channel on entrepreneurs (terms multiplied by Ψ) and workers (terms multiplied by $1-\Psi$).

1. *Macroeconomic effect*: This is the macroeconomic contraction induced by default, captured by the reduction in employment, $\partial L^H / \partial M^H$. The central mechanism through which default generates a macroeconomic contraction is by destroying the financial wealth of entrepreneurs $M^H = \delta B^{HH} + B^{FH}$. This term is multiplied by B^{HH} because larger holdings of home bonds by home entrepreneurs result in a bigger destruction of their wealth, leading to a larger contraction in the demand of labor. The macroeconomic effect of default is negative for both entrepreneurs and workers. For entrepreneurs because it reduces the scale of production and, therefore, total profits. For workers it generates lower employment (although this is partially compensated by a lower dis-utility from working).

[5] The objective function of the government can be micro-founded with a probabilistic voting model where two opportunistic candidates compete for elections. It is easy to show that such voting model results in each politician choosing the policy that maximizes a planner's problem with weights given by the political power of each group (see Azzimonti (2011) for details).

2. *Price effect*: The destruction in entrepreneurs' wealth, with consequent decline in the demand for labor, generates a drop in the wage rate. This is captured by the term $\partial w^H/\partial M^H$. Again, this is multiplied by B^{HH} because the change in M^H is smaller if the holdings of home bonds by home entrepreneurs is lower. This effect is positive for entrepreneurs (since they have to pay less to workers) but it is negative for workers (since they receive lower compensation).

3. *Redistribution effect*: If the government repays less debt, it redistributes wealth from the holders of debt to tax payers. Therefore, the welfare effect for entrepreneurs is negative (since they hold the debt) but it is positive for workers (since they pay less taxes).[6] While the positive effect on workers is multiplied by B^H, the negative effect on entrepreneurs is multiplied by B^{HH} since only part of the home debt is held by home entrepreneurs.

To the extent that part of the home debt is held by foreign entrepreneurs, the third channel redistributes wealth from the foreign country to the home country. This is also a feature of many other sovereign default models studied in the literature. The focus of our paper, however, is not on this channel. Instead, we focus on the first channel: default generates lower financial losses for home entrepreneurs because their portfolios are internationally diversified. The wage redistribution channel is also part of our focus but the importance of this channel is secondary compared to the negative effect on employment.

Although the direct redistributive effect of default is beneficial for workers, the macroeconomic effect has negative consequences for them. Thus, from the perspective of workers, default implies a trade-off: the benefit is the reduction in taxes; the cost is the reduction in income. From the perspective of entrepreneurs, instead, default implies only a cost: in addition to the direct loss of financial wealth, entrepreneurs also earn lower incomes (total profits decline even if entrepreneurs pay lower wages). But the key insight is that the macroeconomic costs are lower when domestic portfolios are diversified; that is, when a large share of domestic financial wealth is held in foreign assets $\frac{B^{FH}}{M^H}$. This mechanism, which is novel in the sovereign default literature, affects the incentive of the government to default. We will explore the empirical relevance of this mechanism in Section 8.

To further characterize the equilibrium, we will make the following assumption about the portfolios holdings at the beginning of period 2.

Assumption 4 *Portfolio holdings satisfy* $B^{HF} = B^{HH}$ *and* $B^{FH} = B^{FF}$.

This assumption implies that half of the debt issued by the home country, B^H, is held by domestic entrepreneurs and half by foreign entrepreneurs. The same for the

[6]The assumption that only workers pay taxes is not essential. This would be true even if taxes were equally paid by workers and entrepreneurs. What matters is that taxes are not proportional to the holding of public debt so that default implies that agents who hold the debt (entrepreneurs) experience a net loss while agents who do not hold the debt (workers) experience a net gain.

debt issued by the foreign country, B^F. Although this assumption seems arbitrary at this stage, we will see later that, when entrepreneurs choose their portfolio optimally in period 1, this becomes an equilibrium outcome.

The following proposition characterizes the optimal repayment rate chosen by the unconstrained home government.

Proposition 5 *Suppose that (i) $\Psi < \frac{2}{3}$, (ii) $z_H < \frac{2-3\Psi}{1-\Psi}\frac{2\nu}{\nu-1}$, and (iii) $B^H > 0$. There exists a unique solution to the unconstrained maximization problem (10). The optimal unconstrained repayment rate $\delta^u(z^H, B^H, B^F)$ is strictly increasing in aggregate productivity z^H and strictly decreasing in B^H and B^F.*

Proof. See Appendix B. ■

The restrictions on the parameter space assumed in the proposition ensure that the objective function is well defined. All results are derived analytically. However, since the analytical solution is cumbersome, the precise expressions are relegated to the appendix.

According to the proposition, the incentive to default—that is, the incentive to repay a lower fraction of the outstanding debt—is higher when the country is in recession. By repaying less debt, the government makes the recession deeper: in addition to the direct impact of lower productivity on employment and output, the destruction of entrepreneurial wealth associated with lower repayment further discourages the demand for labor. Thus, the optimal government policy acts as an amplification mechanism for the aggregate shock. Still, from the government point of view, the policy is welfare improving.

We can now characterize the optimal constrained solution, that is, the solution that is subject to the constraint $\delta \leq 1$. Given the unconstrained solution $\delta^u(z^H, B^H, B^F)$, the constrained optimal government policy is

$$\underline{\delta}(z^H, B^H, B^F) = \min\left\{ \delta^u(z^H, B^H, B^F)\ ,\ 1 \right\}, \tag{11}$$

that is, the minimum between the unconstrained policy and the upper bound 1.

The top panel of Figure 4 illustrates how the fraction repaid $\underline{\delta}(z^H, B^H, B^F)$ depends on domestic debt, B^H, for two realizations of productivity z^H. The bottom panel plots the wealth of home entrepreneurs, M^H (after debt repayment). In constructing these two plots, the foreign debt B^F is kept constant.[7]

For low levels of home debt B^H, the home government fully repays the debt, that is, $\underline{\delta}(z^H, B^H, B^F) = 1$. However, once the stock of debt inherited from period 1 increases above a certain threshold, the government repays less ($\underline{\delta} < 1$). Because the incentive to repay increases with productivity, the threshold is higher when productivity is high (continuous line vs. dashed line). The optimal repayment policy implies

[7]The parameters used in this numerical example are $\nu = 2$, $\Psi = 0.5$, $z^H = 1.5$, $z^F = 1.7$, and $B^F = 0.1$.

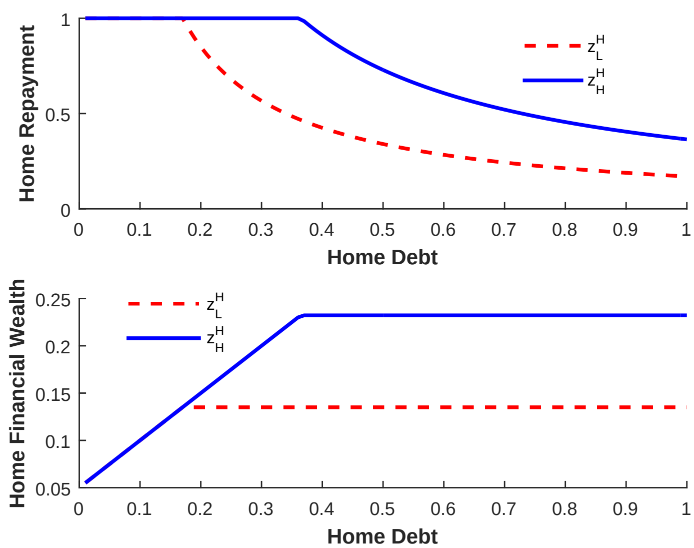

Figure 4: Optimal repayment rate $\underline{\delta}$ (top panel) and post-default financial wealth M^H (bottom panel) as functions of home debt, B^H. Foreign debt B^F is held constant. Parameters: $\nu = 2$, $\Psi = 0.5$, $z_L = 1.5$, $z_H = 1.7$, $B^F = 0.1$, and $B^H \in (0, 1]$.

that post-default financial wealth is increasing in home debt up to a threshold and then becomes flat. This property is important for determining the costs and benefits of issuing debt in the first period as we will see later.

Portfolio diversification and externally induced default. Figure 5 illustrates the importance of portfolio diversification of home residents for the incentive to default. It depicts the optimal repayment rate of the home country, $\underline{\delta}(z^H, B^H, B^F)$, as a function of external debt assets B^F, for a given level of home debt B^H.

When the stock of foreign debt increases, home residents hold more of the foreign debt. This implies that the macroeconomic disruption when the home government defaults is smaller. Thus, even if the home debt and the amount held by foreigners do not change, the home government has more incentives to default if residents hold more of the foreign debt. Notice that we purposely keep the home debt and the faction held by foreigners constant in order to isolate the usual channel through which international diversification affects the incentive to default: if more of the home debt is held by foreigners, default generates a larger redistribution toward home residents. This channel is also present in other models proposed in the literature. However, the mechanism emphasized in this paper and illustrated in Figure 5 is different and novel.

To be more specific, the typical redistribution channel that is present in other sovereign default models depends on B^{HH}/B^H, that is, the fraction of the debt issued by the home country held domestically. The channel emphasized here, instead,

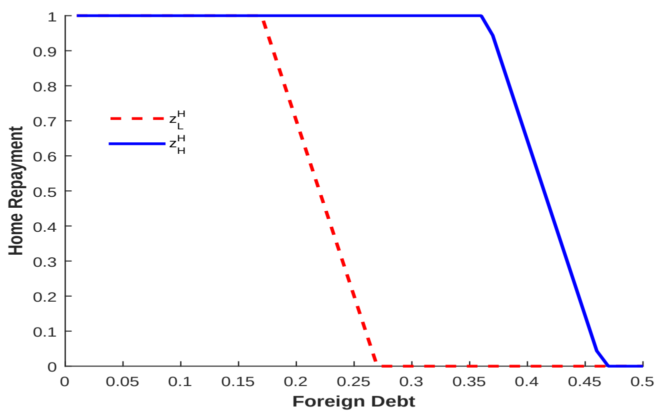

Figure 5: Optimal repayment rate $\underline{\delta}$ as a function of foreign debt, B^F. Home debt B^H is held constant. Parameters: $\nu = 2$, $\Psi = 0.5$, $z_L = 1.5$, $z_H = 1.7$, $B^H = 0.1$, and $B^F \in (0, 0.5]$.

depends on B^{FH}/M^H, that is, the fraction of wealth of home residents held in foreign countries. This is an indicator of portfolio diversification for home residents. In general, these two indices are highly correlated and, therefore, when B^{FH}/M^H increases, B^{HH}/B^H goes down. The theoretical exercise illustrated in Figure 5, however, isolates the portfolio channel from the redistributive channel by keeping the home debt B^H fixed. Since we are assuming that $B^{HH} = B^{HF}$, the cross-country redistribution induced by default does not change. The importance of the exercise is to show that the default of a country could be driven by the debt issued of other countries.

Although in this paper we have considered only financial assets issued by governments, the effect described above could also be driven by the expansion of private financial markets. For example, a financial boom in advanced economies, either private or public, could induce the default of some emerging countries even if financial and real conditions in these countries have not changed. It is in this sense that default could be externally driven.

4.3 Financial integration with bailout

Default by the home country also destroys entrepreneurial wealth held by foreign entrepreneurs and, therefore, it affects adversely employment and output in the foreign country. The macroeconomic consequences of sovereign default are then *exported* to other countries (spillover). In this section we analyze how the macroeconomic spillovers create an incentive for the foreign country to bailout, directly or indirectly, foreign entrepreneurs.

4.4 Domestic bailout

We first consider the case in which the foreign government bailouts its own entrepreneurs if country H defaults. To do so, the F government levies a tax τ on workers and the revenues are transferred to entrepreneurs. We refer to this as *domestic bailout.*

The value of τ is chosen to maximize the country welfare $\Psi U^F + (1-\Psi)W^F$ (see Appendix A for a derivation of the analytical expression of this objective). After substituting all conditions that define a competitive equilibrium, the optimization problem solved by the F country in response to the default of the H country can be written as

$$\underline{V}^F(\mathbf{s}) = \max_{\tau} \Psi \left[M^F + \gamma(z^F)(M^F)^{\frac{1}{\nu}}\right] + (1-\Psi)\left[\alpha(z^F)M^F - B^F - \tau\right] \quad (12)$$

subject to

$$\tau \in \left[0\,,\ \frac{(1-\underline{\delta})B^H}{2}\right]. \quad (13)$$

$$M^F = \frac{\underline{\delta}B^H + B^F}{2} + \tau.$$

The variable M^F is the post-bailout wealth of entrepreneurs in country F given the repayment rate $\underline{\delta}$ chosen by the H country. The terms $\gamma(z^F)$ and $\alpha(z^F)$ are increasing functions of the foreign country shock z^F. The transfer τ is constrained to be non-negative and it cannot be bigger than the financial losses incurred by entrepreneurs when the home country defaults ($\underline{\delta} < 1$). In other words, the F government cannot transfer to its own entrepreneurs more than the losses incurrent as a consequence of the default of the H country, which defines the upper bound on the first constraint to the problem above.

Proposition 6 *Suppose that (i) $\Psi < \frac{1}{2}$, (ii) $z_F < \frac{1-2\Psi}{1-\Psi}\frac{2\nu}{\nu-1}$, and (iii) $\underline{\delta} < 1$. There exists a unique solution to the domestic bailout problem (12). If eq. (13) does not bind and $z^H = z^F$, the optimal τ increasing in aggregate productivity z^F. When $\Psi \geq \frac{1}{2}$, the welfare function is always increasing in τ, implying that the solution is $\tau = (1-\underline{\delta})B^H/2$.*

Proof. See Appendix C. ■

Figure 6 illustrates the optimal bailout transfer τ for particular parameters. The dashed line represents the case in which the home country has low productivity, whereas the solid line corresponds to the case with high productivity. When B^H is low, the home country fully repays the debt and, therefore, the bailout transfer τ is zero. When B^H reaches a certain level, the home country repays a fraction of the debt smaller than 1 (default). In this case the F government compensates the lower repayment by taxing workers and transferring the revenues to entrepreneurs. Under the assumption that $\Psi = \frac{1}{2}$, the F government fully compensates the losses incurred by foreign entrepreneurs. This implies that domestic bailout eliminates the negative spillovers that would affect the F country as a consequence of default from the home country.

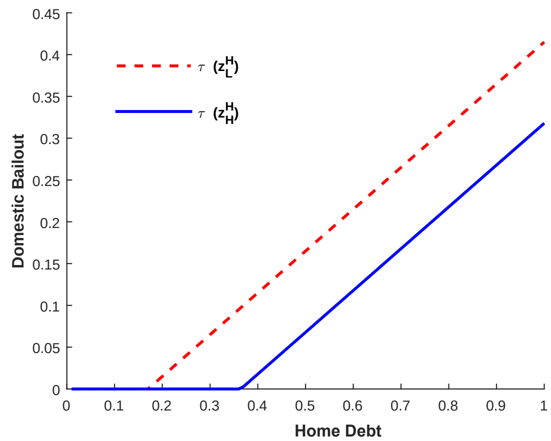

Figure 6: Domestic bailout transfer τ as function of B^H. Dashed line: recession in country H; Solid line: boom in country H. Parameters: $\nu = 2$, $\Psi = 0.5$, $z_L = 1.5$, $z_H = 1.7$, $B^F = 0.1$, $B^H \in (0, 1]$.

The welfare attained by the H country under this scenario—that is, when it repays the optimal fraction $\underline{\delta}$ and the F country implements the optimal domestic bailout—can be written as

$$\underline{V}^H(\mathbf{s}) \quad = \quad \Psi\left[M^H + \gamma(z^H)(M^H)^{\frac{1}{\nu}}\right] + (1-\Psi)\left[\alpha(z^H)M^H - B^H\right], \tag{14}$$

where $M^H = \frac{\underline{\delta} B^H + B^F}{2}$ is different from M^F because entrepreneurs in the home country do not receive the bailout τ. The formal derivation is in Appendix A.

4.5 External bailouts

Because the home government ignores the macroeconomic cost of default incurred by the foreign country, the optimal repayment is not socially efficient. This implies that both countries may gain from renegotiating the repayment of the home debt.

Renegotiation takes place only if country H defaults, that is, if it is optimal for the H country to choose $\underline{\delta} < 1$. With renegotiation, the two countries bargain the fraction of the home debt that should be repaid, δ, together with a transfer τ paid by the F country to H country. We think of this payment as an *external bailout.* To use a compact notation, we denote the renegotiated policies by $\pi = (\delta, \tau)$.

Let's first define some key functions. Given the negotiated policies π, Appendix A shows that the welfare of government $i \in \{H, F\}$ can be written as

$$\overline{V}^i(\mathbf{s};\pi) \quad = \quad \Psi\Big[M + \gamma(z^i)M^{\frac{1}{\nu}}\Big] + (1-\Psi)\Big[\alpha(z^i)M + T^i\Big] \tag{15}$$

with

$$\begin{aligned} M &= \frac{\delta B^H + B^F}{2}, \\ T^i &= \begin{cases} -\delta B^H + \tau, & \text{if } i = H \\ \\ -B^F - \tau, & \text{if } i = F \end{cases}. \end{aligned}$$

The vector $\mathbf{s} = (z^H, z^F, B^H, B^F)$ denotes the state variables in period 2.

We can see that a higher repayment δ has positive effects by increasing entrepreneurs' wealth M. These effects are the same for the home and foreign countries. Higher repayments, however, also imply a cost for the home country due to higher taxes that home workers have to pay ($T^H = -\delta B^H + \tau$). The bailout transfer τ, instead, is a benefit for the home country (since it reduces the tax burden of home workers) but it is a cost for the foreign country, since foreign workers have to pay more taxes to cover the transfer ($T^F = -B^F - \tau$). Effectively, foreign workers help home workers to repay the debt of the home country. This is in contrast to domestic bailout described in the previous subsection, where the additional taxes paid by foreign workers to fund the bailout, are transferred to foreign entrepreneurs.

Because a higher repayment δ has positive effects on the macroeconomy, the foreign government could be willing to pay τ in order to induce a higher repayment from the home government. Since a higher repayment has also positive effects for the home country, the foreign government can convince the H government to repay the debt by 'partially' subsidizing the repayment. This will become clear below. Before doing so, however, we need to define the bargaining problem solved by the two governments in the negotiation of the external bailout.

Negotiation of the external bailout. If the two countries do not reach an agreement, they revert to the equilibrium without renegotiation. The threat value for the H country is the value of defaulting without renegotiation, that is, $\underline{V}^H(\mathbf{s})$ derived in (14). The foreign country can always respond to the default with a domestic bailout. Therefore, the threat value for the F country is $\underline{V}^F(\mathbf{s})$ derived in (12). The bargaining problem can then be written as

$$\max_{\tau,\,\delta \le 1} \quad \Big[\overline{V}^H(\mathbf{s};\pi) - \underline{V}^H(\mathbf{s})\Big]^\eta \Big[\overline{V}^F(\mathbf{s};\pi) - \underline{V}^F(\mathbf{s})\Big]^{1-\eta}, \tag{16}$$

where η is the relative bargaining power for the home country.

The bargaining problem maximizes the weighted product of the *net* renegotiation surpluses of the two countries. The first order condition with respect to τ determines how the bargaining power affects the division of the surplus between the two countries,

$$\frac{\eta}{1-\eta} = \frac{\overline{V}^H(\mathbf{s};\pi) - \underline{V}^H(\mathbf{s})}{\overline{V}^F(\mathbf{s};\pi) - \underline{V}^F(\mathbf{s})}. \tag{17}$$

The optimal repayment is determined by the first order condition with respect to δ. This takes the form

$$\frac{\partial \overline{V}^H(\mathbf{s};\pi)}{\partial \delta} + \frac{\partial \overline{V}^F(\mathbf{s};\pi)}{\partial \delta} \geq 0, \tag{18}$$

and it is satisfied with the inequality sign if the solution for the optimal repayment is binding, that is, $\delta = 1$.

The Nash-bargaining solution implies that the sum of the (net) marginal benefits from repaying an extra unit of debt for both countries must be equal to zero (or equivalently the marginal cost for the home country must be equal to the marginal benefit for the foreign country).

Two properties are worth emphasizing. First, this result is independent of the bargaining weights. Second, it can be easily shown that the Nash-bargaining solution coincides with the solution to a problem in which τ and δ are chosen by a benevolent planner who assigns positive weights to both countries. This equivalence derives from the assumption that the utility from consumption is linear.

Denote by $\underline{\delta}^u(\mathbf{s})$ the optimal and unconstrained repayment *without* renegotiation and by $\bar{\delta}^u(\mathbf{s})$ the optimal and unconstrained repayment *with* renegotiation. We then have the following proposition.

Proposition 7 *Suppose that (i) $\Psi < \frac{1}{2}$, (ii) $2 - \alpha(z^H) - \alpha(z^F)]\frac{1-\Psi}{\Psi} - 2 > 0$, and (iii) $\underline{\delta}(\mathbf{s}) < 1$. The optimal and unconstrained repayment rate $\bar{\delta}^u(\mathbf{s})$ is strictly increasing in z^H and z^F, and strictly decreasing in B^H and B^F. Furthermore, the unconstrained repayment rate with renegotiation is always bigger than the unconstrained repayment without renegotiation, that is, $\bar{\delta}^u(\mathbf{s}) > \underline{\delta}^u(\mathbf{s})$.*

Proof. See Appendix D. ■

What differentiates the optimal repayment with and without renegotiation is that the latter takes into account only the effects of default in the home country whereas the former takes into account the consequences for both home and foreign countries. Debt renegotiation forces the home country to internalize the macroeconomic spillovers of default and, therefore, it induces higher repayment.

The constrained solution, that is, the solution that satisfies $\delta \leq 1$, is

$$\bar{\delta}(\mathbf{s}) = \min\left\{ \bar{\delta}^u(\mathbf{s})\ ,\ 1 \right\}. \tag{19}$$

As in the no-renegotiation case, total debt $B^H + B^F$ is fully repaid for relatively low values of home and foreign debt. Furthermore, higher values of home debt are associated with lower repayment rates. It is easy to show that, when $\Psi \geq \frac{1}{2}$, the objective function in the Nash bargaining problem is strictly increasing. Hence, $\bar{\delta}(\mathbf{s}) = 1$ regardless of the shocks or levels of debt in each country.

Domestic vs External bailouts Figure 7 shows the external bailout transfer for a numerical example. In this example $\Psi = \frac{1}{2}$ and, therefore, the bailout results in full repayment, that is, $\bar{\delta} = 1$.

The top panels of Figure 7 plot total repayment (left panel) and home repayment (right panel) in the equilibrium without renegotiation (no domestic or external bailouts). Total repayment is calculated as the ratio between the post-default worldwide wealth of entrepreneurs over the outstanding worldwide sovereign debt, that is, $\frac{M^H+M^F}{B^H+B^F}$. If this ratio is equal to 1 it means that the worldwide debt is fully repaid. When the ratio is smaller than 1 the worldwide debt is repaid only partially. Home repay, instead, is the ratio of taxes paid by workers in the home country over the debt of the home country, that is, $\frac{T^H}{B^H}$. Without renegotiation this is equal to the optimal repayment ratio $\underline{\delta}(\mathbf{s})$.

The bottom panels of Figure 7 plot the same variables but with an external bailout. As observed above, since in this example $\Psi = \frac{1}{2}$, the worldwide debt is fully repaid under bailout. However, we can see from the last panel that home workers repay only a fraction of the home debt. Part of the repayment is covered by the transfer $\tau > 0$ that the F country makes to the H country.

Does the foreign country gain from participating in the external bailout instead of simply bailing out its own entrepreneurs? Provided that the bargaining Problem (16) has a solution, the answer is obviously yes. This is because the bargaining value for the F country cannot be smaller than its threat value (given by the welfare achieved with domestic bailout).

Figure 8 illustrates this with a numerical example in which $\Psi = \frac{1}{2}$. With this welfare weight, the debt held by foreign entrepreneurs is fully repaid with both domestic and foreign bailouts. So the gain for country F from bailing out country H is only the difference between the transfers that F workers have to pay with external and domestic bailouts. The figure plots these two transfers when home productivity is low (left panel) and when home productivity is high (right panel).

When B^H is small, the home country does not default and, therefore, there is neither domestic or external bailout. Once B^H reaches a certain level, the home country finds optimal to default and the fraction repaid declines with the stock of debt. As a result, bailout transfers increase with B^H. However, the transfer paid by country F under a domestic bailout is bigger than the transfer under an external bailout. This shows why, at least under some conditions, countries may prefer the more common bailout approach instead of a domestic bailout.

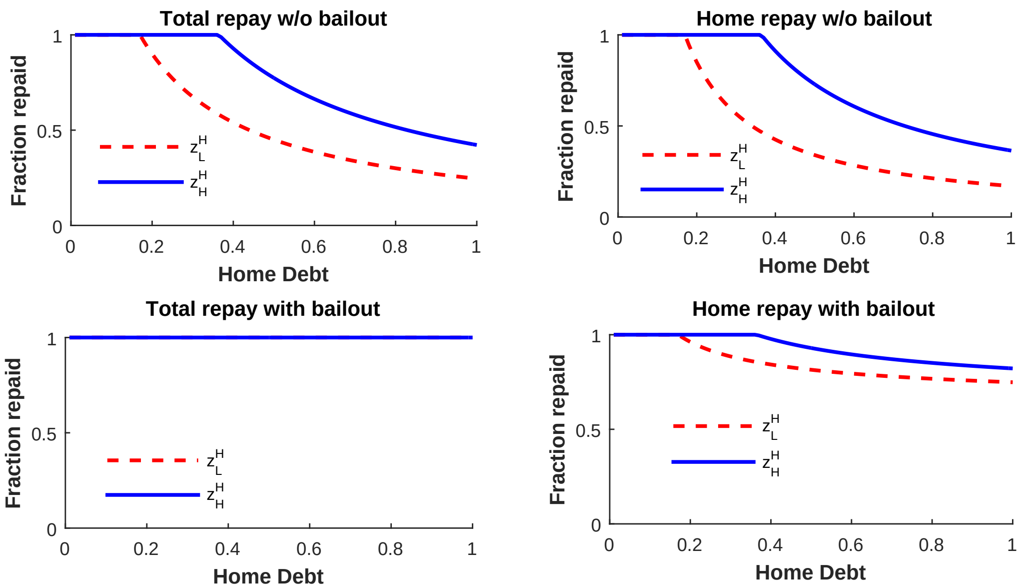

Figure 7: Debt repayments with and w/o bailouts as a function of home debt, B^H. 'Total repay' is the fraction $\frac{M^H+M^F}{B^H+B^F}$. 'Home repay' is the fraction $\frac{T^H}{B^H}$. Parameters: $\nu = 2$, $\Psi = 0.5$, $z_L = 1.5$, $z_H = 1.7$, $B^F = 0.1$, $B^H \in (0, 1]$, and $\eta = 0.5$.

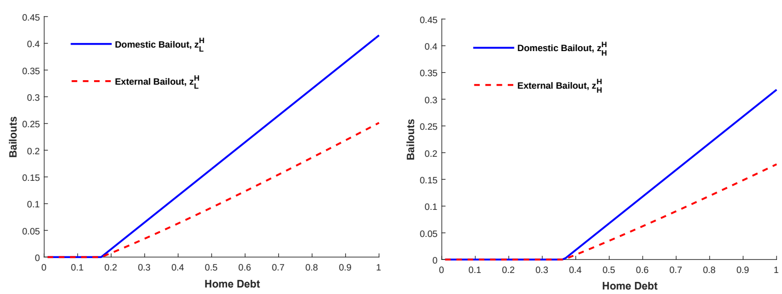

Figure 8: Domestic and external bailouts as a function of B^H. Left panel: z_L^H and right panel z_H^H. Parameters: $\nu = 2$, $\Psi = 0.5$, $z_L = 1.5$, $z_H = 1.7$, $\eta = 0.5$, $B^F = 0.1$, and $B^H \in (0, 1]$.

5 Equilibrium in Period 1

The analysis conducted so far has focused on period 2. Given debt issued by the two governments and the portfolio composition of private agents, we have characterized the optimal repayment policy of the home country with and without external bailouts. We can now move back to period 1 and characterize the optimal issuance of debt. This allows us to derive endogenously also the optimal portfolio composition chosen by entrepreneurs which in the previous section we took as exogenous (see Assumption

4). The issuance of debt and the optimal portfolio compositions will then determine the states in period 2.

To keep tractability, we focus on the optimal issuance of debt chosen by the home government, B^H, and assume that the debt of the foreign government, B^F, is exogenous. This simplifies the analysis because we do not need to consider the strategic interaction between governments in the first period. We will relax this assumption when we introduce the general model in Section 6.

5.1 Equilibrium for given policies

Using the previous notation, we denote with $\delta(\mathbf{s})$ the fraction of debt repaid in period 2 by the home country. The repayment policy depends on the aggregate states $\mathbf{s}$. In general, the aggregate states are $\mathbf{s} = (z^H, z^F, B^{HH}, B^{HF}, B^{FH}, B^{FF})$. However, as we will show below, the optimal portfolio choices in period 1 implies that B^H and B^F are sufficient to determine B^{HH}, B^{HF}, B^{FH} and B^{FF}. Therefore, in what follows we use $\mathbf{s} = (z^H, z^F, B^H, B^F)$.

The equilibrium repayment rate will be denoted by $\delta(\mathbf{s})$. As we have seen this depends on the particular environment: no-renegotiation or external bailout. When necessary to specify the particular environment, we will use $\underline{\delta}(\mathbf{s})$ for the environment without external bailout and $\bar{\delta}(\mathbf{s})$ for the environment with external bailout.

The equilibrium policy function $\delta(\mathbf{s})$ is taken as given by agents and governments when they solve their optimization problem in period 1. In addition, private agents take as given the government policies chosen in period 1, that is, B^H and B^F.

In period 1 workers behave passively: they consume the endowment plus the revenues that the government receive from issuing debt. Their lifetime utility is

$$\begin{aligned} W_1^i(B^H, B^F) &= \ln\left(c_1^i\right) + \beta \mathbb{E}\left[c_2^i(\mathbf{s}) - L^i(\mathbf{s})^{\frac{1}{\nu}}\right] \qquad (20) \\ &\text{with} \\ c_1^i &= e + \frac{B^i}{R^i}, \\ c_2^i(\mathbf{s}) &= e + w^i(\mathbf{s})L^i(\mathbf{s}) + T^i(\mathbf{s}) \end{aligned}$$

where $w^i(\mathbf{s})$ are the equilibrium wage, $L^i(\mathbf{s})$ the equilibrium labor supply and $T^i(\mathbf{s})$ the government transfers (taxes if negative), all in period 2. These objects depend on the states in period 2 as specified in the previous analysis. They also depend on the particular renegotiation environment: with or without external bailout.

Entrepreneurs choose optimally their savings and allocation between home and foreign bonds (portfolio composition) to solve the problem

$$
\begin{aligned}
U_1^i(B^H, B^F) &= \max_{b^{Hi}, b^{Fi}} d_1^i + \beta \mathbb{E} d_2^i \left(\mathbf{s}, b^{Hi}, b^{Fi}\right) \\
& \quad \text{subject to} \\
d_1^i &= a - \frac{b^{Hi}}{R^H} - \frac{b^{Fi}}{R^F} \\
d_2^i \left(\mathbf{s}, b^{Hi}, b^{Fi}\right) &= \delta(\mathbf{s}) b^{Hi} + b^{Fi} + \gamma(z^i) \Big[\delta(\mathbf{s}) b^{Hi} + b^{Fi}\Big]^{\frac{1}{\nu}},
\end{aligned}
\tag{21}
$$

where the term $\gamma(z^i)$ is only a function of own country aggregate productivity.

The following lemma characterizes the optimal portfolio allocation chosen by entrepreneurs.

Lemma 8 *In equilibrium, the optimal portfolio choice of entrepreneurs satisfies*

$$
\frac{b^{Hi}}{b^{Hi} + b^{Fi}} = \frac{B^H}{B^H + B^F}.
$$

Proof. See Appendix E. ■

The lemma says that entrepreneurs choose the same portfolio allocation in the two countries. More specifically, given the savings of entrepreneurs in country i, $b^{Hi} + b^{Fi}$, the fraction allocated to home bonds, $b^{Hi}/[b^{Hi} + b^{Fi}]$, is the same for home and foreign entrepreneurs. Obviously, if entrepreneurs in the two countries choose to allocate the same fraction of savings in home issued bonds, this fraction must be equal to the aggregate relative supply, $B^H/(B^H + B^F)$. This shows that the assumption made in the previous section about the composition of portfolio in period 2 (see Assumption 4) is an equilibrium outcome. This also implies that the sufficient set of aggregate states is $\mathbf{s} = (z^H, z^F, B^H, B^F)$, as we conjectured.

The choice of the same portfolio composition derives from the assumption that the two countries are identical in preferences and technology and entrepreneurs do not pay taxes or receive transfers.[8]

5.2 Optimal issuance of debt

The objective for the home government in period 1 is

$$
V_1^H(B^H, B^F) = \Psi U_1^H(B^H, B^F) + (1 - \Psi) W_1^H(B^H, B^F),
$$

the functions $W_1^H(B^H, B^F)$ and $U_1^H(B^H, B^F)$ are the equilibrium lifetime utilities of workers and entrepreneurs defined, respectively, in (20) and (21). The debt issued by

[8]If entrepreneurs pay taxes or receive transfers, their disposable income would be affected by default decisions. Because the foreign government always repays debt whereas the home government could default, there would be an asymmetry in portfolio decisions. Another required assumption for the portfolio result is the absence of income effects on the labor supply.

the foreign government, B^F, is exogenous in this version of the model and, therefore, it is taken as given by the H government.

The optimal issuance of debt from the home government is shown with a numerical example in Figure 9. The figure plots the social welfare function for the home government as a function of debt B^H, and for a given B^F. The solid line represents the no-bailout case, whereas the dashed line is for the environment with (expected) external bailout in period 2. The optimal debt is the value of B^H which maximizes social welfare.

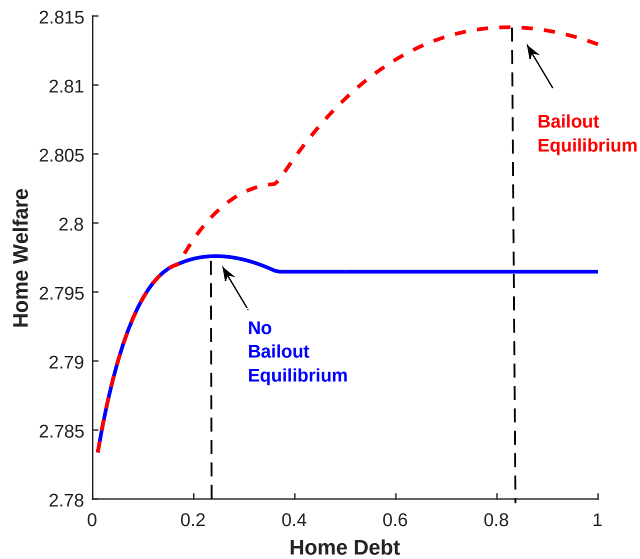

Figure 9: First period welfare $V_1^H(B^H, B^F)$ in the bailout (dotted) and no-bailout (solid) cases as functions of domestic debt B^H (given B^F). Parameters: $\nu = 2$, $\beta = 0.9$, $a = 3$, $e = 2$, $\Psi = 0.5$, $z_L = 1.5$, $z_H = 1.7$, $B^F = 0.1$, $\eta = 0.5$, and $B^H \in (0, 1]$.

In the environment without external bailout, the home government chooses to borrow $B^H = 0.24$. With anticipated external bailout, instead, the optimal debt is $B^H = 0.82$, which is bigger than the debt issued without bailout. With anticipated bailout the home country anticipates that part of the debt will be repaid by the foreign country and, therefore, it has more incentive to borrow. This captures the typical moral hazard mechanism induced by the anticipation of bailouts.

As can be seen from Figure 9, the anticipation of bailout improves welfare for the home country. But does this also improve the welfare of the foreign country? More specifically, would the F government also gain by not committing ex-ante to bailout the H country?

Figure 10 plots the social welfare for country F with and without external bailout for different values of B^H. As can be seen, the welfare with bailout dominates the welfare without bailout (where the F country is able to commit to not bailout the

H country). Thus, anticipated bailouts could be efficient not only ex-post (after a country has chosen to default) but also ex-ante (before the issuance of debt). Although this result cannot be generalized to any arbitrary parameter configuration, it shows that it is possible for anticipated bailouts to be ex-ante optimal.

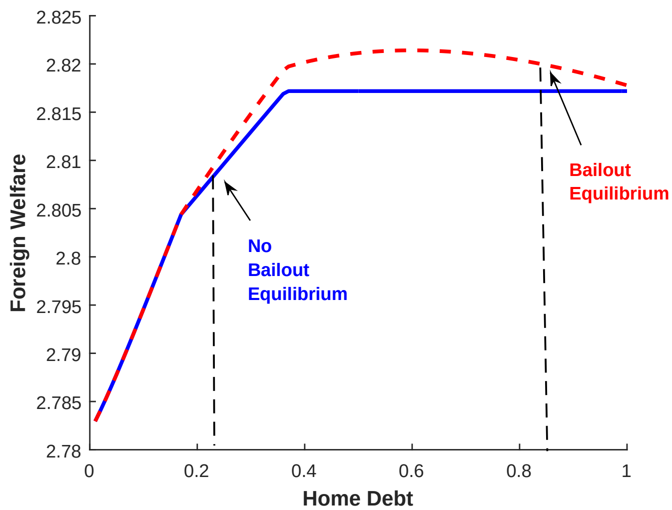

Figure 10: First period welfare $V_1^F(B^H, B^F)$ in the bailout (dotted) and no-bailout (solid) cases as functions of domestic debt B^H (given B^F). Parameters: $\nu = 2$, $\beta = 0.9$, $a = 3$, $e = 2$, $\Psi = 0.5$, $z_L = 1.5$, $z_H = 1.7$, $B^F = 0.1$, $\eta = 0.5$, and $B^H \in (0, 1]$.

Why are anticipated bailouts welfare improving? More debt allows for greater efficiency in period 2. However, with financial integration, when a country issues more debt, the benefits are shared with other countries. This implies that the country does not fully internalize the benefits of more debt and, therefore, it issues too little debt. Anticipated bailouts alleviate this problem. The home country anticipates in period 1 that the foreign country will contribute to the repayment of the debt. This increases the incentive to borrow, bringing the allocation closer to the efficient one. Effectively, renegotiation acts as a counterbalancing mechanism for the under-issuance of debt that results from the fact that countries do not internalize that their issuance of debt brings benefits also to other countries.

6 General model

In this section we consider three main extensions of the model:

1. Concave utilities for both workers and entrepreneurs.

2. Micro-foundation for the cost function $\varpi(l, w, m)$.

3. Optimal issuance of debt and default also for the foreign country.

The concavity of utilities allows us to derive a sharper characterization of the equilibrium in period 1 when both countries choose their borrowing optimally. With linear preferences the equilibrium in period 1 is undetermined. The concavity for entrepreneurs is also important for the micro-foundation of the cost function $\varpi(l, w, m)$. The drawback is that it is more difficult to characterize the properties of the model analytically. The main insights we obtained from the simple model also apply to the more general model studied here. In particular, (i) employment increases with entrepreneurs' wealth M; (ii) the home government has more incentives to default in recessions than in booms; (iii) anticipated bailouts result in higher levels of debt which, importantly, could be welfare improving also ex-ante.

To facilitate the presentation we first analyze the model in which only the home country can default in period 2. The case in which both countries could default will be studied in Section 7.

6.1 Environment without default from F country

The lifetime utility of workers is

$$\ln(c_1) + \beta \mathbb{E} \ln \Big(\varphi(c_2, \ell_2) \Big), \qquad \varphi(c, \ell) = c - \ell^{\nu}$$

Because the dis-utility of working in period 2 is additive to consumption, the labor supply still satisfies eq.(5) and it is fully determined by the wage rate.

The utility of entrepreneurs is also logarithmic, that is,

$$\ln(d_1) + \beta \mathbb{E} \ln(d_2).$$

The production function in period 2 is

$$y_2^i = A(z_2^i, \varepsilon_2) l_2^i,$$

where z_2^i is an aggregate country-specific productivity shock and ε_2 is an idiosyncratic productivity shock specific to an individual entrepreneur. The only modification to the production function is the addition of the idiosyncratic shock.

While the aggregate shock z_2^i is observed at the beginning of the period before making any production decision (as assumed in the previous model), the idiosyncratic shock ε_2 is observed after choosing the input of labor l_2^i. This timing assumption makes the hiring decision risky. Since entrepreneurs are risk averse, the marginal product of labor must exceed the wage rate to compensate for the hiring risk (risk premium).

We continue to assume that the debt repayment of the home country, δ, is decided after the observation of the aggregate shock z^i but *before* entrepreneurs choose the input of labor. The following lemma characterizes the hiring decision in period 2 given the debt repayment δ.

Lemma 9 *Let ϕ^i satisfy the condition $\mathbb{E}_\varepsilon \frac{A(z^i,\varepsilon)-w^i}{1+[A(z^i,\varepsilon)-w^i]\phi^i} = 0$. Entrepreneur's policies in country i are*

$$
\begin{aligned}
l^i &= \phi^i m^i, \\
d^i(\varepsilon) &= \left[1 + \left(A(z^i,\varepsilon) - w^i\right)\phi^i\right] m^i,
\end{aligned}
$$

where $m^i = \left(\delta b^{Hi} + b^{Fi}\right)$ is the entrepreneur's wealth after default.

Proof. See Appendix F. ■

As in the simpler model, the demand for labor l^i depends, linearly, on the post-default wealth of the entrepreneur. This is because labor is risky and when individual wealth is lower, the entrepreneur is less willing to take risks. To reduce the risk, the entrepreneur hires fewer workers. The aggregation of individual decisions then implies that equilibrium employment and wage in period 2 depend positively on the debt repayment policies. This is made precise in the following lemma.

Lemma 10 *In the competitive equilibrium in period 2*

1. *The hiring factor ϕ^i is strictly decreasing in δ.*
2. *The wage rate w^i and aggregate employment L^i are increasing in δ.*

Proof. See Appendix G. ■

We can now move to period 1 taking as given the equilibrium policy rules in period 2. The first period maximization problems for workers and entrepreneurs are analogous to those described in Section 5.1. The next lemma characterizes the entrepreneurs' portfolio decisions in period 1.

Lemma 11 *The entrepreneur's policies in period 1 of country i are*

$$
\begin{aligned}
d_1^i &= a\left(1 - \theta^H - \theta^F\right), \\
b^{Hi} &= \theta^H R^H a, \\
b^{Fi} &= \theta^F R^F a,
\end{aligned}
$$

where θ^H and θ^F solve

$$
\begin{aligned}
1 + \beta &= \beta \mathbb{E}\left(\frac{1}{\theta^H \frac{\delta R^H}{R^F} + \theta^F}\right), \\
\beta &= (1+\beta)(\theta^H + \theta^F).
\end{aligned}
$$

Proof. See Appendix H. ■

Entrepreneurs allocate a fraction $1 - \theta^H - \theta^F$ of their initial endowment a to consumption and the remaining fraction $\theta^H + \theta^F$ is saved. Savings are then invested in home bonds and foreign bonds according to θ^H and θ^F.

Home and foreign entrepreneurs choose the same saving rates and the same composition of portfolios, that is, the same ratio of bonds issued by home and foreign governments. In equilibrium then, and analogously to our benchmark case, it must be that, for each country $i \in \{H, F\}$,

$$\frac{b^{Hi}}{b^{Hi} + b^{Fi}} = \frac{B^H}{B^H + B^F}.$$

Taking into account this property, we can reduce the sufficient aggregate states in period 2 to $\mathbf{s} = (z^H, z^F, B^H, B^F)$, that is, the aggregate shocks and the outstanding debt of the two countries.

6.2 Default decisions in period 2: no-renegotiation

Given the aggregate states $\mathbf{s}$, the home government chooses the repayment rate δ in order to solve

$$\underline{V}^H(\mathbf{s}) \;\; = \;\; \max_{\delta \leq 1} \Psi \mathbb{E}_\varepsilon \ln \Big(d^H(\varepsilon) \Big) + (1 - \Psi) \ln \Big(\varphi(c^H, \ell^H) \Big). \tag{22}$$

The variables h^H, c^H, $d^H(\varepsilon)$ are determined by the competitive equilibrium for given policies. Denote the solution to this problem by $\underline{\delta}(z^H, B^H, B^F)$.

Figure 11 plots the optimal repayment of the home country, $\underline{\delta}(z^H, B^H, B^F)B^H$, as a function of the foreign debt B^F, which is fully repaid by the foreign country. The parameters used in the example are $\nu = 2$, $e = 1$, $\Psi = 0.5$, the idiosyncratic productivity takes values $\{0.9, 1, 1.1\}$ with equal probabilities and the aggregate productivity takes values $z \in \{0.95, 1.1\}$, also with equal probabilities. We assume that $A(z^i, \varepsilon) = z^i + \varepsilon$. We first fix the outstanding debt of the home country, B^H, and then we plot the optimal repayment $\underline{\delta}(z^H, B^H, B^F)B^H$ as we change B^F. In the left panel $B^H = 0.48$ and in the right panel $B^H = 0.51$. Each panel reports the best responses of the home government, separately, when aggregate productivity in the home country is low and when it is high. As in the benchmark case, there is more incentive to default when the country is in a recession.

The figure shows that as external financial assets B^F increase, it is optimal for the home country to repay less. For low levels of foreign debt, the repayment of the home country is flat because the constraint $\delta \leq 1$ binds.

6.3 Domestic bailout in period 2

If the home country defaults, the foreign country could bailout foreign entrepreneurs. In this case the foreign government raises additional taxes from workers and uses

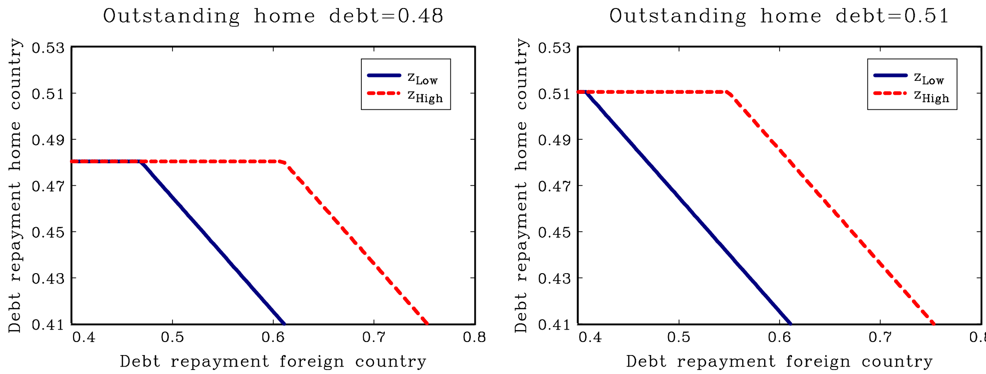

Figure 11: Optimal debt repayment in period 2 from home country as a function of the foreign debt. Parameters: $\nu = 2$, $e = 1$, $\Psi = 0.5$, $\varepsilon \in \{0.9, 1, 1.1\}$ with equal probabilities, and $z \in \{0.95, 1.1\}$ with equal probabilities.

the revenues to pay—partially or in full—the capital losses of foreign entrepreneurs following the default of the home government.

Given the holding of the home debt held by foreign entrepreneur, B^{HF}, the capital losses incurred by foreign entrepreneurs are $[1-\delta(z^H, B^H, B^F)]B^{HF}$. In the domestic bailout the foreign government chooses the lump-sum taxes raised from foreign workers, τ, and redistribute them to foreign entrepreneurs only, to solve the problem

$$\begin{aligned} \underline{V}^F(\mathbf{s}) \quad &= \quad \max_{\tau} \Psi \mathbb{E}_\varepsilon \ln\Big(d^F(\varepsilon)\Big) + (1-\Psi)\ln\Big(\varphi(c^F, \ell^F)\Big) \qquad (23) \\ & \quad\quad \text{subject to} \\ & \quad\quad \tau \in \Big[0\,,\ \Big(1-\delta(z^H, B^H, B^F)\Big)B^{HF}\Big]. \end{aligned}$$

Notice that the taxes paid by foreign workers for the domestic bailout, τ, are in addition to the taxes needed to repay for their government debt B^F.

6.4 External bailout

The renegotiation protocol is identical to the one in the simpler model that we characterized earlier: the two countries bargain the home debt repayment $\bar{\delta}(z^H, z^F, B^H, B^F)$ and the bailout transfer τ paid by country F to country H. This gives rise to the bargaining problem (16), where the threat values are now defined in (22) and (23).

To characterize the bargaining solution we resort to the numerical example used to construct Figure 11. The bargaining share parameter is set to $\eta = 0.5$.

Figure 12 plots the debt repayment. Total repayment captures the ratio of entrepreneurs' financial assets in both countries ($M^H + M^F = \delta B^H + M^F$) over the initial value of debt issued by both countries ($B^H + B^F$). When both countries fully repay their debt, the ratio is equal to 1. Home repayment without bailout is the repayment fraction $\underline{\delta}(z^H, B^H, B^F)$. The home repayment with bailout is $[\bar{\delta}(z^H, B^H, B^F)B^H -$

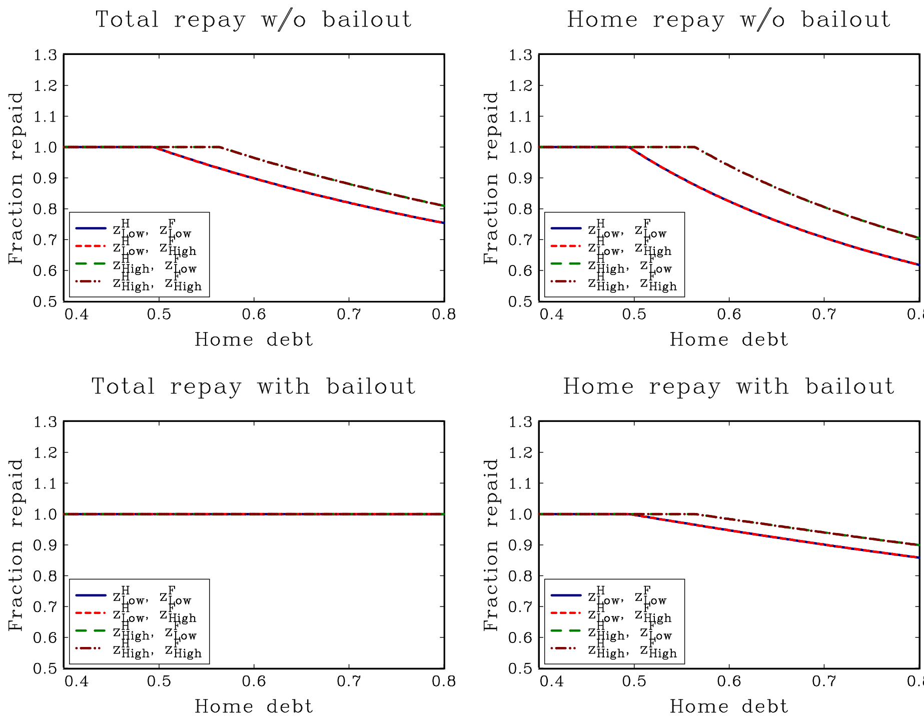

Figure 12: Debt repayments with and w/o bailout. The repayments are plotted as functions of the home debt, when the foreign debt is $B^F = 0.677$. 'Total repay' is the fraction $(M^H + M^F)/(B^H + B^F)$. 'Home repay' denotes T^H/B^H. Each line corresponds to different combinations of productivity in the two countries. Parameters: $\eta = 0.5$, $\nu = 2$, $e = 1$, $\Psi = 0.5$, $\varepsilon \in \{0.9, 1, 1.1\}$ with equal probabilities, and $z \in \{0.95, 1.1\}$ with equal probabilities.

$\tau]/B^H$, that is, the H repayment minus the transfer received from the F country (relative to the debt).

The figure shows that with bailout the total debt $B^H + B^F$ is fully repaid for relatively low values of debt. For very large values of debt, however, the debt would not be fully repaid. Furthermore, the higher the debt of the home country is, the lower the fraction $[\bar{\delta}(z^H, z^F, B^H, B^F)B^H - \tau]/B^H$ repaid by the home country. Although not shown, the repayment of the home country increases with the debt of the foreign country. Starting with a higher foreign debt is similar to giving more bargaining power to the F country.

6.5 Borrowing decisions in period 1

In period 1 the governments of both countries issue bonds. Country H issues B^H at price $1/R^H$ and country F issues B^F at price $1/R^F$. The revenues are distributed to domestic workers.

The issuance of debt in period 1 is made simultaneously and non-cooperatively. Takes as given the debt of the other country, B^{-i}, and the equilibrium policies in period 2, the government of country i solves

$$\max_{B^i} \quad \left\{ \Psi U^i(B^H, B^F) + (1-\Psi) W^i(B^H, B^F) \right\}.$$

The functions $U^i(B^H, B^F)$ and $W^i(B^H, B^F)$ are the equilibrium lifetime expected utilities of workers and entrepreneurs. These functions incorporate the equilibrium policies that will be determined in period 2 and their dependence on the current policies B^H and B^F.

Denote by $g^i(B^{-i})$ the best response function of country i to the debt issued by the other country, B^{-i}. A Nash equilibrium in period 1 is defined as the pair (B^H, B^F) that satisfies the conditions

$$\begin{aligned} B^H &= g^H\Big(B^F\Big), \\ B^F &= g^F\Big(B^H\Big). \end{aligned}$$

For the numerical characterization, we use the same parameters we used in the previous examples. In addition, we set the discount factor to $\beta = 0.9825$ and the first period endowments to $a = e = 1$. The top-left panel in Figure 13 plots the responses of the two countries as a function of the other country's debt under the no-renegotiation regime (no anticipated bailout).

The responses of the two countries are downward sloping. This is because, as the debt of the other country increases, part of which is held by domestic entrepreneurs, there is less need for liquidity. The equilibrium is given by the intersection of the two response functions. It can be noticed that there is a small range of multiplicity, that is, the responses functions intersect on multiple points. The range of multiplicity, however, is relatively small and the findings discussed below are (qualitatively) insensitive to the choice of the particular intersection (equilibrium).[9]

The top-right panel depicts the best responses in the bailout case and the equilibrium is again characterized by the intersection of the two response functions. Relative to the case without renegotiation, we observe two differences. First, the equilibrium debt issued by the home country is much larger whereas that of the foreign country is smaller. Since the home country knows that there will be a bailout in period 2, which implies that part of the debt will be repaid by the foreign country, it has an incentive to borrow more in period 1. Essentially, renegotiation makes the debt cheaper

[9] When B^H is relatively small, the home country never defaults in period 2. Thus, the foreign country's response to increases in B^H simply reflects a lower need to provide liquidity to foreign entrepreneurs. When B^H reaches a certain level, default may occur in period 2. Since the default decision of the home country depends on B^F, the foreign country will take this into account when it chooses the optimal debt. In particular, the foreign government may choose to reduce more its own debt when B^H increases in order to prevent the home country from defaulting in period 2. The reaction function becomes steeper and overlaps with the reaction function of the home country. Once B^H has reached a certain level, however, it is no longer beneficial to reduce B^F to prevent default and the reaction function of the foreign country becomes less steep.

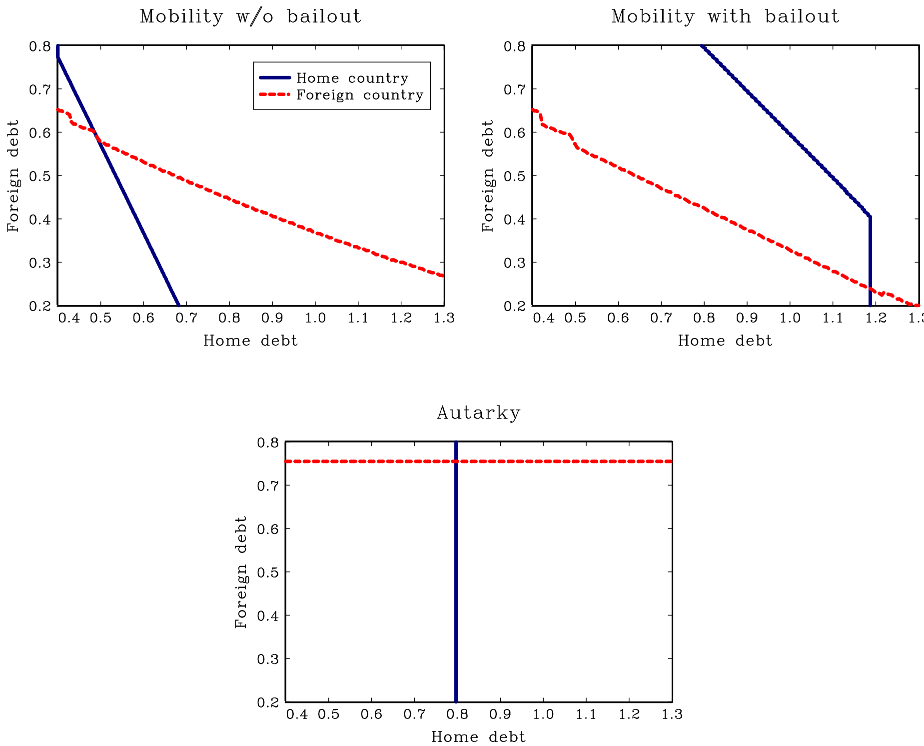

Figure 13: Response functions in period 1. Parameters: $\beta = 0.985$, $\eta = 0.5$, $\nu = 2$, $a = e = 1$, $\Psi = 0.5$, $\varepsilon \in \{0.9, 1, 1.1\}$ with equal probabilities, and $z \in \{0.95, 1.1\}$ with equal probabilities.

for the home country. Second, when there is anticipation of bailout, the worldwide debt level $(B^H + B^F)$ is bigger than without renegotiation. Without renegotiation, each country ignores the benefits that higher liquidity brings to the other country (a positive externality). When countries renegotiate in period 2, they will take this into account, which results in higher overall debt. These properties are qualitatively similar to those studied in the benchmark economy. The new insight from the more general environment is that the foreign country optimally reduces its issuance of debt in period 1. Despite of this, overall liquidity rises.

The bottom panel of Figure 13 plots the response functions in autarky. Obviously, being in autarky, the responses of the two countries are insensitive to the debt issued by the other country. The equilibrium, is still characterized by the intersection of these two functions. As summarized in Table 1, the worldwide debt $(B^H + B^F)$ issued in autarky is even higher than the worldwide debt in the regime with capital mobility when there is renegotiation. Therefore, the anticipation of bailout compensates only partially for the externality in the issuance of debt when markets are integrated.

Although renegotiation is always efficient in period 2, it is not obvious whether the anticipation of bailout also increases welfare in period 1, that is, before the two countries issue B^H and B^F. This is especially important for the foreign country, that

Table 1: Equilibrium debt and welfare for different regimes

	Mobility w/o bailout			*Mobility with bailout*			*Autarky*		
	Home	*Foreign*	*World*	*Home*	*Foreign*	*World*	*Home*	*Foreign*	*World*
Debt	0.49	0.58	1.08	1.19	0.24	1.43	0.80	0.75	1.55
Welfare	-0.448	-0.466	-0.457	-0.476	-0.414	-0.445	-0.410	-0.410	-0.410

is, the country that always repays. In the numerical example considered here we find that the equilibrium with anticipated bailout gives higher expected welfare to the foreign country when compared to the no-renegotiation equilibrium. This is shown in Table 1. The bailout equilibrium also gives higher worldwide welfare (compared to the no-renegotiation equilibrium), where worldwide welfare is defined as the sum of the two countries' welfare. The home country, however, achieves lower welfare.

The intuition for the welfare findings is similar to the one provided in the analysis of the simpler model: in an integrated economy, each country ignores the benefits that the issuance of debt brings to the other country and, as a result, the equilibrium debt is inefficiently low. The anticipation of renegotiation (bailout) increases the incentive to issue more debt in period 1 and compensates, at least in part, the inefficiency associated to the externality. Therefore, bailouts can be efficient ex-ante also in the extended model. The benefits of anticipated bailouts, however, are enjoyed only by the bailing country. This is because, thanks to the threat of domestic bailout, the foreign country can extract most of the benefits from higher worldwide liquidity.

7 The role of commitment

In this section, we relax the assumption that the F country always commits to repay the debt while the H country always defaults if it is optimal to do so. More specifically, we allow both countries to default in period 2 with some probability. We assume that with probability ρ^H the home country keeps its commitment to repay in period 2 and with probability $1-\rho^H$ it will opportunistically choose whether to default. Similarly, with probability ρ^F the foreign country keeps its commitment to repay the debt in period 2 and with probability $1-\rho^F$ it will choose whether to default. The model studied in the previous section is a special case with $\rho^H = 0$ and $\rho^F = 1$.

The stochastic commitment of governments captures the possible turnover of ruling parties with different political orientation or preferences. For example, we could think that in period 2 there is a probability of a new ruling party which assigns higher weight to entrepreneurs.[10]

[10]Because the indirect utility of entrepreneurs is strictly increasing in the debt repaid in period 2, a government that assigns a weight $\Psi^i = 1$ to entrepreneurs would never find it optimal to default.

Since both countries could default in period 2, the repayment decision becomes strategic. Country H chooses the optimal repayment taking as given the repayment of country F and the same is done by country F. We then characterize the equilibrium repayments in period 2 as determined by a Nash game between the two countries.

Denote by $\xi^i \in \{Commit, NotCommit\}$ the commitment state of country $i \in \{H, F\}$. Given the state vector $\mathbf{s} = (\xi^H, \xi^F, z^H, z^F, B^H, B^F)$, the government of country i chooses the repayment rate δ^i. If the government remains committed to repay, that is, $\xi^i = Commit$, then $\delta^i = 1$. However, if $\xi^i = NotCommit$, the government behaves opportunistically and will default if this improves its own welfare.

When $\xi^i = NotCommit$, the government of country i solves

$$\max_{\delta^i \leq 1} \quad \Psi \mathbb{E}_\varepsilon \ln \Big(d^H(\varepsilon) \Big) + (1-\Psi) \ln \Big(\varphi(c^i, \ell^i) \Big), \tag{24}$$

taking as given the repayment of the other country, δ^{-i}. The optimal policy is

$$f^i\left(\mathbf{s}, \delta^{-i}\right) = \begin{cases} 1, & \text{if} \quad \xi^i = Commit \\ \\ \text{Solution to Problem (24)}, & \text{otherwise} \end{cases}$$

The function f^i is the optimal repayment policy of country i, given the policy of the other country. A Nash equilibrium in period 2 is defined by the pair (δ^H, δ^F) that satisfies

$$\begin{aligned} \delta^H &= f^H\left(\mathbf{s}, \delta^F\right), \\ \delta^F &= f^F\left(\mathbf{s}, \delta^H\right). \end{aligned}$$

Given the equilibrium in period 2, we can now move to period 1 and solve for the equilibrium issuance of debt. In determining the optimal issuance of debt, the two governments take expectations of period 2 variables with respect to the commitment states in addition to productivities.

Figure 14 plots the best responses in period 1 for both countries as functions of the other country's debt in the no-renegotiation and bailout regimes. In this numerical example, the probability of commitment is $\rho^H = \rho^F = 0.5$. Even though the specific shapes of the response functions are different from those plotted in Figure 13, we still find that the possibility of bailouts induces higher issuance of debt in period 1.[11]

To show the importance of commitment, we compare the baseline model to two alternative scenarios with a probability of commitment of 60% (high commitment) and 40% (low commitment) for both countries. Table 2 reports the equilibrium debt and social welfare for different levels of commitment. For all levels of commitment, the renegotiation case is associated with higher welfare relative to the no-renegotiation case. Looking at the mobility regime with bailout we observe that lower commitment

Smaller values of Ψ^i would be associated with lower repayment values.

[11]Under stochastic commitment, multiplicity in the no-bailout case typically disappears.

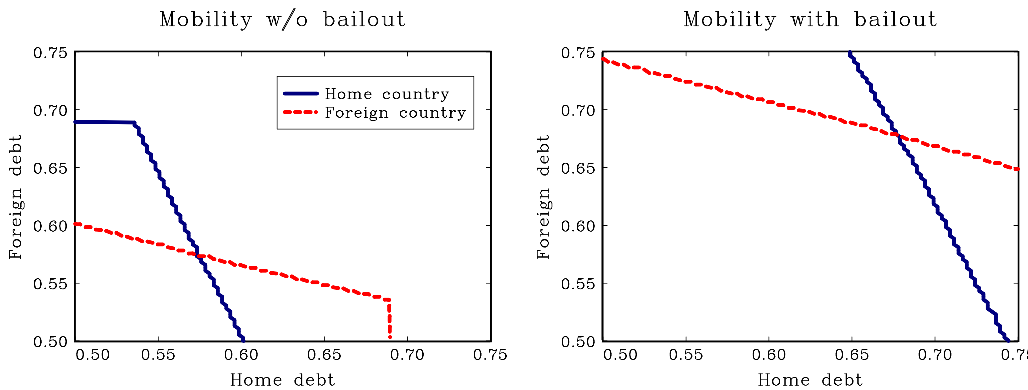

Figure 14: Response functions in period 1 for mobility without bailout and with bailout. Equilibrium determined by the intersection of the two response functions. Parameters: $\rho^H = \rho^F = 0.5$, $\beta = 0.985$, $\eta = 0.5$, $\nu = 2$, $a = e = 1$, $\Psi = 0.5$, $\varepsilon \in \{0.9, 1, 1.1\}$ with equal probabilities, and $z \in \{0.95, 1.1\}$ with equal probabilities.

Table 2: Equilibrium debt and welfare for different commitment

	High commitment ($\rho = 0.6$)		*Baseline* ($\rho = 0.5$)		*Low commitment* ($\rho = 0.4$)	
	Debt	*Utility*	*Debt*	*Utility*	*Debt*	*Utility*
Mobility w/o bailout	0.578	-0.449	0.573	-0.453	0.527	-0.464
Mobility with bailout	0.656	-0.420	0.678	-0.417	0.704	-0.415

increases equilibrium debt and improves welfare. This is because, with lower commitment there is a higher probability that the debt issued by one country is partially repaid by the other country (through a bailout), which increases the incentive to borrow. This compensates in part the under-issuance of debt as the benefits of creating financial assets for one country are shared with the other country. Thus, lower commitment brings the equilibrium closer to the case where each country internalizes the full benefit of issuing debt.

8 Empirical analysis

A important mechanism imbedded in the model is that the composition of domestic portfolios affects the government's decision to default. When the level of external debt assets (e.g. foreign debt held by domestic agents) raises, the domestic government has higher incentive to default. This mechanism is novel in the international literature and in this section we conduct an empirical analysis to test its significance using cross-country data for a panel of 31 countries in the period 1993-2011.

Before moving to the empirical specification, it is useful to summarize what determines the incentive to default in the model. There are two main channels: (i) the *redistribution* channel, which depends on the ratio of home debt held by home residents $\frac{B^{HH}}{B^{H}}$ and (ii) the *portfolio diversification* channel which can be captured by the share of home financial wealth held in foreign assets $\frac{B^{FH}}{M^{H}}$. Recall that B^{HH} is the home debt held by home entrepreneurs, B^{FH} is the foreign debt held by home entrepreneurs and $M^{H} = B^{HH} + B^{FH}$ is the financial wealth of home entrepreneurs. A higher ratio $\frac{B^{FH}}{M^{H}}$, keeping everything else constant, lowers the macroeconomic costs of default and hence increases the incentives to renege the debt. On the other hand, higher values of $\frac{B^{HH}}{B^{H}}$ means that a larger share of the debt is held domestically which makes default less attractive for the home government. Hence, our theory suggests that the empirical specification should include both variables. In addition, the model predicts that the incentive to default is higher in recessions, suggesting the inclusion of proxies for the business cycle.

From model to data: The data counterpart for our variable of interest, B^{FH}, is 'External Debt Assets' obtained from the External Wealth of Nations Database by Lane and Milesi-Ferreti[12]. This annual series, constructed from BIS and IFS data, includes 'debt securities' and 'other investment'. It is expressed in millions of current US dollars.

It is important to emphasize that this variable includes both public and private debt. Ideally, we would like to have a measure that includes only external public debt, as this would correspond more closely to our model. However, we observe that an augmented version of our model that includes private assets would have similar implications. The reason is because an increase in any type of assets that are not defaultable by the domestic government would reduce the macroeconomic cost of default in the home country and hence increases the incentive to default.

Home debt held by foreigners, B^{HF}, is proxied by 'External Government Debt' obtained from Global Financial Data. The variable corresponds to 'International Debt Securities, General Government' Table C1 of the Debt Securities Statistics of the BIS. Home debt held by domestic entrepreneurs B^{HH} is constructed as the difference between total home debt, B^{H}, and the part held by foreigners, B^{HF}. Total government debt is proxied by 'Central Government Debt' obtained from Global Financial Data, which corresponds to 'Total Debt Securities, General Government' in Table C1 of the Debt Securities Statistics of the BIS. Both B^{HF} and B^{HH} are measured in millions of current US dollars. Total (domestic) financial assets, M^{H}, are constructed by adding external debt assets and home debt held domestically, that is, $M^{h} = B^{FH} + B^{HH}$.

[12]The database contains data on foreign assets and foreign liabilities for a large sample of countries for the period 1970-2011. See Lane and Milesi-Ferreti (2007) for details. The data can be downloaded from: http://www.philiplane.org/EWN.html

Empirical specification and results: To check whether there are statistically significant links between the default risk of a country and the portfolio composition of its residents, we regress sovereign interest rate spreads on $\frac{B^{HH}}{B^H}$ and $\frac{B^{FH}}{M^H}$, in addition to other control variables. The interest rate spread of country H in year t, $S_{H,t}$, is computed as the difference between the country's interest rate and the risk free rate

$$S_t^H = R_t^H - \bar{R}_t,$$

where R_t^H corresponds to long-term 'Government Bond Yields' (10 years in most cases) of country H in period t, obtained from Global Financial Data. The risk free rate $\bar{R}_t$ is proxied by the yield of US government bonds of the same maturity.

We estimate the following fixed effect regression equation:

$$S_t^H = \alpha^H + I_t + \beta \cdot \left(\frac{B_t^{FH}}{M_t^H}\right) + \gamma \cdot \left(\frac{B_t^{HH}}{B_t^H}\right) + \theta \cdot X_t^H + \epsilon_t^H. \tag{25}$$

The variables α^H and I_t are, respectively, country and year fixed-effects. Our set of additional controls X_t^H includes GDP growth, inflation rates, and an indicator for whether the country is in default. GDP growth is computed as the difference in the natural logarithm of GDP from Lane and Milesi-Ferretti's database (measured in million of current US dollars). Inflation rates are obtained from the OECD and correspond to the CPI percentage change from last year. Finally, the default dummy is obtained from the Global Crises Dataset constructed by Reinhart, Rogoff, and Trebesch[13]. We use the variable 'Domestic Debt in Default'. The estimation results are shown in Table 3.

The first specification includes only the main variable of interest $\frac{B^{FH}}{M^H}$, as well as country and time fixed effects. Consistent with the model, a larger share of external debt assets is associated with a higher spread, indicating additional default risk. In the second specification, we restrict the sample to include at least 8 years of observations. We can see that, while the coefficient is basically unchanged, the goodness of fit improves, as seen from the higher value of the R^2. The number of countries is only reduced from 31 to 27.

In the third specification, we include GDP growth and $\frac{B^{HH}}{B^H}$ as additional controls. The result is qualitatively similar. As expected, higher GDP growth is associated with lower spreads, which is also consistent with our model. The coefficient on $\frac{B^{HH}}{B^H}$ is statistically insignificant. This suggests that the portfolio composition channel—which is the novel channel emphasized in this paper—is more important than the redistribution channel. Compared to the previous specification, the R^2 increases from 0.2 to 0.31.

Finally, the fourth and final specification includes inflation and the default status dummy as additional controls. The positive relationship between external debt assets

[13] The dataset is available at http://www.hbs.edu/faculty/initiatives/behavioral-finance-and-financial-stability/Pages/global.aspx

Table 3: Cross-country fixed-effect regression. The dependent variable is the spread on long-term government debt.

	(1)	(2)	(3)	(4)
Proxy for $\frac{B^{FH}}{M^H}$	6.5^{*}	3.1^{**}	6.1^{**}	5.21^{**}
	(3.55)	(1.21)	(2.57)	(2.16)
Proxy for $\frac{B^{HH}}{B^H}$			4.29	2.76
			(2.55)	(2.17)
Debt/GDP	0.021	0.012	0.023^{*}	0.027^{**}
	(0.016)	(0.011)	(0.013)	(0.010)
Default dummy	21.61^{***}	18.43^{***}	15.4^{***}	14.79^{***}
	(2.98)	(0.49)	(1.2)	(1.46)
GDP growth			-0.05^{***}	-0.06^{***}
			(0.02)	(0.018)
Lag Inflation				0.11^{**}
				(0.043)
Observations	492	469	469	441
R-squared	0.44	0.46	0.50	0.56
Number of countries	31	27	27	27

The sample period is 1993-2011 and includes the following countries: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, China, Colombia, Denmark, Finland, France, Germany, Greece, Italy, Japan, Mexico, Malaysia, the Netherlands, New Zealand, Norway, Philippines, Poland, Portugal, Russia, Singapore, Spain, South Africa, Sweden, Thailand, and the United Kingdom. Specifications (2)-(4) exclude: Argentina, Colombia, China, and Denmark because they have fewer than 8 observations. Robust standard errors are clustered per country and adjusted for autocorrelation and heteroskedasticity, represented are in parentheses.

* Significant at 10 percent. ** Significant at 5 percent. *** Significant at 1 percent.

and spreads is robust to the inclusion of these controls. The significant increase in the R^2 indicates that this specification is the one that best fits the data. Including dummies for banking crises or stock market crashes (both available in Reinhart, Rogoff, and Trebesch's database) slightly improves the goodness of fit, but does not change the size of the coefficient for $\frac{B^{FH}}{M^H}$.

9 Conclusion

We have shown that the macroeconomic spillovers caused by sovereign default are crucial for understanding equilibrium bailouts. Although anticipated bailouts generate the typical moral hazard problem leading to higher debt, this may counterbalance the under issuance of debt caused by the lack of policy coordination across countries.

We show this result in a model in which government debt is held by the private sector for liquidity and insurance purposes and higher supply of debt improves allocation efficiency. With globalized financial markets, part of the debt issued by one country is acquired by residents of other countries and, therefore, it creates benefits for foreigners. Because governments do not internalize the benefits for other countries, they issue too little debt. Anticipated bailouts could correct for this inefficiency because they reduce the expected cost of debt.

We have also shown that, when financial markets are integrated, sovereign default could be induced by increased borrowing of other countries. This is because, when foreign countries issue more debt, part of the debt is held by domestic agents, making their portfolio more diversified. More diversified portfolios decrease the macroeconomic cost of domestic default and increase the incentive of a government to default. From this perspective, the debt problems experienced by some European countries during 2011 may have been exacerbated by the increased debt (both private and public) issued by 'safe' industrialized countries since the early 1980s. The empirical analysis conducted in the paper shows that the incentive of a country to default is affected by the international diversification of domestic portfolios. A channel that has not been fully explored in the international literature but is central in the analysis of our paper.

Appendix

A Government Welfare given Policy

Let policy be denoted by a triplet $\pi = \{\delta, T^H, T^F\}$, where δ is the repayment rate of the home country and T^i denotes transfers to workers in country i. Using Lemma 2 and eq. (4), we can obtain an analytical expression for the utility of workers in period 2 as a function of the shock z^i and the stock of debt issued by each country, B^H and B^F

$$W^i(z^i, B^H, B^F; \pi) = e + \alpha(z^i)M + T^i, \tag{26}$$

with $\alpha(z^i) = \frac{\nu-1}{2\nu} z^i$ and

$$M = \frac{\delta B^H + B^F}{2}. \tag{27}$$

Under a domestic bailout, $M^F = \frac{\delta B^H + B^F}{2} + \tau$ for the foreign country and $M^H = M$.

Replacing the results from Lemma 2 into eq. (6), the entrepreneurs' welfare in $t = 2$ becomes

$$U^i(z^i, B^H, B^F; \pi) = M + \gamma(z^i)M^{\frac{1}{\nu}}, \tag{28}$$

with $\gamma(z^i) = \left(\frac{z^i}{2}\right)^{\frac{1}{\nu}+1} \left(\frac{1}{\nu}\right)^{\frac{1}{\nu}}$. The objective function of the government in country i is

$$V^i(z^i, B^H, B^F; \delta) = \Psi \left\{ M + \gamma(z^i)M^{\frac{1}{\nu}} \right\} + (1 - \Psi) \left\{ e + \alpha(z^i)M + T^i \right\}. \tag{29}$$

B Proof of Proposition 5

Uniqueness: The objective function of the government, eq. (29) derived in Appendix A , becomes

$$V^H(z^H, B^H, B^F; \pi) = \Psi \left\{ M + \gamma(z^H)M^{\frac{1}{\nu}} \right\} + (1 - \Psi) \left\{ e + \alpha(z^H)M - \delta B^H \right\}. \tag{30}$$

since $T^H = -\delta B^H$ and $T^F = -B^F$. This function is strictly concave in δ from the assumption that $\nu > 1$,

$$\frac{\partial V^{H^2}}{\partial^2 \delta} = \left(\frac{B^H}{2}\right)^2 \Psi \frac{\gamma(z^H)}{\nu} \left(\frac{1}{\nu} - 1\right) M^{\frac{1}{\nu}-2} < 0.$$

Therefore, if a solution to the unconstrained problem exists, then it is unique. We denote it by $\delta^u(z^H, B^H, B^F)$.

Existence and characterization: The FOC of the unconstrained problem satisfies

$$\Psi\left(1+\frac{\gamma(z^H)}{\nu}M^{\frac{1}{\nu}-1}\right)+(1-\Psi)\left(\alpha(z^H)-2\right)=0.$$

Using eq. (27) and simplifying,

$$\delta^u(z^H,B^H,B^F)=\left\{2P\left(z^H\right)-B^F\right\}\frac{1}{B^H} \tag{31}$$

where

$$P\left(z^H\right)=\left\{\left[\frac{1-\Psi}{\Psi}\left(2-\alpha(z^H)\right)-1\right]\frac{\nu}{\gamma(z^H)}\right\}^{\frac{\nu}{1-\nu}}. \tag{32}$$

The solution to this problem is well defined if $\frac{1-\Psi}{\Psi}\left(2-\alpha(z^H)\right)-1>0$. It is easy to see (after some manipulations) that assumptions (i) and (ii) in the Proposition provide sufficient conditions for this to be the case. To show that δ^u is increasing in z^H note that

$$\frac{\partial P\left(z^H\right)}{\partial z^H}=\frac{\nu^2 P^{\frac{2\nu-1}{\nu}}}{(1-\nu)}\left\{\left[\frac{1-\Psi}{\Psi}\left(2-\alpha(z^H)\right)-1\right]\left(-\frac{1}{\gamma^2}\frac{\partial\gamma}{\partial z^H}\right)+\frac{1-\Psi}{\Psi\gamma}\frac{\partial\alpha}{\partial z^H},\right\}>0$$

since $\frac{\partial\alpha}{\partial z^H}>0$ and $\frac{\partial\gamma}{\partial z^H}>0$. Hence,

$$\frac{\partial\delta^u(z^H,B^H,B^F)}{\partial z^H}=\frac{2}{B^H}\frac{\partial P\left(z^H\right)}{\partial z^H}>0.$$

That δ^u is decreasing in B^H and B^F follows from differentiating eq. (31) with respect to these variables. ■

C Proof of Proposition 6

Replacing $M^F=\frac{\delta^u B^H+B^F}{2}+\tau$ into eq. (12), the first order condition of the unconstrained problem with respect to τ becomes

$$\Psi\left(1+\frac{\gamma(z^F)}{\nu}(M^F)^{\frac{1}{\nu}-1}\right)+(1-\Psi)\left(\alpha(z^F)-1\right)=0.$$

Assumptions (i) and (ii) are necessary conditions for the equation to have a well-defined interior solution. It is easy to show, using the steps from Appendix B, that the objective function eq. (12) is strictly concave in τ (given the assumption that $\nu>1$). Hence, if the solution exists it is unique. We denote the constrained solution by τ^u.

Financial assets in the unconstrained case, M^{Fu}, satisfy

$$M^{Fu}(z^F) = \left\{ \left[\frac{1-\Psi}{\Psi} \left(1-\alpha(z^F)\right) - 1 \right] \frac{\nu}{\gamma(z^F)} \right\}^{\frac{\nu}{1-\nu}} .$$

Using eq. (31) computed above, we find that

$$\tau^u(z^F, z^H) = M^{Fu}(z^F) - P\left(z^H\right),$$

where $P\left(z^H\right)$ satisfies eq. (32). The solution to this problem is well defined if $\frac{1-\Psi}{\Psi}\left(1-\alpha(z^H)\right) - 1 > 0$, as this implies a positive level of financial assets. It is easy to see (after some manipulations) that the assumption in the Proposition provides sufficient conditions for this to be the case, establishing existence. That financial assets and bailouts and increasing in z^F follows from

$$\frac{\partial M^{Fu}(z^F)}{\partial z^F} = \frac{\nu^2 (M^{Fu})^{\frac{2\nu-1}{\nu}}}{(1-\nu)} \left\{ \left[\frac{1-\Psi}{\Psi} \left(1-\alpha(z^F)\right) - 1 \right] \left(-\frac{1}{\gamma^2} \frac{\partial \gamma}{\partial z^F} \right) + \frac{1-\Psi}{\Psi\gamma} \frac{\partial \alpha}{\partial z^F}, \right\} > 0$$

Note that

$$\left\{ \left[\frac{1-\Psi}{\Psi} \left(1-\alpha(z^F)\right) - 1 \right] \frac{\nu}{\gamma(z^F)} \right\}^{\frac{\nu}{1-\nu}} > \left\{ \left[\frac{1-\Psi}{\Psi} \left(2-\alpha(z^H)\right) - 1 \right] \frac{\nu}{\gamma(z^H)} \right\}^{\frac{\nu}{1-\nu}}$$

implies $M^{Fu}(z^F) > P\left(z^H\right) = M(z^H)$ when $z^F = z^H$ and the constraints are not binding. From the definition of $\tau^u(z^F, z^H)$, it follows that transfers are also increasing in z^F.

Finally, when $\Psi = \frac{1}{2}$, the default decision is constrained: $\tau = \frac{B^H}{2}\left(1 - \underline{\delta}\right)$ and $M^F = \frac{B^H + B^F}{2}$. This happens because the objective eq. (12) is strictly increasing. ■

D Proof of Proposition 7

The first order condition of Problem 16 delivers, after some manipulations, eq. (18) where

$$\frac{\partial \overline{V}^H(\mathbf{s}; \bar{\pi})}{\partial \delta} = \Psi \frac{B^H}{2} \left(1 + \frac{\gamma(z^H)}{\nu} M^{\frac{1}{\nu}-1} \right) + (1-\Psi) \left(\alpha(z^H) \frac{B^H}{2} - B^H \right), \quad \text{and}$$

$$\frac{\partial \overline{V}^F(\mathbf{s}; \bar{\pi})}{\partial \delta} = \Psi \frac{B^H}{2} \left(1 + \frac{\gamma(z^F)}{\nu} M^{\frac{1}{\nu}-1} \right) + (1-\Psi) \left(\alpha(z^F) \frac{B^H}{2} \right),$$

Assumptions (i) and (ii) ensure that an interior solution is feasible. Adding the two equations above and simplifying, we obtain the solution to the unconstrained problem:

$$\overline{\delta}^u(\mathbf{s}) = \left[2\bar{P}(\mathbf{s}) - B^F \right] \frac{1}{B^H}, \tag{33}$$

with $\bar{P}(\mathbf{s})$ satisfying eq. (34),

$$\bar{P}\left(\mathbf{s}\right)=\left\{\left[\left[2-\alpha(z^{H})-\alpha(z^{F})\right]\frac{1-\Psi}{\Psi}-2\right]\frac{\nu}{\gamma(z^{H})+\gamma(z^{F})}\right\}^{\frac{\nu}{1-\nu}}. \tag{34}$$

The unconstrained repayment rate is increasing in z^i, since

$$\frac{\partial\overline{\delta}^{u}(\mathbf{s})}{\partial z^{i}}=\frac{\nu^{2}\bar{P}^{\frac{2\nu-1}{\nu}}}{(1-\nu)[\gamma(z^{H})+\gamma(z^{H})]^{2}}\left\{\frac{\partial\alpha(z^{i})}{\partial z^{i}}\frac{1-\Psi}{\Psi}[\gamma(z^{H})+\gamma(z^{H})]+\right.$$
$$\left.\left[\left(2-\alpha(z^{H})-\alpha(z^{F})\right)\frac{1-\Psi}{\Psi}-2\right]\frac{\partial\gamma(z^{i})}{\partial z^{i}}\right\}>0$$

From eq.(33),

$$\frac{\partial\overline{\delta}^{u}(\mathbf{s})}{\partial B^{H}}=-\frac{\overline{\delta}^{u}(\mathbf{s})}{B^{H}}<0\quad\text{and}\quad\frac{\partial\overline{\delta}^{u}(\mathbf{s})}{\partial B^{F}}=-\frac{1}{B^{H}}<0.$$

Finally, $\overline{\delta}^{u}(\mathbf{s})>\underline{\delta}^{u}(\mathbf{s})$ follows from the fact that $\bar{P}(\mathbf{s})>P\left(z^{H}\right)$.

■

E Proof of Lemma 8

Entrepreneurs of country i choose the portfolio mix b^{Hi} and b^{Fi} in order to maximize Problem 21 taking prices $1/R^j$ for $j\in\{H,F\}$ and government policy $\{\delta(\mathbf{s}),B^H,B^F\}$ as given. Their first order conditions with respect to b^{ji} are

$$-\frac{1}{R^{j}}+\beta\mathbb{E}_{\mathbf{s}}\left\{\delta^{j}(\mathbf{s})+\gamma(z^{i})\frac{1}{\nu}\left(m^{i}\right)^{\frac{1}{\nu}-1}\delta^{j}(\mathbf{s})\right\}=0, \tag{35}$$

where i denotes the residency of the entrepreneur, j the origin of debt, $m^i=\delta(\mathbf{s})b^{Hi}+b^{Fi}$ are individual financial assets, $\delta^H=\underline{\delta}(\mathbf{s})$ debt repayment in the no-renegotiation case, $\delta^H=\bar{\delta}(\mathbf{s})$ the debt repayment rate with bailouts, and $\delta^F=1$. Because (i) the process determining z^i is identical across countries, (ii) the repayment rate of B^H is the same regardless of residency, and (iii) financial assets are internationally mobile, the FOC is the same for entrepreneurs in both countries. This implies that it will be optimal for them to choose $b^{HH}=b^{HF}$. Aggregating this across agents, $B^{HH}=B^{HF}$. Because in equilibrium $B^H=B^{HH}+B^{HF}$, then it must be the case that $B^{Hi}=\frac{B^H}{2}$. A similar reasoning delivers $B^{Fi}=\frac{B^F}{2}$. ■

F Proof of Lemma 9

Let $m^i=b^{Hi}\delta+b^{Fi}$. In the second period, the entrepreneur chooses l^i (given m^i and z^i) to maximize

$$\max_{l^{i}}\mathbb{E}_{\varepsilon}(\ln d^{i})\qquad\text{with}\qquad d^{i}=(A(z^{i},\varepsilon)-w^{i})l^{i}+m^{i}.$$

Given $\delta^F = 1$ and $\delta^H = \delta(\mathbf{s})$, the FOC is

$$\mathbb{E}_\varepsilon \left(\frac{A(z^i, \varepsilon) - w^i}{d^i} \right) = 0 \tag{36}$$

Guess $l^i = \phi^i m^i$. Under that guess, $d^i = ([A(z^i, \varepsilon) - w^i]\phi^i + 1)\, m^i$. Using the fact that m^i is independent of ε, we obtain

$$\mathbb{E}_\varepsilon \frac{A(z^i, \varepsilon) - w^i}{[A(z^i, \varepsilon) - w^i]\phi^i + 1} = 0, \tag{37}$$

which confirms the guess is correct given the definition of ϕ^i in the Lemma.

■

G Proof of Lemma 10

Given $M^i = \delta B^{Hi} + B^{Fi}$, the wage rate w^i (derived from the aggregate labor supply of workers) and the labor demand factor ϕ^i (derived from the FOC of entrepreneurs) are implicitly determined by the equations (37) and

$$w^i = \nu \left(\phi^i M^i\right)^{\nu - 1}, \tag{38}$$

From Lemma 9, the aggregate labor is given by $L^i = \phi^i M^i$. Replace eq. (38) into eq. (37) to obtain

$$\mathbb{E}_\varepsilon \frac{A(z^i, \varepsilon) - \nu\left(\phi^i M^i\right)^{\nu-1}}{1 + \left[A(z^i, \varepsilon) - \nu\left(\phi^i M^i\right)^{\nu-1}\right]\phi^i} = 0,$$

Denote the LHS of the equation above by $X(\phi^i, \delta)$. Using the implicit function theorem,

$$\frac{\partial \phi^i}{\partial \delta} = -\frac{\partial X/\partial \delta}{\partial X/\partial \phi^i} = -\frac{(\nu-1)w^i B^{Hi}}{M^i} \frac{\mathbb{E}_\varepsilon G^{-2}(A(z^i, \varepsilon) - w^i)^{-2}}{\mathbb{E}_\varepsilon G^{-2}\left[(A(z^i, \varepsilon) - w^i)^{-2}\frac{(\nu-1)w^i}{\phi^i} + 1\right]} < 0,$$

as $G \equiv \left[A(z^i, \varepsilon) - \nu\left(\phi^i M^i\right)^{\nu-1}\right]^{-1} + \phi^i > 0$. This establishes the first result.

Differentiate eq.(38) to obtain $\frac{\partial w^i}{\partial \delta}$. After some algebraic manipulations,

$$\frac{\partial w^i}{\partial \delta} = (\nu - 1)\frac{w^i B^{Hi}}{M^i}\left[\frac{M^i}{\phi^i}\frac{\partial \phi^i}{\partial M^i} + 1]\right]$$

where $\frac{M^i}{\phi^i}\frac{\partial \phi^i}{\partial M^i} \leq 0$ is the elasticity of the entrepreneurs' labor share ϕ^i with respect to M^i. We will show that wages are increasing in M^i by contradiction. Suppose that $\frac{\partial w^i}{\partial \delta} < 0$. Since $M^i \geq 0$ (by assumption), it must be the case that $\frac{M^i}{\phi^i}\frac{\partial \phi^i}{\partial M^i} < -1$. Alternatively,

$$\frac{\mathbb{E}_\varepsilon G^{-2}(A(z^i, \varepsilon) - w^i)^{-2} w^i(\nu - 1)}{\mathbb{E}_\varepsilon G^{-2}\left[(A(z^i, \varepsilon) - w^i)^{-2} w^i(\nu - 1) + \phi^i\right]} > 1.$$

But this would imply that $\mathbb{E}_\varepsilon G^{-2}\phi^i < 0$, a contradiction. Finally, using the fact that $L^i = \left(\frac{w^i}{\nu}\right)^{\frac{1}{\nu-1}}$, we can show that

$$\frac{\partial L^i}{\partial \delta} = \frac{L^i}{(\nu-1)w^i}\frac{\partial w^i}{\partial \delta} \geq 0.$$

■

H Proof of Lemma 11

Given $\delta^F = 1$ and $\delta^H = \delta(\mathbf{s})$, the entrepreneurs' maximization problem in the first period is

$$\max_{\{d_1^i, b^{Fi}, b^{Hi}\}} \ln d_1^i + \beta \mathbb{E}_{\mathbf{s}} \ln d_2^i.$$

$$d_1^i = a - \frac{b^{Hi}}{R^H} - \frac{b^{Fi}}{R^F}$$

$$d_2^i = [(A(z^i,\varepsilon) - w^i)\phi^i + 1]m^i,$$

Their FOC with respect to b^{ji} are

$$\frac{1}{d_1^i}\frac{1}{R^j} = \beta \mathbb{E}_{\mathbf{s}} \frac{\delta^j}{d_2^i} \tag{39}$$

From eq. (39) of $j = H$ and $j = F$ we obtain

$$R^H = \eta R^F \qquad \text{where} \qquad \eta = \frac{\mathbb{E}_{\mathbf{s}} \frac{1}{d_2^i}}{\mathbb{E}_{\mathbf{s}} \frac{\delta^H}{d_2^i}} \geq 1.$$

Guess the following

$$b^{Hi} = \theta^{Hi} R^H a, \quad \text{and} \quad b^{Fi} = \theta^{Fi} R^F a$$

Under that guess

$$d_1^i = a(1 - \theta^{Hi} - \theta^{Fi})$$

Moreover, $m^i = a(\theta^{Hi} R^H \delta^H + \theta^{Fi} R^F)$. Multiplying by ϕ^i and subtracting 1 from both sides of eq. (37), we get

$$\mathbb{E}_\varepsilon \frac{1}{[A(z^i,\varepsilon) - w^i]\phi^i + 1} = 1 \tag{40}$$

Replacing the guesses in eq. (39) for $j = F$

$$\frac{1}{1 - \theta^{Fi} - \theta^{Hi}} = \beta \mathbb{E}_{\mathbf{s}} \left\{ \frac{R^F}{\theta^{Hi} R^H \delta + \theta^{Fi} R^F} \mathbb{E}_\varepsilon \left[\frac{1}{[A(z^i,\varepsilon) - w^i]\phi^i + 1} \right] \right\}$$

From eq. 40, we know that given the aggregate state, the term involving $\mathbb{E}_\varepsilon$ is equal to 1. Using the fact that $R^H = \eta R^F$,

$$\frac{1}{1-\theta^{Fi}-\theta^{Hi}} = \beta \mathbb{E}_{\mathbf{s}} \left[\frac{1}{\theta^{Hi}\eta\delta^H + \theta^{Fi}} \right] \tag{41}$$

Replacing the guesses into eq. (39) for $j = H$, and following the same steps we obtain

$$\frac{1}{1-\theta^{Fi}-\theta^{Hi}} = \beta \mathbb{E}_{\mathbf{s}} \left[\frac{\delta^H \eta}{\theta^{Hi}\eta\delta^H + \theta^{Fi}} \right] \tag{42}$$

Multiply both sides of eq. (41) by θ^{Fi}, and both sides of eq. (42) by θ^{Hi}, and add the resulting expressions. This delivers, after some algebra,

$$\theta^{Hi}(1+\beta) = \beta - (1+\beta)\theta^{Fi} \tag{43}$$

We can replace eq. (43) into eq. (41) and obtain

$$\beta \mathbb{E}_{\mathbf{s}} \left[\frac{1}{\frac{\beta-(1+\beta)\theta^{Fi}}{1+\beta}\eta\delta^H + \theta^{Fi}} \right] = 1+\beta \tag{44}$$

This determines θ^{Fi} as a function of η. Notice that since η is not country specific, the equation implies that $\theta^{FH} = \theta^{FF} \equiv \theta^F$. From eq. (43), we get that $\theta^{HH} = \theta^{HF} \equiv \theta^H$ as well. Hence, the portfolio allocation is the same in both countries. ■

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