Bargaining over Mandatory Spending and Entitlements

Marina Azzimonti
Stony Brook University
and NBER

Laura Karpuska
Stony Brook University

Gabriel Mihalache
Stony Brook University

February 2020

Abstract

Do mandatory spending rules improve society's welfare? To answer this, we analyze an infinite-horizon dynamic political-economy model with two parties which disagree on how to split a fixed budget between public and private goods. We study the welfare implications of introducing two types of budget rules, *mandatory spending* on public goods and *entitlement programs*, the latter imposing constraints on the private goods' allocations that can be implemented. We model budget rules following the literature on legislative bargaining with an endogenous status quo. Under a mandatory spending rule on public goods, expenditures are governed by criteria determined by enacted law. In particular, previous year's spending bill is applied in the current year unless explicitly changed by a majority of policymakers. Entitlement programs, on the other hand, impose restrictions on the provision of private transfers through eligibility rules and generosity formulas that can only be modified with bi-partisan support. We find that entitlement programs induce over-provision of private goods and under-provision of public goods, whereas the opposite is true under a mandatory spending rule on public goods. We show that mandatory spending rules are typically associated with larger welfare gains than entitlement programs. The desirability of the rule, however, depends on the degree of political turnover: (i) with high enough political turnover, both budget rules are better than discretion, but (ii) entitlement programs can generate welfare losses when political persistence is large. This happens because entitlement rules actually increase the volatility of private and public consumption, and reduce public goods' provision significantly. Finally, we describe conditions under which budget rules would arise in a bargaining equilibrium.

Keywords: Dynamic legislative bargaining, endogenous status quo, political economy, mandatory spending, budget rules, fiscal rules, entitlements.

JEL Classification: C7, D6, E6

1 Introduction

Prior to the Great Depression, nearly all federal expenditures in the United States were discretionary. That is, spending did not occur in a given year unless Congress provided funding through an annual appropriations' bill. Following the Social Security Act of 1935, an increasing percentage of the federal budget became devoted to financing *mandatory spending programs*. Their key characteristic is that they need to be established under authorization laws and can only be changed with approval form a majority (or super-majority) of members of Congress. While there is a wide range of goods financed by the government which are mandatory, the two most important ones are the entitlement programs Social Security and Medicare. These social welfare programs have specific criteria set by Congress, such as eligibility requirements and benefit generosity, which require bi-partisan support to be modified. In this paper, we analyze the welfare implications of alternative mandatory spending programs in a dynamic infinite horizon political-economy model featuring disagreement over the distribution of resources.

We depart from a situation where two parties representing different constituencies must decide how to divide a fixed budget between (pure) public goods and private transfers. We do not restrict the set of instruments that policymakers have access to, and focus instead on the set of allocations that can be implemented in a political equilibrium with bargaining. Every period, a party is selected at random to make a budget proposal. The proposer makes a take-it-or-leave-it offer to the opposition. If the other party accepts the offer, then it is implemented. If the proposal is rejected, a status-quo default allocation is implemented instead. Mimicking the early decision-making process in the US government, absent budget rules (e.g. under discretion) little to no public spending occurs when a budget proposal is rejected. We point to two sources of Pareto inefficiency arising in this case. Statically, the proposer under-provides public goods relative to a planning problem solution. Dynamically, the equilibrium under discretionary spending exhibits too much volatility of consumption, as the proposer only provides private transfers to their constituents.

We then consider two alternative budget rules that, in order to capture mandatory spending programs, impose constraints on the choice of allocations and require bi-partisan support to be changed. The first one is mandatory spending on public goods, and establishes that last year's spending bill is applied in the current year unless explicitly changed by a majority of policymakers. In our model, this amounts to the determination of an endogenous status quo in the provision of public goods. We show that this rule induces over-provision of public goods relative to the first best: because both types of agents benefit from public goods, it is easier to find a compromise involving increases in its provision. Moreover, introducing this rule constitutes a Pareto improvement relative to discretion. Intuitively, the mandatory spending rule introduces inertia, which in turn reduces the volatility of private consumption. We will sometimes refer to this rule, simply, as *mandatory spending*, and under the understanding that it imposes constraints on the provision of public goods. The second type of budget rule considered is *entitlement programs*. In the United States, entitlements are a subset category of mandatory spending, which through restrictions on eligibility and generosity, impose constraints on private transfers. As with mandatory spending, modifications of the rules determining entitlements require bi-partisan support. In contrast, these targeted programs affect each type of agent's private consumption differentially. Because a proposer would like to redirect resources to their constituents, but not to the opposition, this program allows her to secure a good bargaining position in the future by over-providing private transfers. In the long run, entitlement programs divert resources away from public goods, enjoyed by a majority of the population, and are associated with larger fluctuations in both private and public goods. Because of this, introducing an entitlement program may generate welfare losses to some groups in society. Finally, we find that a proposer would always have incentives to introduce an entitlement rule because of the advantage associated with securing greater consumption of private goods for her constituents. The opposition party, on the other hand, may be worse off. Because of this, we also consider a 'reform stage' where parties must agree whether to introduce the rule or not. This one-time bargaining process must result in allocations that, at least, weakly increase the level of welfare attained by both parties. We show that political turnover is an important determinant of the welfare gains associated with the introduction of budget rules, with mandatory spending rules on public goods generating the largest gains.

Our paper makes two contributions to the existing literature. First, we model entitlements in a bargaining environment with concave utilities. This has important implications for the welfare gains associated to introducing budget rules because, in contrast to standard bargaining models with quasi-linear utility, risk-averse agents prefer smooth consumption profiles. Because we relaxed the linearity assumption, characterization of the symmetric Markov-perfect equilibrium in the infinite-horizon dynamic game requires a numerical approximation. Our second contribution is methodological, in that we propose a numerical method that can robustly compute the equilibrium for a wide range of parameters. Computation is complex, because under an entitlement rule, we have a multidimensional state space. Our method is inspired by Duggan and Kalandrakis (2012b), and uses advances in the quantitative macroeconomics literature, such as those in Gordon (2019).

A more detailed literature review can be found in the next section. Section 3 defines the economic environment and theoretically characterizes first-best allocations for arbitrary Pareto weights. In Section 4 we define a political equilibrium. We first analyze the case under discretion, or 'winner take all' solution in the spirit of Persson and Tabellini (1990), in Section 4.1. The bargaining protocol and the Markov-perfect equilibrium for a generic budget rule are described in Section 5. Section 6 characterizes a two-period model example to illustrate how these rules affect the equilibrium. The infinite-horizon dynamic model is solved quantitatively in Section 8 for a benchmark economy. Section 8.3 analyzes the robustness of our findings to different values of political turnover. Finally, Section 9 concludes.

2 Literature Review

Our paper contributes to the literature on the optimal provision of public goods in the presence of political fictions, e.g. Persson and Tabellini (2000)). As in Lizzeri and Persico (2001), we analyze how alternative institutional arrangements can improve on allocations obtained under 'winner-take-all' systems. While they focus on proportional systems, we consider mandatory spending rules instead.

We study the provision of public goods and private transfers in a legislative bargaining model similar to Baron and Ferejohn (1989), Battaglini and Coate (2008), or Battaglini and Coate (2007). In these papers, without mandatory spending rules, the bargaining process ends once an agreement is reached (see also Rubinstein (1982)). Our work follows, instead, an emerging strand of the literature on bargaining with an endogenous status quo, as in Epple and Riordan (1987) and Kalandrakis (2004). In this approach, a status quo allocation remains in place unless some political group proposes an alternative allocation that is acceptable for the opposition. This mechanism creates a dynamic, strategic link between the groups by impacting the trade-off current politicians have when choosing an allocation.

Mandatory spending programs in unidimensional policy spaces have been studied at least since Baron (1996), who analyzed the provision of a public good under a mandatory spending rule. While legislators act strategically, the median ideal point is still delivered in his paper because the analysis is performed in a unidimensional space. Kalandrakis (2004) studies a split-the-dollar game with three players and an endogenous reversion point. In his environment, the focus is on private transfers, but there is no public good provision. Other papers that make contributions on legislative bargaining with unidimensional policies are Baron and Ferejohn (1989), Baron and Kalai (1993), Kalandrakis (2004), Battaglini et al. (2012), Duggan and Kalandrakis (2012a), Dziuda and Loeper (2018), Dziuda and Loeper (2016), Anesi (2010), Anesi and Seidmann (2013), Anesi and Duggan (2018) Diermeier et al. (2017), Richter (2014), Piguillem and Riboni (2011), Baron and Bowen (2015), Bowen et al. (2017), Karakas (2017) and Grechyna (2017). Our main departure from these papers is that in our model government policies affect both, public and private goods. That is, that we work with a multidimensional policy space.

The closest paper to ours is Bowen et al. (2014), who analyze the welfare implications of mandatory spending rules on public goods in a multidimensional policy space. Their main finding is that a mandatory spending rule can restore Pareto efficiency. A key underlying assumption in their model is the linearity in the utility of private goods. Because of linearity, fluctuations necessary to deliver the Samuelson level of public goods arising in the bargaining solution are inconsequential. We relax this assumption by considering concave utility functions, and show that: (i) mandatory spending rules on public goods do not restore Pareto efficiency (they do involve Pareto improvements, though) and (ii) entitlement rules, which impose constraints on private transfers, have different welfare implications, that the type of mandatory spending rule matters. This finding is relevant because the largest mandatory spending programs in the United States are entitlements (e.g. budget rules on private transfers, rather on public goods). Concavity is important for these results because, in our model, agents prefer smooth consumption profiles in public as well as private goods.

Our paper is also related to the literature studying the effects of power alternation on government policy, which includes Persson and Svensson (1989), Alesina and Tabellini (1990), or Azzimonti (2011). These papers emphasize that political turnover introduces inefficiencies in a political equilibrium. We contribute to this literature by considering alternative budgetary rules that can improve on the allocations in a model with legislative bargaining. [1]

The discussion of rules versus discretion in fiscal policy, such as in Amador et al. (2006), is also salient to our results. For example, Halac and Yared (2014) study the optimal level of discretion in fiscal policy when the economy faces persistent shocks. They show that when shocks are not i.i.d., an ex-ante optimal fiscal rule can create incentives for governments to accumulate maximal debt, becoming immiserated. The literature has also focused on the welfare implications of balanced budget rules, as in Azzimonti et al. (2016), in environments without mandatory spending rules but where borrowing is allowed instead. We depart from these papers by considering bargaining over an endogenous status quo, but restricting the government ability to issue debt. Allowing for sovereign debt would be an interesting extension to our work. An excellent summary of the recent literature on budget rules in economies with 'winner take all' political systems can be found in Yared (2019).

3 Environment

Consider a discrete-time infinite horizon economy populated by two types of agents of equal measure, A and B. They value private goods c and public goods g according to a standard instan-

[1] Another way to study the implications of political fluctuations instead of legislative bargaining was also introduced by Chatterjee and Eyigungor (2020). They add an adjustment cost for changes in the legislation to study "incumbency disadvantage".

taneous utility $u(c, g)$. We assume that preferences are additively separable and logarithmic,

$$u(c, g) = \ln(c) + \ln(g). \tag{1}$$

The government has a fixed budget Y every period which can be divided between the public good g_t and the amount of private goods consumed by each agent, $c_{A,t}$ and $c_{B,t}$. This specification presupposes that the government has access to a complete set of policy instruments, so that any desired allocation can be decentralized through a set of lump-sum taxes or transfers. Thus, the focus is not on the distortionary effects caused by an incomplete set of policy instruments (such as distortionary taxes), but on the effects of political disagreement. The government budget constraint is

$$c_{A,t} + c_{B,t} + g_t = Y. \tag{2}$$

In addition, the government is subject to the following constraints

$$c_{it} \geq \overline{x}_c \quad \text{and} \quad g_t \geq \overline{x}_g \quad \forall t. \tag{3}$$

The constants $\overline{x}_c$ and $\overline{x}_g$ represent bounds on private and public consumption that cannot be changed by policymakers. These capture constitutional constraints on the minimum levels of private consumption, as well as a lower bound on the provision of public goods and services which cannot be affected by budgeting decisions. Technically, $\overline{x}_c$ and $\overline{x}_g$ ensure that welfare is well-defined when a policymaker chooses not to allocate resources to agents in one of the groups.

Before describing the political environment, it is useful to characterize the set of Pareto optimal allocations, to be used as a benchmark.

First-Best Allocations

A benevolent planner chooses sequences of private and public consumption in order to maximize a weighted sum of the lifetime utility of agents,

$$\max_{\{c_{A,t}, c_{B,t}, g_t\}_t} \sum_{t=0}^{\infty} \beta^t \Big\{ (1-\lambda) u(c_{A,t}, g_t) + \lambda u(c_{B,t}, g_t) \Big\},$$

subject to the resource constraint, eq. (2) and the lower bounds on allocations, eq. (3). The parameter $\lambda \in [0, 1]$ denotes the Pareto-weight of type-B agents. The solution is characterized in Proposition 1.

Proposition 1 *Suppose that $\bar{x}_c = \bar{x}_g$. Let $\lambda_1 = \frac{2\bar{x}_c}{Y}$ and $\lambda_2 = \frac{Y - 2\bar{x}_c}{Y}$. The Pareto efficient allocations satisfy*

$$g_t^* = \begin{cases} \frac{Y - \bar{x}_c}{2 - \lambda}, & \text{if} \quad \lambda \in [0, \lambda_1] \\ \frac{Y}{2}, & \text{if} \quad \lambda \in [\lambda_1, \lambda_2] \\ \frac{Y - \bar{x}_c}{1 + \lambda}, & \text{if} \quad \lambda \in [\lambda_2, 1]. \end{cases}$$

$$c_{B,t}^* = \begin{cases} \bar{x}_c, & \text{if} \quad \lambda \in [0, \lambda_1] \\ \frac{\lambda Y}{2}, & \text{if} \quad \lambda \in [\lambda_1, \lambda_2] \\ \frac{(Y - \bar{x}_c)\lambda}{1 + \lambda}, & \text{if} \quad \lambda \in [\lambda_2, 1]. \end{cases}$$

$$c_{A,t}^* = Y - c_{B,t}^* - g_t^*.$$

Proof. See Appendix 10.1. □

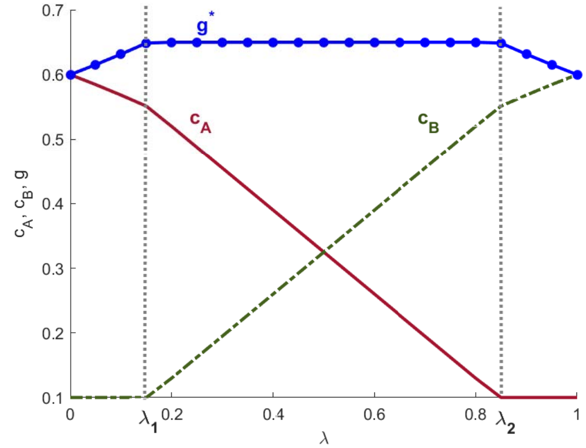


Figure 1: Pareto Optimal allocations. Parameters: $Y = 1.3$ and $\overline{x}_c = 0.1$.

Because there is a fixed endowment and no uncertainty, the solution is time-invarient.

Figure 1 depicts optimal levels of consumption of private and public goods as functions of λ for a numerical example. When $\lambda < \lambda_1$, the planner gives so little weight to type-B agents that the minimum consumption constraint binds, implying $c_B = \bar{x}_c$. In the case of $\lambda = 0$, the remaining of the budget is split equally between g and c_A. As λ raises, the planner increases the consumption of the pure public good at the expense of agent A's consumption. When λ is higher than λ_1 the constraint on c_B stops binding, implying that it is optimal to provide agent B with a level of consumption above $\bar{x}_c$. For intermediate values of λ, as the weight on B rises, the planner chooses to increase B's consumption while decreasing A's consumption, but keeping the provision of the public good constant at g^*. For large values of λ (e.g. above λ_2), no further reductions on c_A are feasible as the minimum-consumption constraint starts binding. Given that the utility function of agents are identical, the private allocations are symmetric.

Corollary 1 *For any Pareto Efficient allocation with $c_A, c_B > \bar{x}_c$, $g^* = \frac{Y}{2}$ is the unique Samuelson level of public good provision.*

The corollary states that when the lower bounds on consumption are not binding, the provision of public goods satisfies the 'Samuelson rule.' The Samuelson rule requires that the social marginal benefit of providing the public good (e.g. the sum of private marginal benefits) equals its social marginal cost. In the literature, this is typically referred to as the *efficient level* of the public good (see Bowen et al. (2014)).

4 Political Equilibrium

Because members of the two groups disagree on how to split resources, political parties will naturally arise in this environment. We assume that there are two parties: A and B representing agents of each group. Allocations are chosen by a representative of one of two groups, like in citizen candidates model (see Osborne and Slivinski (1996) or Besley and Coate (1997)). The identity of the active ruler is immaterial as individuals are homogenous within a group. Each period, an incumbent chooses current-period allocations for public and private goods in order to maximize the utility of her constituency knowing that she will be re-elected with (exogenous) probability p.

We focus on Stationary Markov Perfect Equilibria, referred here as MPE. A MPE is a Subgame Perfect Equilbria (SPE) in which strategies are restricted to be stationary Markovian. A strategy profile is stationary Markovian if, for any two ex post histories that terminate in the same state, the strategies that follow are the same.[2] This restriction is not without loss of generality. After all, stationary Markovian strategies ignore all details of a history of plays. Much of the applied work of dynamic games has focused on the use of stationary Markovian strategies not only for their simplicity, but also for their prediction power as Folk-like results can be prevented. Moreover, it seems reasonable in a variety of settings to require strategies to rely only on relevant information at the time of the play. In fact, if the dynamic interaction of politicians is seen as an infinite game played by a sequence of legislators who face uncertainty their reelection, the restrictions of Markovian strategies is justifiable. See Bhaskar et al. (2013) for a discussion of the restriction to Markov strategies.

Before discussing the political equilibrium with budget rules, it is enlightening to characterize the solution under discretion, where parties alternate in power stochastically.

4.1 Policy under Discretion

This environment, commonly used in the literature (see Persson and Tabellini (2000), among others), is typically referred to as the 'winner takes all' election model. Absent budget rules, the policymaker does not need the other party's approval to implement a given policy.

Since power alternation follows a Markov-process, it is easier to write the problem of an incumbent recursively. Moreover, since parties are completely symmetric, we can focus on a Symmetric MPE where decisions depend only on whether a given party is in power or not, but do not depend on the identity of the party. Let V_i^D denote the value function of an incumbent type $i \in \{A, B\}$ when in power and W_i^D when out of power. Then,

$$V_i^D = \max_{\{c_i, c_j, g\}} \left\{ \ln(c_i) + \ln(g) + \beta[pV_i^D + (1-p)W_i^D] \right\}, \tag{4}$$

subject to equations (2) and (3).

Because the endowment is fixed and the government is subject to a balanced budget, there is no dynamic state variable in this economy. Therefore, the problem of an incumbent choosing policies under discretion is static.[3]

Proposition 2 *Under discretion, the allocations chosen by incumbent type i satisfy*

$$g^D = \frac{Y - \bar{x}_c}{2}, \quad c_i^D = g^D, \quad \textit{and} \quad c_j^D = \bar{x}_c \quad \textit{for } j \neq i.$$

Proof. Maximize eq. (4) subject to eqs (2) and (3), and simplify. □

The policymaker expropriates the other group as much as possible, providing them with the minimum feasible level of consumption when in power, $c_j^D = \bar{x}_c$. The remaining of the budget is split evenly between the public good and the consumption of group i. This results follows from the assumption that g and c have the same weight (and curvature) in the utility function of agents. Note that while the two parties disagree on the composition of spending, they both choose the same provision of public goods, $g^D = \frac{Y - \bar{x}_c}{2}$.

[2]For a mode precise definition of stationary Markovian strategies see Mailath and Samuelson (2006).

[3]Note that there is no need to specify W at this stage because current allocations do not affect future choices.

4.2 Political Inefficiencies

Suppose that party B is the incumbent this period. Figure 2 shows B's consumption and the provision of public goods for two cases: (i) under discretion and (ii) the first-best, for different levels of λ. Statically, the choice under discretion delivers the same outcome as the one chosen by a planner who assigns a weight of 1 to group B (and zero to the other group). This can be seen in the figure because $c_B^D = c_B^*$ and $g^D = g^*$ at the right extreme of the graph, when $\lambda = 1$.

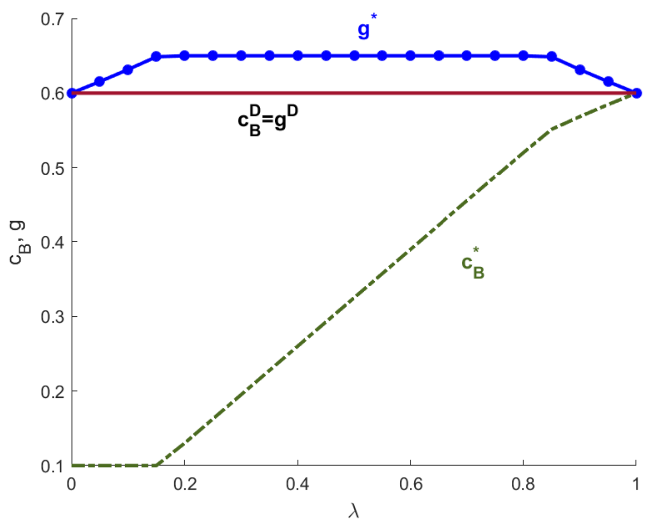


Figure 2: Pareto Optimal Allocations vs Discretion. Parameters: $Y = 1.3$ and $\overline{x}_c = 0.1$.

However, to the extent that the planner assigns positive weight to both agents, namely $\lambda \in (0, 1)$, the solution under discretion exhibits under-provision of public goods and over-provision of private goods,

$$g^D < g^* \quad \text{and} \quad c^D > c^*$$

where $c^D = c_B^D + c_A^D$ and $c^* = c_B^* + c_A^*$. The solution under discretion never satisfies the Samuelson rule. This happens because the incumbent equates the marginal cost of public goods to her private marginal benefit, whereas the planner would equate it to the social marginal benefit. In other words, incumbent i ignores the welfare gains to group j of providing g and this results in over-provision of private goods and under-provision of public goods, regardless of the identity of the incumbent.

Dynamically, the solution is Pareto inefficient under political turnover even when $\lambda = 1$. This is shown in the next Corollary.

Corollary 2 *For any $p \in (0, 1)$, the allocations under discretion are Pareto inefficient.*

Proof. Under discretion, private consumption allocations fluctuate between $c_i = \frac{Y - \bar{x}}{2}$ and $c_j = \bar{x}$, whereas they are constant under the planner (even for cases where $\lambda = 1$ or $\lambda = 0$). $\square$

With political turnover, the levels of consumption enjoyed by each group fluctuate between $\frac{Y - \bar{x}_c}{2}$ and $\bar{x}_c$ depending on the identity of the incumbent. Curvature in the utility function of agents implies that they would prefer a smooth consumption sequence to a volatile one. The planner redistributes resources, but keeps allocations constant over time. In a political equilibrium, the

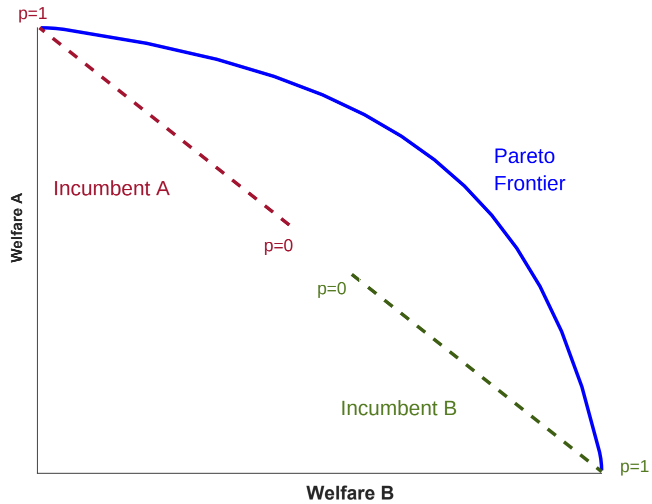


Figure 3: Pareto Frontier vs Solution under Discretion. Parameters: $Y = 1.3$ and $\overline{x}_c = 0.1$.

volatility induced by power alternation reduces lifetime utility, and it is a dynamic source of inefficiency in this model.

Figure 3 shows the pairs of lifetime utilities attained by each type of agent in the first-best and under discretion. The horizontal axis depicts the welfare of type B agents, whereas the vertical axis measures the lifetime welfare of the other group. The solid line represents the Pareto Frontier, which is obtained by evaluating the Pareto efficient allocations characterized in Proposition 1 into the lifetime utility of each type of agent for different values of λ. The upper-left value corresponds to $\lambda = 0$. The dashed brown line represents welfare combinations for the solution under discretion at different values of p, assuming that A is the incumbent. That is, each point in the line is a pair (W_B^D, V_A^D) for a specific value of p. The point at which the welfare under discretion is identical to the one under a planner is that where $p = 1$ (e.g. under no power alternation). As we decrease the probability of re-election of the incumbent, welfare moves further inside the Pareto Frontier. The dashed green line corresponds to the case where B is the incumbent; as p increases, welfare for that group rises. This case corresponds to pairs (V_B^D, W_A^D) for different p values.

A key message from this analysis is that there are two sources of Pareto inefficiency under discretion. The first one is a static inefficiency which arises because, to the extent that $\lambda \in (0, 1)$, the incumbent under-provides public goods relative to the planner. The second one is dynamic and arises because private consumption fluctuates with the identity of the incumbent, relative to a constant value under a planner.

Budget rules that alleviate the first source of inefficiency have been studied previously, e.g. Bowen et al. (2014): a proposed way to improve on the allocation is to introduce mandatory spending on public goods g. The second source of inefficiency (e.g. fluctuations in private consumption) is well understood in the political economy literature but less so in the mandatory spending literature, as preferences are typically assumed to be linear in private consumption in models where public goods are also valued. The reason being that linearity in consumption simplifies the bargaining problem. Under concavity, it is also relevant to consider how mandatory spending on private goods, or *entitlements*, affect welfare. In what follows, we will consider two types of budget rules: mandatory spending and entitlements, and analyze under which assumptions one takes us closer to the Pareto frontier.

5 Political Equilibrium with Budget Rules

In this section, we introduce budget rules, which are constraints on the allocations that can be implemented. In contrast to the previous section, where the incumbent was a dictator within the period, government policies can no longer be chosen unilaterally by the incumbent. Any policy proposal needs the approval of the other party. If such approval is not obtained, a pre-determined allocation is implemented. This decision-making process mimics bargaining over the government budget in legislatures.

We consider two types of budget rules. The first one establishes constraints on the choice of the pure public good g, and will be referred to as *mandatory spending*. Under this rule, expenditure on g is governed by criteria determined by enacted law. This implies that last year's spending bill is applied in the current year, unless explicitly changed by a majority of policymakers. With only two parties, this is equivalent to an unanimity rule. The second type of budget rule considered is *entitlement programs*. These impose restrictions on the provision of private goods. Examples of entitlement programs are Social Security and Medicare. As with mandatory spending, the formulas that currently determine entitlements can only be modified with bi-partisan support.

5.1 Bargaining Protocol under Budget Rules

The relevant state variable in this economy is the triplet $\mathbf{s} = \{\bar{c}_A, \bar{c}_B, \bar{g}\}$ determined by the allocations of private and public goods consumed last period. We can model enacted law as a function $\xi(\mathbf{s}, r)$ that maps which component of current spending—if any—needs bi-partisan support to be changed when the budget rule is of type r. Under discretion, $\xi(\mathbf{s}, d) = \{\bar{x}_c, \bar{x}_c, \bar{x}_g\}$, so there is no mapping from previous period spending into current allocations. The only restriction is that the allocation respects the lower bounds determined in eq. (3). Under a mandatory spending rule, $\xi(\mathbf{s}, m) = \{\bar{x}_c, \bar{x}_c, \bar{g}\}$. This implies that the allocation of c_A and c_B is discretionary (as long as it is above the minimum level $\bar{x}_c$), but spending on g is not. Note that any feasible $\bar{g}$ must satisfy $\bar{g} \geq \bar{x}_g$ so the rule ξ is de-facto imposing a restriction in the choice set of the policymaker when it holds with strict inequality. Under an entitlement spending rule, $\xi(\mathbf{s}, e) = \{\bar{c}_A, \bar{c}_B, \bar{x}_g\}$. Hence, while the provision of public goods is discretionary (over and above the lower bound $\bar{x}_g$), changes in c_A and c_B from the amount chosen last period need the approval of both parties.

To fix ideas, suppose that the incumbent is of type B. At the outset of this period, incumbent B proposes an allocation of the government budget Y denoted by $\boldsymbol{x}_B = \{c_A, c_B, g\}$ such that eqs. (2) and (3) hold. Party A can accept or reject the proposal, $d_A(\mathbf{s}, r) \in \{0, 1\}$, where $d_A(\mathbf{s}, r) = 1$ denotes that the proposal has been accepted. The tie-breaking rule favors any proposed allocation, i.e., if the respondent is indifferent between the status quo and a new proposed policy, we assume that the respondent accepts. If the proposal is accepted, the proposed allocation is implemented and becomes the new state, $\mathbf{s}' = \mathbf{x}_B$. If the proposal is rejected, the status quo allocation determined in $\mathbf{s}$ is implemented according to the budget rule, and next period's state is simply $\mathbf{s}' = \mathbf{s}$. The probability of retaining proposal power is determined by a symmetric Markov-chain, with p denoting the probability of being the proposer next period. Under discretion, the problem is identical to the one studied in Section 4.1. In particular, the decision of any policymaker is completely static. With budget rules, on the other hand, there is a dynamic component as the outcome of the bargaining game becomes the endogenous status quo for next period.

5.2 Markov Perfect Equilibrium

Given a budget rule r, equilibrium policy functions chosen by incumbent type B will be denoted by $\mathcal{G}_B(\mathbf{s}, r)$ for public goods, $\mathcal{C}_{B,B}(\mathbf{s}, r)$ for the consumption of proposer B, and $\mathcal{C}_{A,B}(\mathbf{s}, r)$ for private consumption of the opposition party A. The associated continuation utilities are $V_B(\mathbf{s}', r)$ if the incumbent remains a proposer next period and $W_B(\mathbf{s}', r)$ if out of power.

The problem of the proposer can be written as

$$\max_{\boldsymbol{x}_B=\{c_A, c_B, g\}} \ln(c_B) + \ln(g) + \beta\Big\{pV_B(\boldsymbol{x}_B, r) + (1-p)W_B(\boldsymbol{x}_B, r)\Big\} \tag{5}$$

where we used the fact that, if the proposal is accepted, $\mathbf{s}' = \boldsymbol{x}_B$. The constraints are eqs. (2) and (3), and the *acceptance constraint*

$$\ln(c_A) + \ln(g) + \beta\Big\{(1-p)V_A(\boldsymbol{x}_B, r) + pW_A(\boldsymbol{x}_B, r)\Big\} \geq K_A\Big(\xi(\mathbf{s}, r)\Big) \tag{6}$$

where $K_A\Big(\xi(\mathbf{s}, r)\Big)$ denotes the dynamic payoff to the opposition party when the proposal is rejected; that is, the payoff from keeping the status quo $\mathbf{s} = \{\bar{c}_A, \bar{c}_B, \bar{g}\}$ given the budget rule r. Under a mandatory spending rule, we have that

$$K_A\Big(\xi(\mathbf{s}, m)\Big) = \ln(\bar{x}_c) + \ln(\bar{g}) + \beta\Big\{(1-p)V_A(\mathbf{s}, m) + pW_A(\mathbf{s}, m)\Big\}, \tag{7}$$

whereas under an entitlement rule $r = e$, we have

$$K_A\Big(\xi(\boldsymbol{s}, e)\Big) = \ln(\bar{c}_A) + \ln(\bar{x}_g) + \beta\Big\{(1-p)V_A(\boldsymbol{s}, e) + pW_A(\boldsymbol{s}, e)\Big\}. \tag{8}$$

The acceptance constraint ensures that the proposal is accepted if and only if the payoff from the proposal is weakly higher than the payoff under the status quo $\boldsymbol{s}$, for a given budget rule r. The expression makes it clear that the budget rule defines a lower bound for the welfare level attained by party A. While the two expressions above seem almost identical, their key difference is that in one case the rule constrains decisions through a public good while in the other it constrains choices through a private good. Because a given amount of the public good is enjoyed by both parties every period (regardless of which one is in power), whereas private goods are not, the effect of mandatory spending rules and entitlement rules will be markedly different. For example, consider a situation where the status quo under an entitlement rule is such that $\bar{c}_A$ is extremely high. Incumbent B is constrained to spend a significant part of the budget on a good that she does not enjoy. Her only way to induce the opposition to accept a reduction in A is by increasing the provision of the public good g. Given that total resources are limited, this means reducing c_B. If, on the other hand, the status quo under a mandatory spending rule is such that $\bar{g}$ is high, reducing g may be 'cheaper' because it would only require a small increase in c_A. Recall that under a mandatory spending rule, there is no pre-determined level of private consumption, so A's default private consumption level, if the proposal is rejected is given by the lower bound $\bar{x}_c$. The difference between the two budget rules will become clearer in the two-period example studied in Section 6.

Finally, we have that in the MPE, the value function of a type-B incumbent satisfies

$$V_B(\mathbf{s}, r) = \ln\left(\mathcal{C}_{B,B}(\mathbf{s}, r)\right) + \ln\left(\mathcal{G}_B(\mathbf{s}, r)\right) + \beta\Big\{pV_B\Big(\Pi_B(\mathbf{s}, r), r\Big) + (1-p)W_B\Big(\Pi_B(\mathbf{s}, r), r\Big)\Big\} \tag{9}$$

with next period's status quo given by today's equilibrium choices by incumbent B, namely $\Pi_B(\mathbf{s},r) = \{\mathcal{C}_{A,B}(\mathbf{s},r), \mathcal{C}_{B,B}(\mathbf{s},r), \mathcal{G}_B(\mathbf{s},r)\}$. The value function of type B when out of power satisfies

$$W_B(\mathbf{s},r) = \ln\left(\mathcal{C}_{B,A}(\mathbf{s},r)\right) + \ln\left(\mathcal{G}_A(\mathbf{s},r)\right) + \beta\Big\{(1-p)V_B\Big(\Pi_A(\mathbf{s},r),r\Big) + pW_B\Big(\Pi_A(\mathbf{s},r),r\Big)\Big\} \quad (10)$$

as policies are chosen by party A in such case, with $\Pi_A(\mathbf{s},r) = \{\mathcal{C}_{A,A}(\mathbf{s},r), \mathcal{C}_{B,A}(\mathbf{s},r), \mathcal{G}_A(\mathbf{s},r)\}$. We can now formally define the Markov perfect equilibrium for the bargaining game.

Definition 1. *A MPE is a set of value functions* $\{V_B(\mathbf{s},r), V_A(\mathbf{s},r), W_B(\mathbf{s},r),\ W_A(\mathbf{s},r)\}$, *policy functions* $\Pi_B(\mathbf{s},r) = \Big\{\mathcal{C}_{A,B}(\mathbf{s},r),\ \mathcal{C}_{B,B}(\mathbf{s},r), \mathcal{G}_B(\mathbf{s},r)\Big\}$, *and* $\Pi_A(\mathbf{s},r) = \Big\{\mathcal{C}_{A,A}(\mathbf{s},r),\ \mathcal{C}_{A,B}(\mathbf{s},r), \mathcal{G}_A(\mathbf{s},r)\Big\}$ *and acceptance rules* $d_B(\mathbf{s},r)$ *and* $d_A(\mathbf{s},r)$, *such that*

- *Proposer B chooses allocations* $\boldsymbol{x}_B = \{c_A, c_B, g\}$ *to maximize eq. (5) subject to the budget constraint eq. (2), the feasibility constraints eq. (3) and the acceptance constraint, eq. (11). Given the value functions* $V_B(\mathbf{s},r)$ *and* $W_B(\mathbf{s},r)$, *the acceptance decision* $d_A(\mathbf{s},r)$, *and the rules chosen by party A if in power* $\Pi_A(\mathbf{s},r)$, *these define the policy functions* $\Pi_B(\mathbf{s},r) = \Big\{\mathcal{C}_{A,B}(\mathbf{s},r), \mathcal{C}_{B,B}(\mathbf{s},r), \mathcal{G}_B(\mathbf{s},r)\Big\}$. *Policies* $\Pi_A(\mathbf{s},r)$ *are analogously defined.*

- *Given the policy functions* $\Pi_A(\mathbf{s},r)$ *and* $\Pi_B(\mathbf{s},r)$, *the value functions* $V_B(\mathbf{s},r)$ *and* $W_B(\mathbf{s},r)$ *satisfy equations (9) and (10), respectively. The value functions* V_A *and* W_A *are analogously defined.*

- *Given* $V_B(\mathbf{s},r)$ *and* $W_B(\mathbf{s},r)$, *for any proposal* $\boldsymbol{x}_B$ *and status quo* $\mathbf{s}$, *the acceptance strategy* $d_A(\mathbf{s},r) = 1$ *if and only if:*

$$\ln(c_A) + \ln(g) + \beta\Big\{(1-p)V_A(\boldsymbol{x}_B,r) + pW_A(\boldsymbol{x}_B,r)\Big\} \geq K_A\Big(\xi(\mathbf{s},r)\Big)$$

  with $K_A\Big(\xi(\mathbf{s},r)\Big)$ *defined by eq. (7) under* $r = m$ *and by eq. (8) when* $r = m$. *The acceptance rule* $d_B(\mathbf{s},r)$ *is analogously defined.*

The first condition states that policy rules are the ones that solve the problem of the proposer, given continuation utilities and an acceptance rule for the opposition party. The second condition defines value functions as a fixed point using policy functions under an accepted proposal. The last condition determines that the opposition party accepts the proposal whenever its welfare is at least as large as under the staus quo. Value functions, policy functions, and decision rules depend on the relevant state variables (e.g. the levels of past consumption of private and public goods) and the type of rule in place. The rule affects the acceptance constraint directly through the function $K_A\Big(\xi(\mathbf{s},r)\Big)$. Because this sometimes constrains the proposer (on what policies will be accepted by the opposition), rules affect all the functions determined in the MPE.

6 Two-Period Model

Before characterizing the infinite-horizon version of the model, it is illustrative to consider a two-period version of the model. Given that this is a two-person, two-period, complete information extensive form game, we can focus on its unique subgame perfect equilibrium (SPE). The second-period strategies don't depend on histories except for the status quo. The problem can be solved backwards. We start with the full characterization of the second-period optimum strategies of party i and show they are statically optimal (e.g. they are the same as the first-best allocations) under both, mandatory spending and entitlement rules. The infinite-horizon dynamic model is specified in Section 8.

6.1 Second-Period Characterization.

The second-period incumbent takes the status quo $\mathbf{s}_2 = \{\bar{c}_{A,1}, \bar{c}_{B,1}, \bar{g}_1\}$ as given. Because the economy ends this period, there is no continuation utility. The analysis allows us to understand how the status quo affects the choice set and the relative bargaining power of the two groups, while ignoring the dynamic consequences of this choice.

Incumbent B proposes $\boldsymbol{x}_{B,2} = \{c_{A,2}, c_{B,2}, g_2\}$, given status quo $\boldsymbol{s}_2$ and the rule r in order to maximize its static payoff,

$$\max_{\boldsymbol{x}_{B,2}} \ln(c_{B,2}) + \ln(g_2) \quad \text{s.t.}$$

$$\ln(c_{A,2}) + \ln(g_2) \geq K_{A,2}\Big(\xi(\boldsymbol{s}_2, r)\Big), \tag{11}$$

$$\text{and eqs (2) and (3),}$$

where $K_{A,2}$ denotes the value of utility for party A if the proposal is rejected given budget rule type r. The constraint ensures that the proposal is accepted if and only if the payoff from the proposal is higher than the payoff under the status quo $\boldsymbol{s}_2$. Note that in the last period, $K_{A,2}\Big(\xi(\boldsymbol{s}_2, m)\Big) = \ln(\bar{x}_c) + \ln(\bar{g}_1)$ under mandatory spending, and $K_{A,2}\Big(\xi(\boldsymbol{s}_2, e)\Big) = \ln(\bar{c}_{A,1}) + \ln(\bar{x}_g)$ under an entitlement rule. In the analysis that follows, we will assume that $\bar{x}_c = \bar{x}_g \equiv \bar{x}$.

Proposition 3 *Define $s_L = \frac{Y-\bar{x}}{2}$ and $s_H = \frac{Y}{2}$. In the last period, the unique equilibrium proposal for incumbent B satisfies:*

$$\mathcal{G}_{B,2}(\boldsymbol{s}_2) = \begin{cases} \frac{Y-\bar{x}}{2}, & \text{if} \quad s < s_L \\ s, & \text{if} \quad s \in [s_L, s_H) \\ \frac{Y}{2}, & \text{if} \quad s \geq s_H. \end{cases}$$

$$\mathcal{C}_{B,B,2}(\boldsymbol{s}_2) = \begin{cases} \frac{Y-\bar{x}}{2}, & \text{if} \quad s < s_L \\ Y - s - \bar{x}, & \text{if} \quad s \in [s_L, s_H) \\ \frac{Y^2-4\bar{x}s}{2Y}, & \text{if} \quad s \geq s_H. \end{cases}$$

$$\mathcal{C}_{A,B,2}(\boldsymbol{s}_2) = Y - \mathcal{C}_{B,B,2}(\boldsymbol{s}_2) - \mathcal{G}_{B,2}(\boldsymbol{s}_2),$$

where $s = \bar{g}_1$ under mandatory spending and $s = \bar{c}_{A,1}$ under entitlements.

Proof. See Appendix 10.2. □

Figure 4 illustrates the allocations in an example where the incumbent is of type B. When $s < s_L$, the proposer solves an unconstrained problem and simply equates her marginal utility of private consumption with her (private) marginal utility of public consumption. This happens because $s_L = g^D = c^D$ is the optimum unconstrained level of provision of private and public goods under discretion. When $s_2 \in [s_L, s_H)$, the unconstrained allocations are no longer enough to guarantee the opposition's minimum welfare under the status quo, implying that constraint (11) becomes binding. In order to induce the opposition to accept a proposal, incumbent B needs to either provide more entitlements (e.g. increase $c_{A,2}$) or more public goods. Given that B also enjoys consuming public goods, it is best to rise g, as we see by the fact that $\mathcal{C}_{A,B,2}$ (the solid line in the figure) remains at the lower bound whereas $\mathcal{G}_{B,2}$ (the solid line with circles) rises. Once the

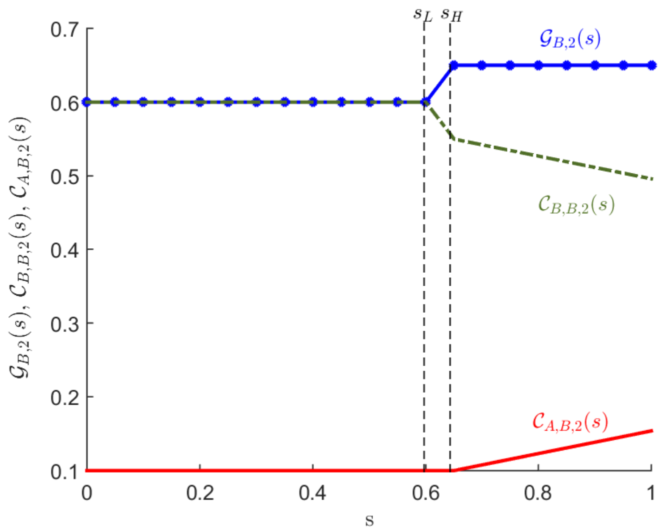


Figure 4: Second period allocation under incumbent B. Parameters: $Y = 1.3$ and $\overline{x} = 0.1$.

status quo is large enough, $s \geq s_H = g^*$, party B starts providing private goods to the opposition (dotted line), rather than increasing the public good.

A few remarks are warranted. First, the allocations do not depend on the type of the budget rule in a one-shot game. This results from the homogeneity of preferences over private and public goods, and the fact that $\bar{x}_c = \bar{x}_g$. Mathematically, for a status quo equal to s, the acceptance constraint under entitlements is identical to the one under mandatory spending,

$$K_{A,2}\Big(\xi(\boldsymbol{s}_2, e)\Big) = K_{A,2}\Big(\xi(\boldsymbol{s}_2, m)\Big) = \ln(s) + \ln(\bar{x}).$$

Second, the resulting allocations are Pareto efficient in the second period for any value of s. That is, for any status quo, there exists a Pareto-weight $\lambda \in [0, 1]$ such that the allocations in the political equilibrium with budget rules is Pareto optimal. Note that the equilibrium may involve significant inequity. For example, when B is the incumbent, for any $x < s_L$, the allocation corresponds to that with $\lambda = 1$. This means that whenever the status quo is relatively low, the incumbent can implement the allocation under discretion, which corresponds to that where she is given all the weight in the planning problem. At the other extreme, when $s \geq s_H$, the incumbent proposes the Samuelson level of the public good and a level of consumption to the opposition above the lower bound $\bar{x}$. Hence, a higher value of s implies greater bargaining power to the opposition party.

For completeness, it is useful to characterize the policy rules that would be chosen by incumbent type A if in power in the second period. These are:

$$\mathcal{G}_{A,2}(\boldsymbol{s}_2) = \begin{cases} \frac{Y-\bar{x}}{2}, & \text{if} \quad s < s_L \\ s, & \text{if} \quad s \in [s_L, s_H) \\ \frac{Y}{2}, & \text{if} \quad s \geq s_H. \end{cases}$$

$$\mathcal{C}_{A,A,2}(\boldsymbol{s}_2) = \begin{cases} \frac{Y-\bar{x}}{2}, & \text{if} \quad s < s_L \\ Y - s - \bar{x}, & \text{if} \quad s \in [s_L, s_H) \\ \frac{Y^2-4\bar{x}s}{2Y}, & \text{if} \quad s \geq s_H. \end{cases}$$

$$\mathcal{C}_{B,A,2}(\boldsymbol{s}_2) = Y - \mathcal{C}_{A,A,2}(\boldsymbol{s}_2) - \mathcal{G}_{A,2}(\boldsymbol{s}_2),$$

where $s = \bar{g}_1$ under mandatory spending and $s = \bar{c}_{B,1}$ under entitlements. Note that due to symmetry, the levels of public goods chosen by both incumbent types are the same under a mandatory spending rule, $\mathcal{G}_{A,2}(\boldsymbol{s}_2) = \mathcal{G}_{B,2}(\boldsymbol{s}_2)$. In addition, when $r = m$, we also have that the incumbent's consumption does not depend on her type, $\mathcal{C}_{A,A,2}(\boldsymbol{s}_2) = \mathcal{C}_{B,B,2}(\boldsymbol{s}_2)$, which in turn implies that $\mathcal{C}_{A,B,2}(\boldsymbol{s}_2) = \mathcal{C}_{B,A,2}(\boldsymbol{s}_2)$. Under an entitlement rule, these equalities hold only in the case where $\bar{c}_{A,1} = \bar{c}_{B,1}$, but not for arbitrary combinations of entitlement levels.

6.2 First-Period Characterization.

We now characterize first-period allocations. We depart from a situation where the proposer has discretion. Hence, in principle, she could choose the allocations characterized in Section 4.1. If the proposer remained in power with certainty, even in the presence of budget rules, it would actually choose the allocations under discretion. This is the case because those achieve the highest level of welfare for the proposer at every point in time. There would be no gain to restrict or distort allocations to modify tomorrow's status quo. When there is uncertainty about the identity of the proposer tomorrow, on the other hand, the current policymaker may have incentives to choose an allocation which differs from that under discretion. This would, through the constraints imposed by the budget rule, tie the hands of its successor forcing it to provide a minimum level of welfare. By sacrificing the optimal consumption mix today, it is possible to ensure a good bargaining position in the future. Moreover, because mandatory spending and entitlements change the continuation utility differently, the proposer would not be indifferent between the two types of budget rules in period 1.

Consider a situation where incumbent B proposes allocation $\boldsymbol{x}_{B,1} = \{c_{A,1}, c_{B,1}, g_1\}$ under discretion. The proposer understands that, if accepted, this allocation becomes the status quo next period, $\boldsymbol{s}_2 = \boldsymbol{x}_{B,1}$. The latter, in turn, affects tomorrow's welfare according to the mapping $\xi(\boldsymbol{s}_2, r)$. Recall that the function ξ represents enacted law under the budget rule $r \in \{e, m\}$. The incumbent's maximization problem is

$$\max_{\boldsymbol{x}_{B,1}} \quad \ln(c_{B,1}) + \ln(g_1) + \beta \{pV_B(\boldsymbol{x}_{B,1}, r) + (1-p)W_B(\boldsymbol{x}_{B,1}, r)\} \tag{12}$$

$$\text{s.t. eq. (2) and eq. (3),}$$

where we have used the fact that $\boldsymbol{s}_2 = \boldsymbol{x}_{B,1}$. The value functions V_B and W_B can be obtained by evaluating the solution characterized in Proposition 3 into the utility in the second period.

Regardless of the rule, because the current proposer can make choices under discretion, it will set $c_{A,1} = \bar{x}$. There is no gain (current or future) in providing additional consumption to the opposition. Using this result, and the budget constraint, eq. (2), we can write B's consumption in the first period as $c_{B,1} = Y - g_1 - \bar{x}$. Then, the incumbent chooses the optimal mix between the public good and her private consumption, internalizing that this choice affects her continuation utility. To understand the different trade-offs, it is intuitive to present the results under the alternative budget rules separately.

6.3 *Mandatory Spending*

We first characterize the solution under a mandatory spending rule, $r = m$. While the proposer chooses a triplet $\boldsymbol{x}_{B,1} = \{c_{A,1}, c_{B,1}, g_1\}$ today, only g_1 will affect the continuation utility through $\xi(\boldsymbol{x}_{B,1}, m) = \{\bar{x}, \bar{x}, g_1\}$. Using Proposition 3, we can write tomorrow's utility if the proposer today remains a proposer tomorrow as function of g_1, $V_B(g_1, m)$ as

$$V_B(g_1, m) = \begin{cases} 2\ln\left(\frac{Y-\bar{x}}{2}\right), & \text{if} \quad g_1 < s_L \\ \ln(Y - g_1 - \bar{x}) + \ln(g_1), & \text{if} \quad g_1 \in [s_L, s_H) \\ \ln\left(\frac{Y^2-4\bar{x}g_1}{2Y}\right) + \ln\left(\frac{Y}{2}\right), & \text{if} \quad g_1 \geq s_H. \end{cases}$$

If party A becomes the proposer in the second period, B's continuation utility is

$$W_B(g_1, m) = \begin{cases} \ln(\bar{x}) + \frac{Y-\bar{x}}{2}, & \text{if} \quad g_1 < s_L \\ \ln(\bar{x}) + \ln(g_1), & \text{if} \quad g_1 \in [s_L, s_H) \\ \ln\left(\frac{2\bar{x}g_1}{Y}\right) + \ln\left(\frac{Y}{2}\right), & \text{if} \quad g_1 \geq s_H. \end{cases}$$

Replacing the equations above into the optimization problem eq. (12), we can compute optimality condition w.r.t. g_1 under a mandatory spending rule. The first order condition in period 1 for the proposer is

$$\underbrace{\frac{1}{Y - g_1 - \bar{x}}}_{MU_c} - \underbrace{\frac{1}{g_1}}_{MU_g} = \underbrace{\beta\left(p\frac{\partial V_B\left(g_1, m\right)}{\partial g_1} + (1-p)\frac{\partial W_B\left(g_1, m\right)}{\partial g_1}\right)}_{\text{wedge}_g > 0}$$

In the absence of budget rules, the proposer would make the left hand side of the equation equal to zero, as in the solution under discretion. With mandatory spending over public goods, the proposer finds it optimal to distort that solution because, by increasing g_1 enough it can ensure having enough bargaining power—if out of power—to induce the opposition to choose an allocation in the second period satisfying $\mathcal{C}_{B,A,2}(g_1, m) > \bar{x}$. An important aspect of this trade-off is that increasing g_1 affects the continuation utility regardless of whether the incumbent is the proposer tomorrow or not. This happens because the public good is non-rival and non-excludable.

Because this problem is fully symmetric, the rules chosen by A and B—when in power—are identical. The unique solution to this problem is characterized in Proposition 4.

Proposition 4 *The unique proposal strategy for incumbent $i \in \{A, B\}$ under a mandatory spending rule* *is:*

$$\mathcal{G}_1(m) = \begin{cases} \frac{\alpha(p)-\sqrt{\alpha(p)^2-4\delta\kappa(p)}}{2\kappa(p)}, & \text{if } p \in [0, p^*] \\ \\ \frac{(1+\beta)(Y-\overline{x}_c)}{2+\beta(1+p)}, & \text{if } p \in (p^*, 1]. \end{cases}$$

$$\mathcal{C}_{i,1}(m) = Y - \overline{x}_c - \mathcal{G}_1(m).$$

$$\mathcal{C}_{j,1}(m) = \overline{x}_c,$$

with $p^* = 1 - \frac{2(1+\beta)\overline{x}_c}{\beta Y}$ *and* $\kappa(p)$*,* δ *and* $\alpha(p)$ *defined in Appendix 10.3.*

Proof. See Appendix 10.3 □

The solution above determines s_2, and hence the constraint on tomorrow's policymaker. Note that the solution depends on the probability of remaining in power. To understand the intuition better, ignore the role of discounting by assuming that $\beta = 1$. If incumbent B knows that she will be the proposer with certainty next period, then she simply sets mandatory spending to be $\mathcal{G}_1(m) = \frac{(Y-\overline{x}_c)}{2}$, which is equal to the value under discretion g^D. Under uncertainty, the proposer sets $\mathcal{G}_1(m) > g^D$, understanding that this results in too much public good provision today and in the future from her own perspective. The former is a current cost because it distorts the allocation relative to the value under discretion. The gain arises in the future: by forcing over-provision of the public good tomorrow, the only way in which an A proposer would be able to lower g_2 would be by compensating B with more private goods $\mathcal{C}_{B,A,2}(g_1, m) > \bar{x}$. By over-providing public goods, the current proposer ensures a better bargaining position next period. At an extreme, when $p = 0$, we have that $\mathcal{G}_1(m) = \frac{2(Y-\overline{x}_c)}{3}$ which is significantly above g^D.

We showed in previous sections that the solution under discretion was inefficient because the proposer was not providing enough public goods. Consider a case where $\lambda \in (\lambda_1, \lambda_2)$ so that the planner would like both agents to consume above $\bar{x}$. In that case, we showed that $g^D < g^*$. In other words, that the level of public good provision under discretion was below the efficient Samuelson level g^*. In the analysis above, we argued that under a mandatory spending rule, it was possible to induce higher provision of public goods: $\mathcal{G}_1(m) > g^D$. Is it the case, then, that this rule restores Pareto efficiency? The answer is no. The reason being that under political uncertainty there is a range of values for p such that $\mathcal{G}_1(m) > g^*$. Hence, when there is enough proposer turnover (low p), implementing this rule may result in over-provision of public goods not only relative to discretion but also relative to the first-best. The result is shown in the following corollary for a special case with $p = 0$, and also illustrated numerically in Figure 5 for $p \in [0, 1]$.

Corollary 3 *Let* $\lambda \in [\lambda_1, \lambda_2]$*,* $\beta = 1$*, and* $p = 0$*. Then, there is over-provision of public goods in the first period relative to the Samuelson level* $\mathcal{G}_1(m) > g^*$*.*

Proof. Replace $p = 0$ in the expression for public good provision of Proposition 4 and compare the resulting expression with $g^* = \frac{Y}{2}$. □

The right panel of Figure 5 shows, using a numerical example, how $\mathcal{G}_1(m)$ varies with the probability of being the proposer tomorrow p. The first thing to note is that $\mathcal{G}_1(m)$ is always higher than g^D (dashed red line) when there is some uncertainty (e.g. for $p < 1$). The dotted blue line in the picture depicts the Samuelson level of public good provision, g^* (e.g. when $\lambda \in (\lambda_1, \lambda_2)$).

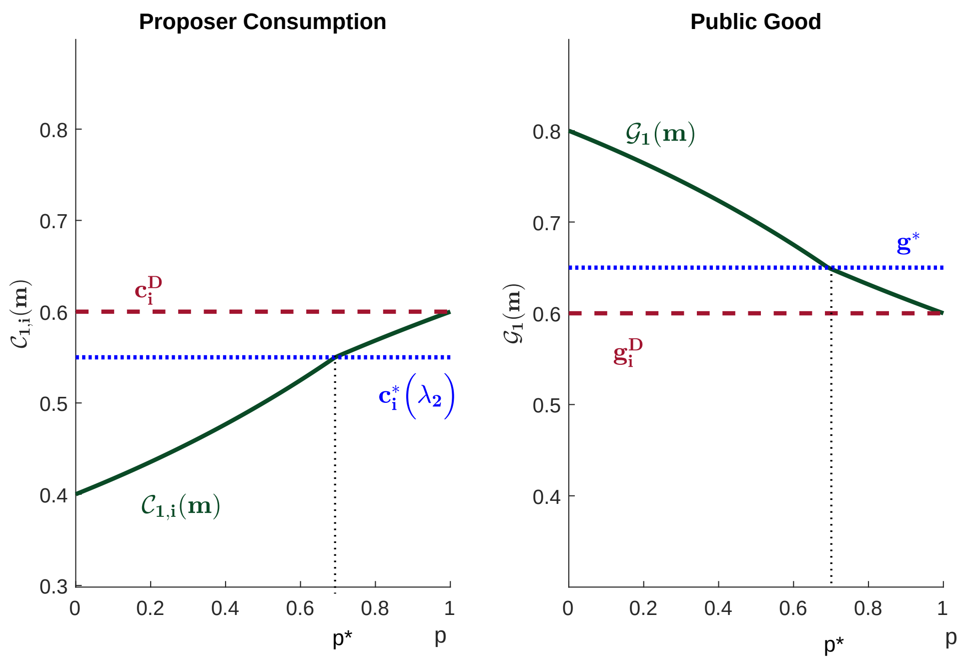


Figure 5: Allocations under a Mandatory Spending Rule. Parameters: $Y = 1.3$, $\beta = 1$, and $\overline{x}_c = 0.1$.

When $p = p^*$, $\mathcal{G}_1(m) = g^*$ but the consumption to the opposition j is inefficiently low since $\mathcal{C}_{j,1}(m) = \overline{x}$ (whereas a Planner would like it to be above $\bar{x}$). When uncertainty is large enough, so $p < p^*$, then in addition to providing too little consumption to group j, the proposer over-provides public goods relative to the first-best, $\mathcal{G}_1(m) > g^*$. The left panel of the graph depicts the level of consumption of the proposer under a mandatory spending rule (green solid line) and under discretion (dashed red line). We can see that a mandatory spending rule reduces the incentives of the proposer to provide private goods to herself for any $p < 1$, since the solid line is always below the dashed line. In the graph, we plot the Pareto optimal allocation $c^*_{1,i}$ for the case where $\lambda = \lambda_2$. At the other extreme, when $\lambda \to \lambda_1$, consumption $c_i^* \to \bar{x}$. We can see that for arbitrary values of $p \in (0, 1)$, we will have that $\mathcal{C}_{i,1}(m) \neq c_i^*$.

It is worth mentioning that this result is in contrast to Bowen et al. (2014), who showed that a mandatory spending rule could restore efficiency. The difference is mainly due to our assumption of concavity in private consumption. In our setup, agents would like to smooth the consumption of private and public goods over time. If incumbent i expects too little consumption in the second period (e.g. $\mathcal{C}_{i,j,2}(g_1, m) = \bar{x}$), she is willing to distort g_1 inter-temporally to ensure a higher level of private consumption in the future. This is driven by a high marginal utility in future consumption. With linear utility in c such effect is absent. While in their environment a mandatory spending rule always restores the Samuelson level of public good provisions, this only happens in our environment for a specific value of p.

6.4 *Entitlements*

We now characterize the solution under an entitlement rule, $r = e$. As before, a type B proposer chooses the triplet $\boldsymbol{x}_{B,1} = \{c_{A,1}, c_{B,1}, g_1\}$ today, but now both $c_{A,1}$ and $c_{B,1}$ become status quo values for tomorrow's private consumption if the proposal is accepted. They affect the continuation utility of both agents through $\xi(\boldsymbol{x}_1, e)$.

Using Proposition 3 given the budget rule $r = e$, we can write the continuation utility of proposer B if she stays in power tomorrow as

$$V_B(c_{A,1}, e) = \begin{cases} 2\ln\left(\frac{Y-\bar{x}}{2}\right), & \text{if} \quad c_{A,1} < s_L \\ \ln(Y - c_{A,1} - \bar{x}) + \ln(c_{A,1}), & \text{if} \quad c_{A,1} \in [s_L, s_H) \\ \ln\left(\frac{Y^2 - 4\bar{x}c_{A,1}}{2Y}\right) + \ln\left(\frac{Y}{2}\right), & \text{if} \quad c_{A,1} \geq s_H. \end{cases}$$

This is because only $c_{A,1}$, the level of entitlements received by the opposition, may constrain future decisions when B remains in power. If, on the other hand, A becomes the incumbent tomorrow, then $c_{B,1}$ will constrain A's decisions instead. The continuation utility for B in such case would be

$$W(c_{B,1}, e) = \begin{cases} \ln(\bar{x}) + \frac{Y-\bar{x}}{2}, & \text{if} \quad c_{B,1} < s_L \\ \ln(\bar{x}) + \ln(c_{B,1}), & \text{if} \quad c_{B,1} \in [s_L, s_H) \\ \ln\left(\frac{2\bar{x}c_{B,1}}{Y}\right) + \ln\left(\frac{Y}{2}\right), & \text{if} \quad c_{B,1} \geq s_H. \end{cases}$$

Proposer B chooses allocations in the first period to maximize eq. (12), which can be re-written as

$$\max_{\{c_{A,1}, c_{B,1}\}} \; \ln(c_{B,1}) + \ln(Y - c_{A,1} - c_{B,1}) + \beta \left\{ pV(c_{A,1}, e) + (1-p)W(c_{B,1}, e) \right\},$$

subject to the lower bound constraints. Inspection of the problem above reveals that it is optimal to set $c_{A,1} = \bar{x}$, as higher values of consumption to the opposition party do not increase B's welfare.[4] The first order condition with respect to $c_{B,1}$ is

$$-\underbrace{\frac{1}{c_{B,1}}}_{MU_c} + \underbrace{\frac{1}{Y - c_{B,1} - \bar{x}}}_{MU_g} = \underbrace{\beta(1-p)\frac{\partial W(c_{B,1}, e)}{\partial c_{B,1}}}_{\text{wedge}_c > 0}. \tag{13}$$

As before, the budget rule creates a wedge in the optimal decision under discretion of agent B. The reason being that by choosing $c_{B,1} \neq c^D$ it is possible to affect the status quo inherited by the opposition if group A becomes the proposer next period, therefore increasing her own welfare, $W(c_{B,1}, e)$, in that state of the world. The solution to the first period allocations under an entitlement rule is characterized in Proposition 5.

Proposition 5 *The unique proposal strategy for incumbent $i \in \{A, B\}$ under an entitlement rule is:*

$$\mathcal{G}_1(e) = \frac{Y - \bar{x}}{2 + \beta(1-p)},$$

$$\mathcal{C}_{1,i}(e) = \frac{\Big(1 + \beta(1-p)\Big)[Y - \bar{x}]}{2 + \beta(1-p)},$$

$$\mathcal{C}_{1,j}(e) = \bar{x}.$$

[4] This is the case because we assumed that the current status quo is discretion, but would not hold under a more general initial condition.

Proof. See Appendix 10.4 □

Because of symmetry and the fact that we depart from discretion, the identity of the first period proposer is irrelevant for the solution.

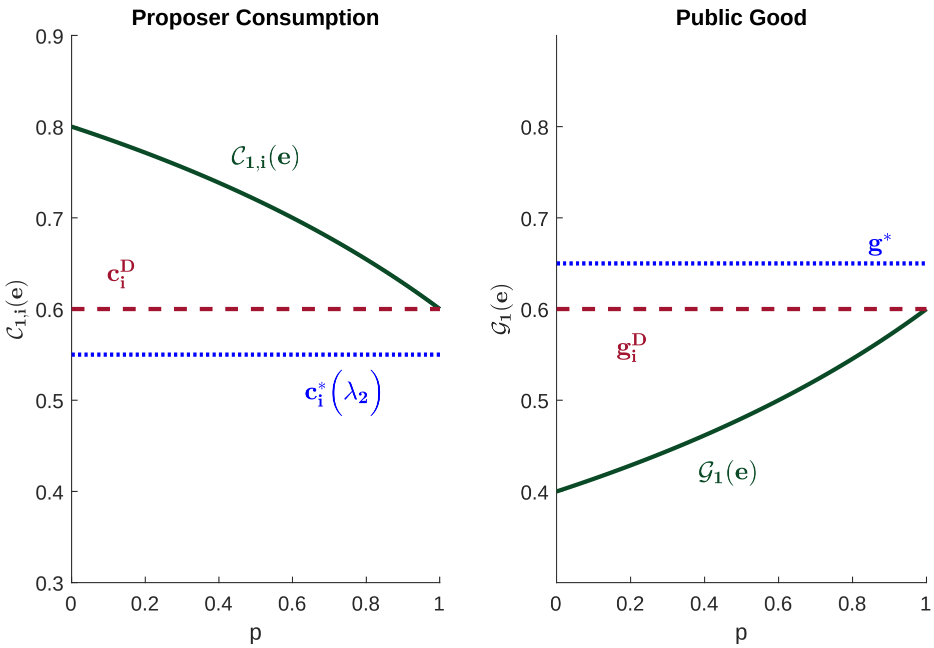


Figure 6: Allocations under an Entitlement Rule. Parameters: $Y = 1.3$, $\beta = 1$, and $\bar{x}_c = 0.1$.

Figure 6 depicts allocations under the entitlement rule (solid line), discretion (dashed line), and the efficient level (dotted line) for $\lambda = \lambda_2$. The left panel shows that the proposer always has incentives to increase consumption relative to her optimal choice under discretion if $p < 1$. As long as she faces uncertainty, the proposer will use the entitlement rule to ensure consumption above $\bar{x}$ when out of power. This comes at the cost of under-providing public goods (see right panel of the picture) relative to her preferred value of g under certainty. The net benefit derived from distorting the allocations under discretion diminishes in p, and as a result $\mathcal{C}_{1,i}(e)$ decreases with the probability of retaining proposal power, p.

Finally, note that with an entitlement rule there is always over-provision of private consumption and under-provision of public goods. This is the case because $c^*(\lambda) < c^*(\lambda_2)$ for all $\lambda \in (\lambda_1, \lambda_2)$. This conclusion is drastically different from the one obtained under a mandatory spending rule. To understand why, it is useful to revisit her optimality condition under an entitlement rule eq. (13). Relative to the expression under mandatory spending, we see that the wedge created by the budget rule involves only changes in W. That is, it only affects welfare when decisions are made by the opposition. A key difference between mandatory spending and entitlements is that with a spending rule on public goods, the current proposer is setting the status quo of a good that will be enjoyed regardless of whether she is the proposer tomorrow or not. Choosing too low a value of g may end up actually reducing her de-facto bargaining power. With an entitlement rule, on the other hand, the proposer can set an arbitrarily high value for c_i, which could become very binding for the opposition if in power, but that would be relatively costless to reduce if i remains in power.

7 Infinite Horizon Model

The previous analysis illustrates how the proposer can strategically use budget rules to position herself advantageously if the opposition gains proposal power in the future. It also makes clear that the proposer is not indifferent between mandatory spending rules and entitlement rules. We found that under entitlement rules, there is always under-provision of public goods, whereas the opposite is true under a mandatory spending rule. A shortcoming of that analysis is that it was made under the extreme assumption that the proposer in period 1 is completely unconstrained. It could manipulate the status quo freely because she was not subject to a budget rule to begin with. In addition, because the economy ends in the second period, the incumbent does not need to consider the reaction of the opposition to the continuation of the play.

In this section, we relax these assumption by considering an arbitrary initial status quo and allowing the economy to have an infinitely large number of periods. By doing so, we can also study the dynamic behavior of private and public consumption and the welfare implications of mandatory spending and entitlement rules in the long run.

8 Quantitative Analysis

Unfortunately, it is not possible to obtain an analytical solution to the proposer's problem in the infinite horizon model. The main reason, and in contrast to most of papers in the bargaining literature, is that agents are risk-averse. Our analysis from now on is thus numerical. Another well-known issue in dynamic legislative bargaining games with endogenous status quo is that standard algorithms are not always successful in computing Markov-perfect equilibria. This paper is no exception, as standard value-function iteration procedures do not converge for arbitrary parameterizations of the model.

The computation of the Markov equilibria of models in this class is notoriously challenging, as documented by Duggan and Kalandrakis (2012b), Martin (2009), Chatterjee and Eyigungor (2012), and others. A common strategy, also adopted here, is to slightly perturb the choices of the proposing agent through the introduction of small, independent and identically distributed shocks. These shocks may apply to fundamentals, as in Chatterjee and Eyigungor (2012), or to the agent's payoff directly, as in Eyigungor and Chatterjee (2019) or Sanchez et al. (2018). We follow Gordon (2019) and use the functional forms and assumptions employed with discrete choice methods. The resulting randomization over options with payoff of comparable value greatly eases the computation of the model, induces smooth value functions and policy functions, and induces near-monotone convergence via standard value function iteration[5]. Appendix 11 documents the model augmented with taste shocks and our solution algorithm.

We consider an annual model where the size of the budget is normalized to $Y = 1.3$. This is without loss of generality. The discount factor is set to $\beta = 0.96$, consistent with a 4% interest rate, implying a standard value for the degree of impatience in the literature. The probability of retaining proposal power is $p = 0.5$ in our benchmark case, implying that political power follows a de-facto iid process. This assumption is relaxed in Section 8.3, where we analyze the effects of persistence in our findings. The minimum consumption levels are set to $\bar{x}_c = \bar{x}_g \equiv \bar{x} = 0.1$, as in the two period model. This choice is not without loss of generality. The larger the value of $\bar{x}$, the less important budget rules will be for welfare. As $\bar{x} \to 0$, almost any proposal is accepted by the opposition because, due to the logarithmic utility function assumption, not reaching an agreement

[5] The method requires the introduction of a parameter governing the importance of these taste shocks for the agent's behavior. We set this parameter to the smallest value consistent with convergence within 1,000 iterations, at $\rho = 5e^{-4}$.

becomes extremely painful. As in the finite-horizon model, we first describe the equilibrium under a mandatory spending rule, and then under an entitlement rule.

8.1 Mandatory Spending Rules

Under a mandatory spending rule, the only relevant state variable for an incumbent is the status-quo level of public spending, $\bar{g}$. As in the two period model, given the symmetry between the two parties, we can focus attention to a symmetric MPE. Such equilibrium has the property that policy rules are independent of the proposer type. For public goods, this implies $\mathcal{G}_B(\bar{g}, m) = \mathcal{G}_A(\bar{g}, m) \equiv \mathcal{G}(\bar{g}, m)$. For private goods, this implies that $\mathcal{C}_{A,A}(\bar{g}, m) = \mathcal{C}_{B,B}(\bar{g}, m) \equiv \mathcal{C}_i(\bar{g}, m)$ and $\mathcal{C}_{A,B}(\bar{g}, m) = \mathcal{C}_{B,A}(\bar{g}, m) \equiv \mathcal{C}_j(\bar{g}, m)$, where i denotes the incumbent and j the opposition. Hence, all that matters to determine private consumption of each type is whether they are currently an incumbent or the opposition.

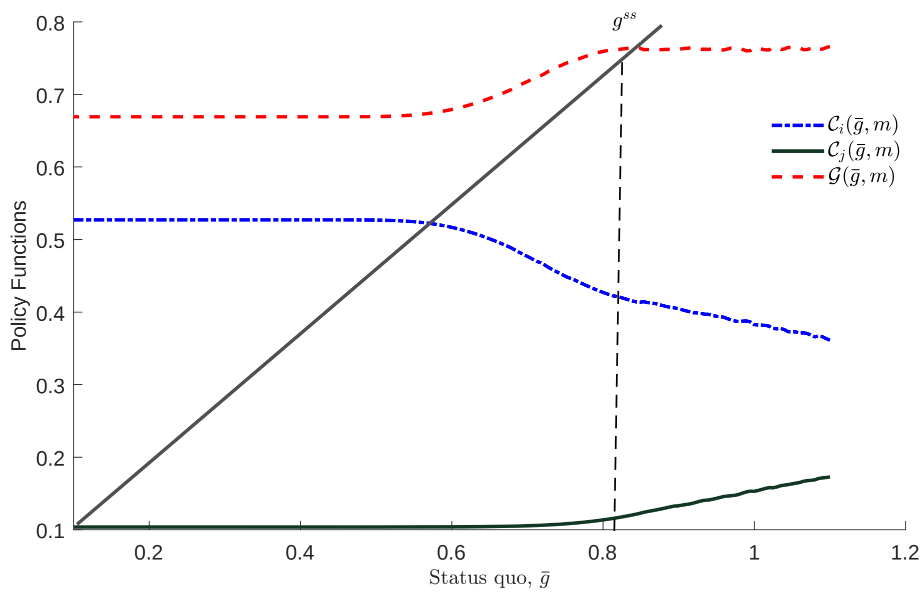


Figure 7: Policy Functions under a Mandatory Spending Rule.

Figure 7 depicts the policy functions of incumbent i as a function of $\bar{g}$ under rule $r = m$. The top dashed (red) line represents the policy function for public goods $\mathcal{G}(\bar{g}, m)$, the dashed-dotted (blue) line the incumbents' consumption policy rule, $\mathcal{C}_i(\bar{g}, m)$, and the bottom (green) line the amount of consumption assigned to the opposition, $\mathcal{C}_j(\bar{g}, m)$. When the status-quo $\bar{g}$ is low enough, the acceptance constraint does not bind, and, as a result, the incumbent assigns the opposition the minimum level of consumption $\mathcal{C}_j(\bar{g}, m) = \bar{x}$. This is similar to the behavior in the two period model, depicted in Figure 4. In contrast to that model, the level of public goods provided is higher than under discretion, $\mathcal{G}(\bar{g}, m) > g^D = 0.6$, whereas private consumption is lower, $\mathcal{C}_i(\bar{g}, m) < c^D = 0.6$ for low $\bar{g}$. This happens because even though the acceptance constraint does not bind in the present, it may bind in the future. The current incumbent knows that she will be replaced with probability $1 - p$ by the opposition, in which case she will receive minimum private consumption levels. In order to secure for herself a more favorable welfare level in such a scenario, the current incumbent proposes a higher value of g than she would under no political turnover.

For higher values of the status quo, the acceptance constraint may bind this period. In this case, proposer i must provide the opposition with higher private consumption than the minimum

$\bar{x}$ in order for j to accept higher levels of g. This can be seen by the fact that the solid green line is increasing in $\bar{g}$ for relatively high values of the status quo, and so is the policy function $\mathcal{G}$. This comes at the expense of lower private consumption for the proposer, as seen by the fact that the dashed-dotted blue line is decreasing in $\bar{g}$.

Evolution under $m-$rule: In the plot, we have included a 45^o line (solid black line), to show that g eventually converges to a unique steady state g^{ss}. The steady state satisfies $\mathcal{G}(\bar{g}, m) = \bar{g}$ and can be found—graphically—at the intersection between the 45^o line and the policy function $\mathcal{G}(\bar{g}, m)$. It exists because proposers follow the same rule in the MPE. It is worth noticing that, in this example, the steady state is not only larger than the value under discretion $g^{ss} > g^D$, but it is also higher than the Samuelson level $g^{ss} > g^*$. In other words, there is over-provision of public goods in the long run. This finding is in sharp contrast with that in Bowen et al. (2014), who found that introducing a mandatory spending rule could lead to the efficient provision of public goods. From the graph, we can also see that g^{ss} is a stable steady state: if we start with a status quo below g^{ss} the incumbent will propose to increase g, whereas in cases where $\bar{g} > g^{ss}$, it will propose to cut public spending levels. This has important implications for the behavior of private consumption, as shown next.

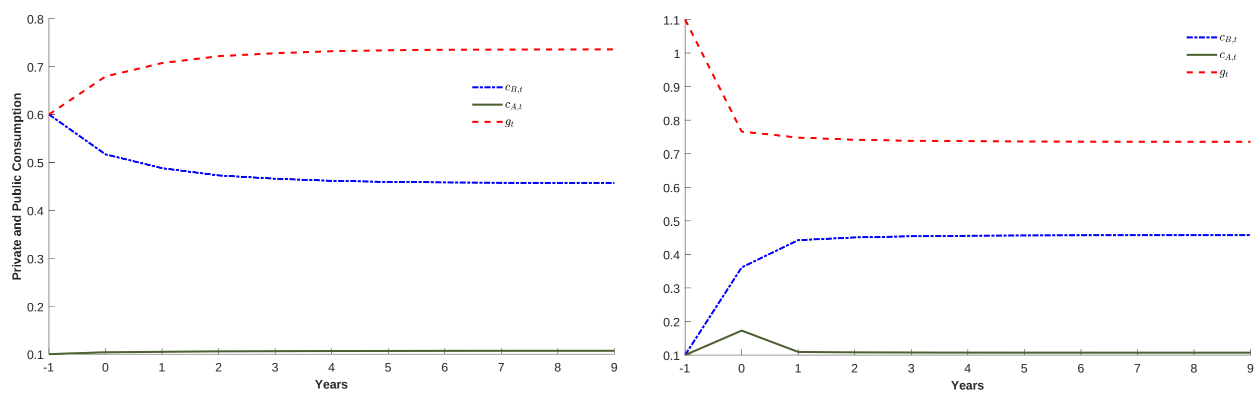


Figure 8: Simulated private and public consumption, B in power.

In Figure 8, we show the effect of alternative status quo levels by simulating an economy where B is an incumbent who inherits allocations from period -1, and makes proposals in all subsequent periods. In the left panel, we start from a situation with $\bar{g} = g^D = 0.6$. Because the status quo in period 0 is lower than the value desired by the proposer, B is able to increase public goods at the expense of her own consumption, while keeping the opposition at the lower bound $c_A = \bar{x}$. Party A agrees to this because it is in her best interest to increase g, so the incentives of the two parties are aligned. After a few periods, g converges to its steady state. In the right panel, we assume that $\bar{g}$ is initially at its highest possible value (e.g. when $c_A = c_B = \bar{x}$). In such case, incumbent B would like to decrease g to increase her own consumption above $\bar{x}$. Party A would not agree to a decline in public good provision unless she is a compensated with higher private consumption. We observe that c_B jumps above the minimum level for one period, and then eventually goes back to $\bar{x}$. This illustrates how high status quo values of g can give the opposition de-facto bargaining power under a mandatory spending rule.

In this example, we assumed that B was the proposer every period (i.e. even though B faced political risk, she happened to stayed in power through the simulation). Under political turnover, we would observe volatile private consumption, due to the changing identity of the proposer, but public goods eventually converging to g^{ss}. In Figure 9, we show a simulation where proposers alternate in power according to p. The left panel depicts a time series of the provision of public goods (as proportions of Y) in the long-run for three cases: the first-best (dark solid line), discretion (blue dashed line), and the equilibrium under a mandatory spending rule. The budget rule induces a higher provision of public goods relative to discretion, but well above what would be socially optimal.

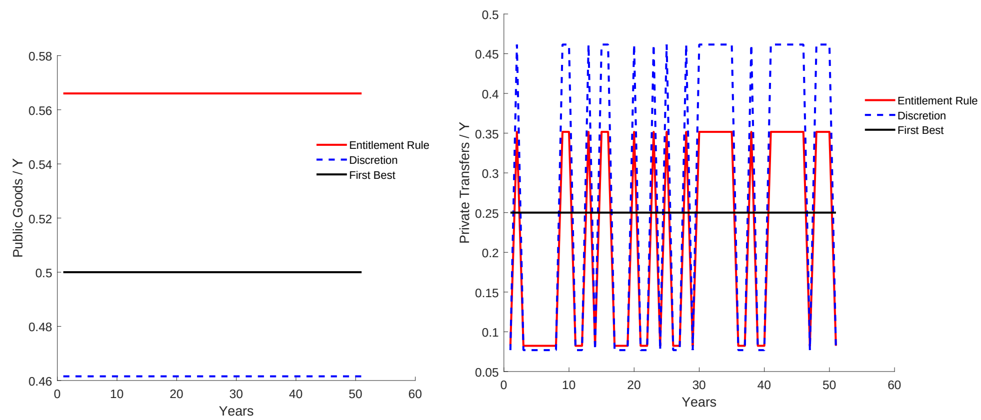


Figure 9: Mandatory spending rule: Simulated allocations (w/ turnover).

In the right-panel, we show the evolution of private transfers (as proportions of Y) to B, in the ergodic set, for the same institutional environments. Private transfers oscillate between two values: out-of-power consumption $\bar{x}$ and proposer consumption c^{ss}, with $c^{ss} = Y - \bar{x} - g^{ss}$. Because the lower bound is the same as in the discretion case, whereas proposer consumption is smaller, $c^{ss} < c^{D}$, the volatility in private consumption under a mandatory spending rule is actually smaller than under discretion. Whether it is beneficial to introduce the rule or not depends on whether the gains associated to lower volatility exceed the losses incurred by having inefficiently high public good provision.

For this particular example, it turns out that introducing a mandatory spending rule—from a situation under discretion—is Pareto improving. In Figure 10 we observe that both parties attain higher levels of welfare when a mandatory spending rule is implemented (black square), relative to the welfare pair under discretion (red diamond). The rule does not eliminate all the inefficiencies: the associated welfare allocations are still at a considerable distance from the Pareto frontier. In Section 8.3, we analyze how this finding changes with the probability of re-election.

8.2 Entitlement Rules

In this section, we consider how entitlement rules affect the evolution of public and private consumption in the Markov-Perfect equilibrium. The problem becomes more complicated because we now have to account for two relevant state variables, $\bar{c}_A$ and $\bar{c}_B$, namely the level of entitlements received by each agent. In the two-period model, we only needed to keep track of the level of consumption promised to the opposition. In the infinite horizon model, on the other hand, both states are relevant when choosing a proposal. To fix ideas, suppose that B is the incumbent. The entitlement promised to A matters because it directly affects how likely the opposition is to accept a reform (as in the two-period example). The value of $\bar{c}_B$ is also important because, if the proposal

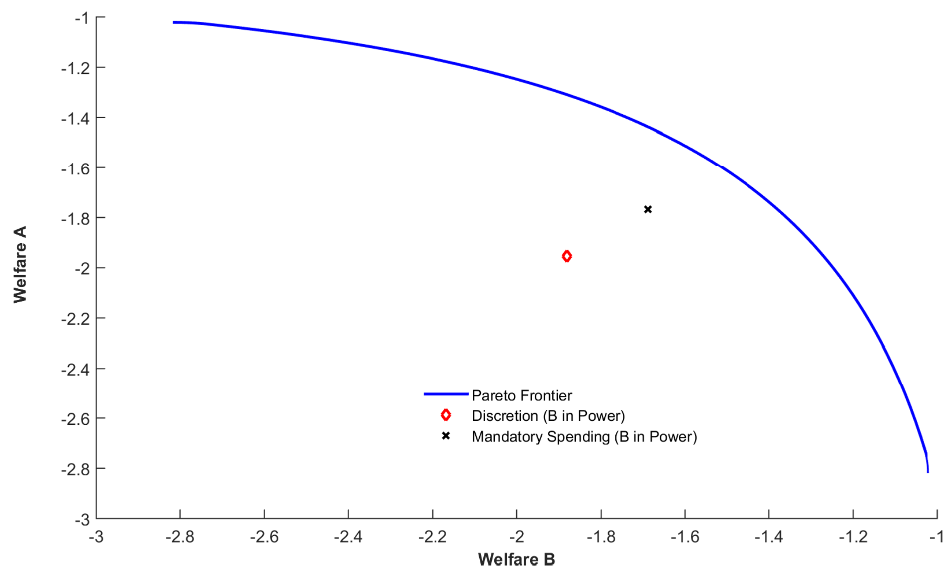


Figure 10: Mandatory Spending Rule, Discretion, and the Pareto Frontier

is rejected, $\bar{c}_B$ determines next period's status quo. This, in turn, affects B's bargaining power if A were to become the proposer tomorrow. Through continuation utilities, then, $\bar{c}_B$ affects today's decisions.

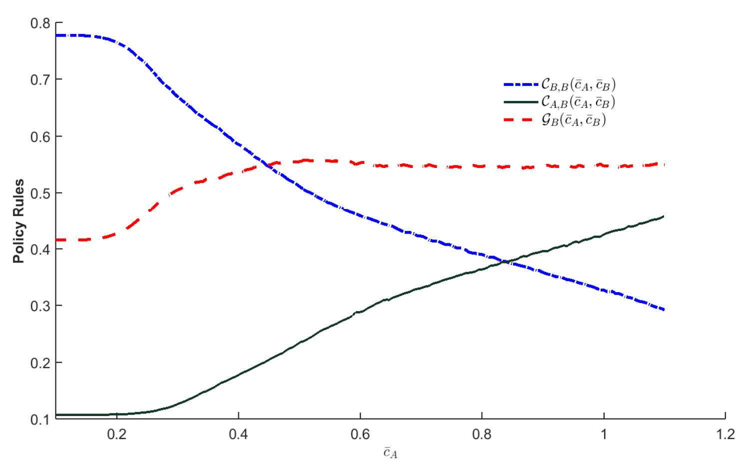


Figure 11: Policy functions under an Entitlement Rule as a function of $\bar{c}_A$ (with $\bar{c}_B = 0.1$).

Figure 11 depicts B's equilibrium policy rules as functions of $\bar{c}_A$, fixing $\bar{c}_B = \bar{x} = 0.1$. When $\bar{c}_A$ is close to the minimum, B chooses extremely high private consumption (e.g. well above the discretionary level of $c^D = 0.6$) and relatively low public good provision (e.g. well below $g^D = 0.6$), without having to increase A's consumption. As entitlements to the opposition grow, the incumbent must sacrifice private consumption, as seen by the fact that the dashed-dotted blue line—representing $\mathcal{C}_{B,B}(\bar{c}_A, 0.1)$—is downward sloping. Party A is willing to accept the proposal as long as it is accompanied with higher values of public goods, which can be seen by the fact that the dashed red line—representing $\mathcal{G}_B(\bar{c}_A, 0.1)$—increases with $\bar{c}_A$. As A's promised entitlements grow even larger, B can only change the status quo towards higher private consumption to herself by increasing A's private consumption and sacrificing public good provision.

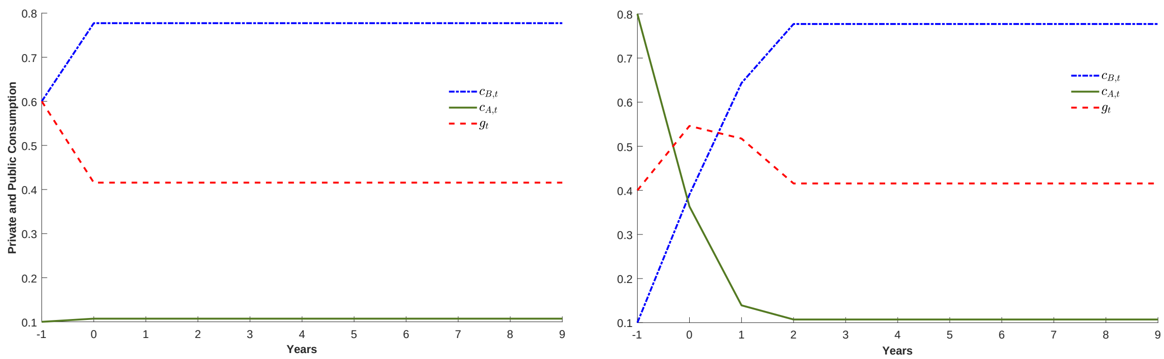


Figure 12: Simulated private and public consumption over time.

In Figure 12, we show a simulation similar to the one in the previous section, but under an entitlement rule. We assume that party B is the proposer for 9 periods. In the left panel, the status quo inherited from period -1 is given by discretion, with $\bar{c}_B = c^D = 0.6$ and $\bar{c}_A = 0.1$. Because A is entitled only to the minimum level of consumption, it is optimal for B to choose a high value of c_B immediately at the expense of lower g. Note that such value is above c^D, because in case of being replaced, higher $\bar{c}_B$ ensures a good bargaining position for B when A is the proposer. If, instead, A is entitled to a high level of consumption—as in the right panel of the figure where $\bar{c}_A = 0.8$—, then the only way to pass a proposal involving a reduction in A's consumption is by temporarily increasing g. To the extent that B remains in power, she will slowly increase her entitlement to private goods through rises in private goods. Eventually, B will be able to secure herself a good enough bargaining position (through high $\bar{c}_B$) to start reducing both g and c_A, in order to finance increases in c_B. If B is in power long enough, she would reach a level of private consumption significantly higher than what she would choose under discretion.

The dynamics of private consumption and public good levels are significantly different than in the mandatory spending rule. This, in turn, affects the ergodic set to which the economy converges. Because of the higher dimensionality of the state space, we can no longer characterize the stationary equilibrium using policy functions (something we could do with the mandatory spending rule). To compute the ergodic set, we simulated the economy for $1,000,000$ years, and eliminated the first $1,000$ periods. It is worth noticing that the economy converges to the same set regardless of initial conditions in our benchmark economy. Figure 13 depicts a scatter plot of private consumption pairs for each period in the simulation (marked blue circles), together with the pairs that would be obtained under discretion (marked with red $\times$). Notice that we span a significant portion of the state-space, whereas under the mandatory spending rule, private goods jumped between two values. This implies that the evolution of consumption is smoother with an entitlement rule and that the distribution of attained values is wider. In Figure 14, we show the histogram of B's private consumption levels (left panel) and the public good provision (right panel) for our current simulation. For comparison, the histogram of c_B was bimodal and of g unimodal with a mandatory spending rule.

In Figure 15 we show the time-series for a 60-period sample simulation in the ergodic set. As

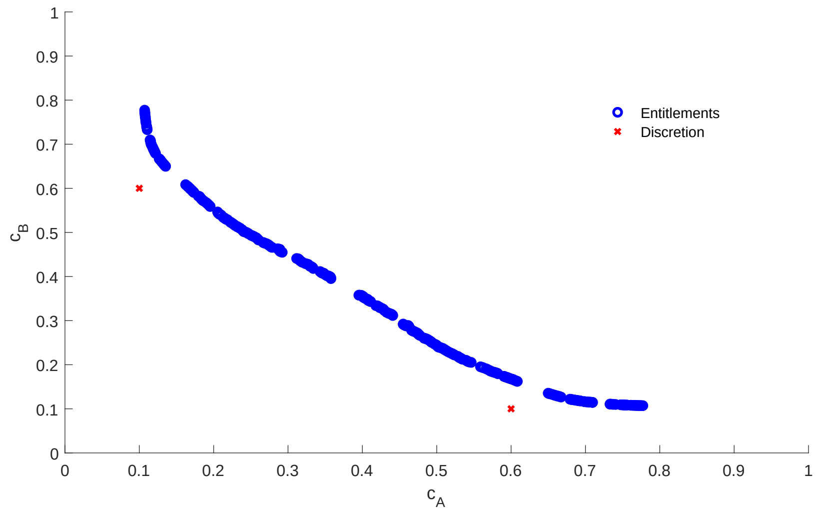


Figure 13: Scatter plot of private consumption pairs in the simulation.

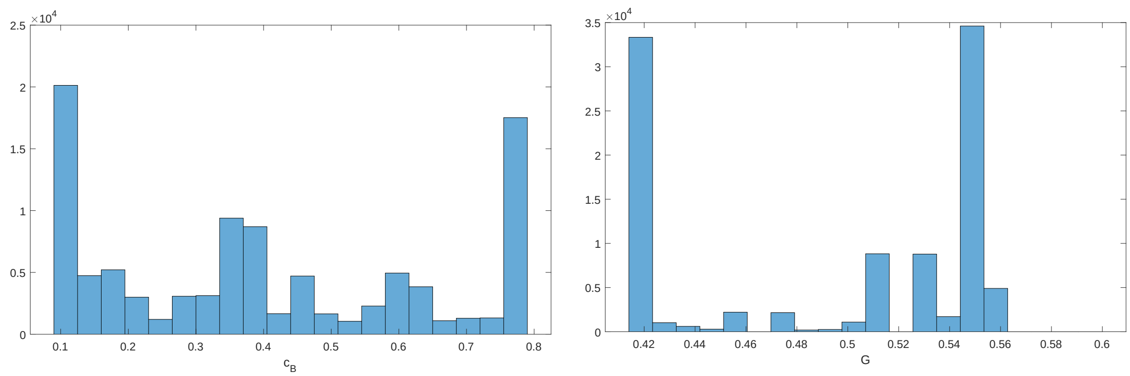


Figure 14: Histograms of private consumption (left) and public consumption (right) in the simulation.

before, the left panel depicts the behavior of public good provision whereas the right panel shows the amount of private transfers received by group B. In contrast to the mandatory spending rule case, we now see that the budget rule induces volatility in the provision of g. In contrast, under discretion, g is constant. Moreover, now the proposer chooses to devote a smaller proportion of the budget, one average, to providing public goods. In terms of public goods, then, this rule induces inefficiencies in terms of the mean and variance of g. In addition, entitlement rules induce more over-provision and volatility of private transfers than what would arise under discretion.

It is worth asking whether introducing an entitlement rule would then result in a Pareto improvement relative to the discretionary case. In Figure 16, we show welfare pairs for the two agents under discretion (marked with a red diamond) and an entitlement rule (marked with a black square) for our benchmark economy. Assuming that party B is the proposer in the two scenarios, we can see that the entitlement rule makes both agents better off. Inspection of the welfare levels reveals that for this parameterization, the gain for B is relatively larger than the gain for A. This happens because the initial condition is beneficial for the proposer.

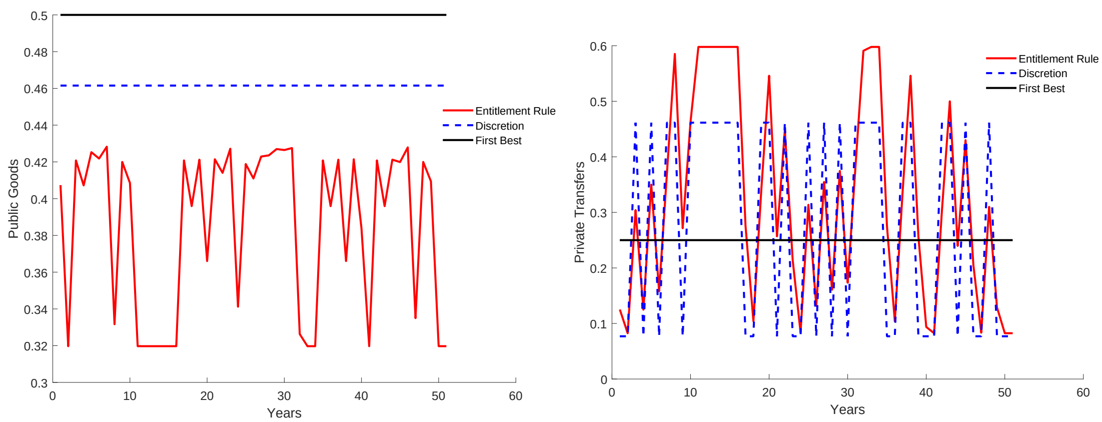


Figure 15: Entitlements: Simulated allocations (w/ turnover).

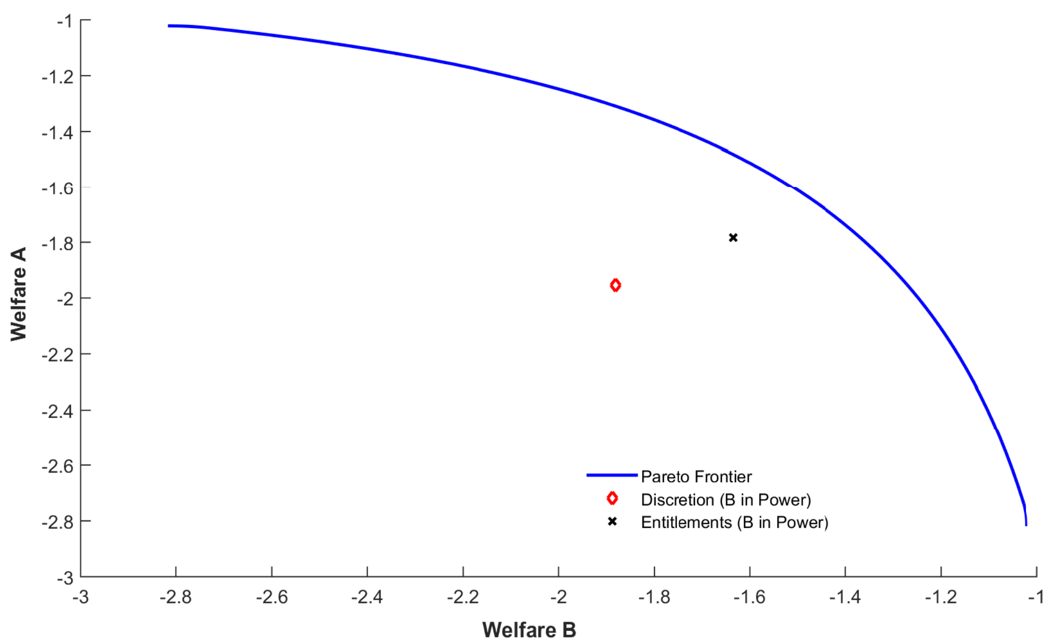


Figure 16: Entitlement Rule, Discretion, and the Pareto Frontier

8.3 The Effects of Political Persistence

The welfare effects of alternative budget rules presented in the previous section were obtained for a parameterization which abstracted from incumbency advantage. That is, under the assumption that $p = 0.5$. In this section, we consider how political turnover affects the welfare gains of mandatory spending and entitlement rules. As before, we compare an economy with no rules, to one where rule r is in place and the initial status quo is given by the values under discretion. When B is the current proposer, this implies that the initial state is $s_0 = (\bar{x}, c^D, g^D)$.

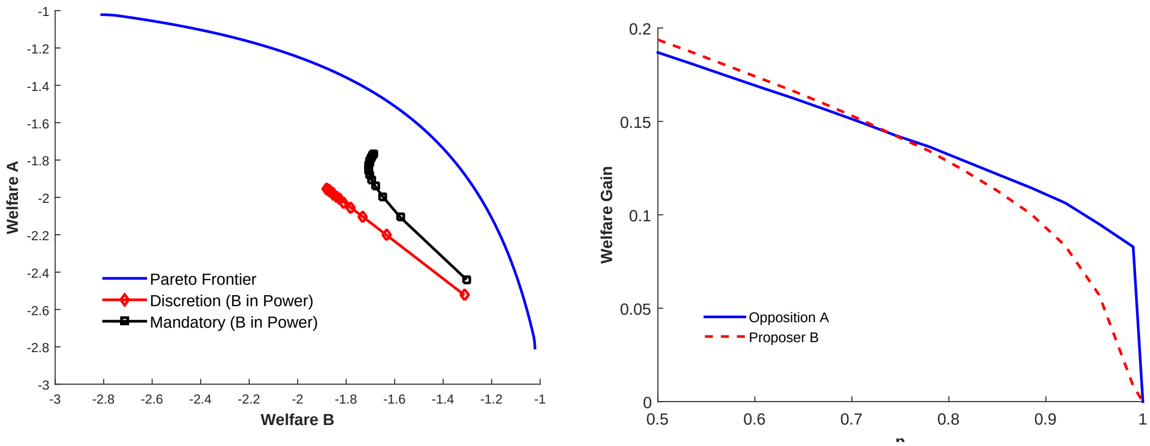


Figure 17: Mandatory spending rules and political persistence.

In the left panel of Figure 17, we extend Figure 10 to allow for alternative degrees of political persistence. We plot the Pareto frontier (in blue), the alternative welfare pairs under discretion (in red, with diamonds) and the pairs obtained under a mandatory spending rule on public goods (in black, with squares). We learn two lessons from this figure. First, mandatory spending is always better than discretion. Second, the rule is more effective the higher the degree of turnover. In other words, the distance between the two lines is maximal when $p = 0.5$.

In the right panel of Figure 17, we plot the welfare gains of imposing a mandatory spending rule relative to a world under discretion. The dashed line represents the gains attained by the proposer, namely $V_B(s_0, m) - V^D$, with $s_0 = (\bar{x}, c^D, g^D)$. While these gains decrease with p, they are always positive. The solid blue line correspond to the gains attained by the opposition, namely $W_A(s_0, m) - V^D$. For high turnover values (i.e. low p), the gains are larger for the proposer. As the proposer is more likely to remain in power (i.e. p rises), the opposition experiences larger gains from adopting the rule. This happens because the rule constrains the proposer to provide a higher level of public goods in equilibrium. Because both parties are better off, the figure indicates that a mandatory spending rule generates a Pareto improvement for any level of political turnover. Moreover, we can infer that if B proposes to adopt a mandatory spending rule with status quo s_0, A would not oppose this initiative.

In the left panel of Figure 18, we consider the welfare combinations associated to a an entitlement program (in black, with squares). In contrast to a mandatory spending rule, we can now see that there are values of p for which discretion yields higher welfare for at least one of the two parties. This becomes more evident when we plot the welfare gains of imposing an entitlement program for each party, depicted in the right panel of Figure 18. For the proposer, adopting an entitlement rule is always beneficial: the dashed red line is always above 0. For the opposition, on the other hand, the rule involves lower welfare than discretion for any $p > 0.75$. The reason is the following: when p is large, but still smaller than 1, the proposer knows that with a very small probability it can be replaced by A and then be out of power for a long period of time. In anticipation of that unlikely, but unfortunate state of the world, B chooses a very large level of c_B at the expense of g, while keeping $c_A = \bar{x}$. Under discretion, A can obtain the same value of private consumption, but a much larger provision of public goods. Because of this, discretion would be preferable for A. In other words, imposing an entitlement rule in an environment where policymakers remain in power for long periods of time can be welfare-reducing for one of the parties due to the distortions in public good provision it implies.

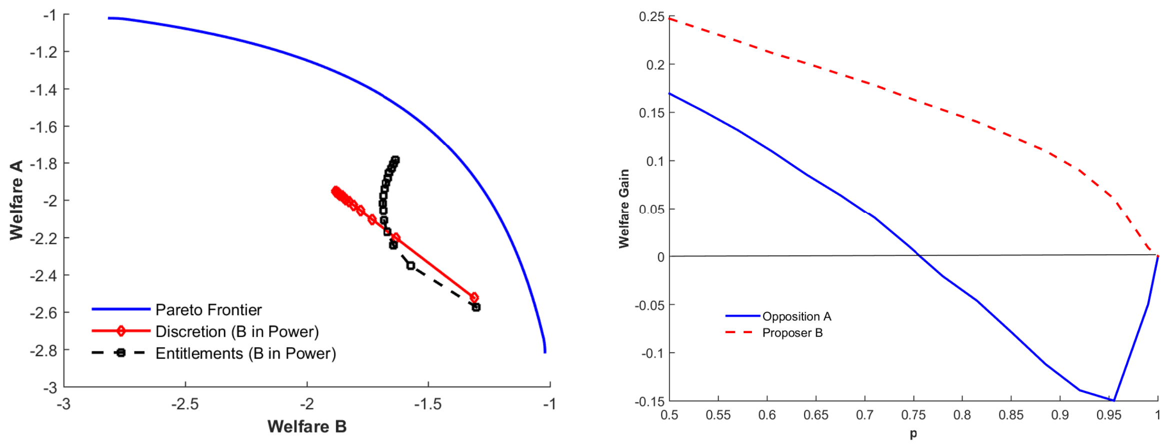


Figure 18: Entitlement programs and political persistence.

This experiment illustrates how different these budget rules can be in terms of welfare in the full dynamic infinite horizon model. The results are mostly driven by continuation utilities.

8.4 Institutional Reform

We have shown that entitlement rules improve the proposer's welfare, but possibly at the expense of the opposition party. This happens because the proposer starts from an advantageous initial status quo at the time of introducing the budget rule. An obvious question is, then, whether implementing the rule would be feasible: If the opposition is worse off, it would never support introducing the rule in the first place.

In this section, we consider a situation where both parties need to agree on whether to introduce a budget rule through a once-and-for-all bargaining process. That is, at time 0, a party is chosen at random and proposes: (i) introducing the budget rule and (ii) an allocation. The other party may accept or reject this proposal. If accepted, the allocation becomes the status quo for the following period—as determined by the rule—, and the game follows the same protocol as in the previous sections. If the proposal is rejected, the economy continues under discretion forever after. Suppose that B is selected to make the proposal in period 0. Her optimization problem is

$$\max_{x_B=\{c_A,c_B,g\}} \ln(c_B) + \ln(g) + \beta\Big\{pV_B(\boldsymbol{x}_B, r) + (1-p)W_B(\boldsymbol{x}_B, r)\Big\}, \tag{14}$$

subject to eq. (2), eq. (3), an incentive compatibility constraint (stating that she must be better off by introducing rule r)

$$\ln(c_B) + \ln(g) + \beta\Big\{pV_B(\boldsymbol{x}_B, r) + (1-p)W_B(\boldsymbol{x}_B, r)\Big\} \geq V_B^D, \tag{15}$$

and the *reform acceptance constraint*

$$\ln(c_A) + \ln(g) + \beta\Big\{(1-p)V_A(\boldsymbol{x}_B, r) + pW_A(\boldsymbol{x}_B, r)\Big\} \geq V_A^D. \tag{16}$$

Equation (16) establishes that in order for the proposal $\boldsymbol{x}_B$ to be accepted by party A, it needs to deliver at least as much welfare as would be attained under discretion V_A^D (where the latter is defined by eq. 4). There are two important differences between this problem and the ones solved before. First, the proposer can strategically choose the initial status quo (before, we were considering s_0, which is the allocation under discretion). Second, the proposer is constrained to make both herself and the opposition at least as well off as under discretion.

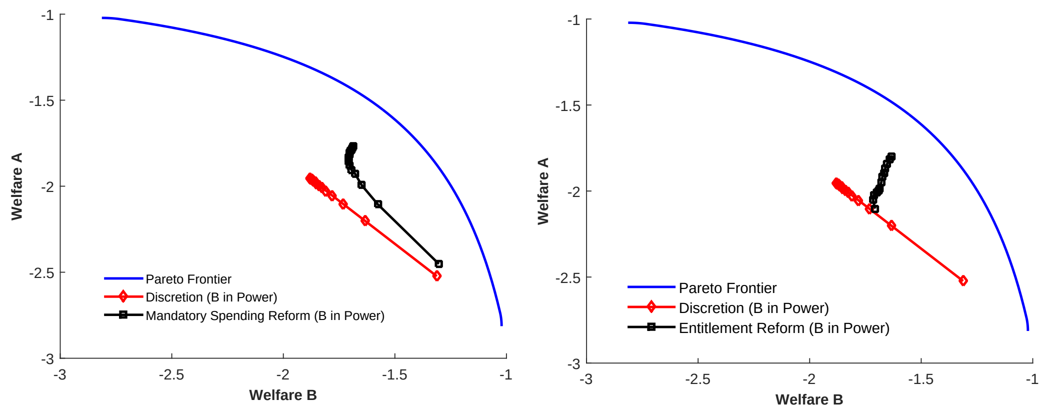


Figure 19: Equilibrium reform.

The left panel of Figure 19 shows the welfare obtained through a mandatory spending rule reform. The welfare achieved via the introduction of an entitlement program is displayed on the right. The first reform does not make significant changes to welfare compared to the previous exercise, mainly because it already made both agents better off. The only change relative to Figure 17 is that the proposer can now choose an even better initial status quo than s_0. The main difference arises when considering the adoption of an entitlement program. The proposer is now constrained to give enough welfare to the opposition in order for the reform to pass. This can no longer be achieved when incumbency advantage is substantial (i.e. as $p \to 1$). This is why the black-squared line converges to the red-diamond line for cases with high political persistence.

9 Conclusion

We described the welfare effects of introducing mandatory spending rules in an environment where agents disagree on the distribution of a fixed endowment. We showed that mandatory spending rules on public goods imply a Pareto improvement relative to a scenario where all spending is discretionary, while entitlement rules might only be beneficial to the party proposing their introduction. These findings are characterized theoretically in a two period example, and then quantitatively in the infinite horizon economy. Computing long-run allocations is important because the evolution of the endogenous status quo associated to each rule affects equilibrium allocations and the associated welfare levels attained by different groups in society. Our model can be extended in several interesting dimensions. First, we assumed that the two parties are identical and solved for a symmetric Markov-perfect equilibrium. A natural extension would consider the effects of political polarization, e.g. parties that disagree on the value of private vs public goods. Second, we assumed away the distortionary costs of taxation that may be associated to implementing the equilibrium allocations. In our model, we assumed that the government could implement any allocation without incurring deadweight losses, other than those implied by the bargaining process itself. If implementing specific allocations changed the size of the budget, this would plausibly impact the welfare gains of alternative budget rules. Finally, we assumed the government was subject to a balanced budget. It would be interesting to augment our model to consider the possibility of issuing public debt.

References

Alesina, A. and G. Tabellini (1990, 07). A Positive Theory of Fiscal Deficits and Government Debt. *The Review of Economic Studies 57*(3), 403–414.

Amador, M., I. Werning, and G.-M. Angeletos (2006). Commitment vs. flexibility. *Econometrica 74*(2), 365–396.

Anesi, V. (2010). Noncooperative Foundations of Stable Sets in Voting Games. *Games and Economic Behavior 70.2*, 488–493.

Anesi, V. and J. Duggan (2018, may). Existence and indeterminacy of Markovian equilibria in dynamic bargaining games. *Theoretical Economics 13*(2), 505–525.

Anesi, V. and D. J. Seidmann (2013). Bargaining in standing committees with an endogenous default. *Review of Economic Studies 82*(3), 825–867.

Arellano, C., Y. Bai, and G. Mihalache (2019). Monetary Policy and Sovereign Risk in Emerging Economies (NK-Default). Technical report.

Azzimonti, M. (2011, August). Barriers to investment in polarized societies. *American Economic Review 101*(5), 2182–2204.

Azzimonti, M., M. Battaglini, and S. Coate (2016). The costs and benefits of balanced budget rules: Lessons from a political economy model of fiscal policy. *Journal of Public Economics 136*(February), 45–61.

Baron, D. and E. Kalai (1993). The simplest equilibrium of a majority-rule division game. *Journal of Economic Theory 61*, 290–301.

Baron, D. P. (1996). A Dynamic Theory of Collective Goods Programs. *The American Political Science Review 90*(2), 316–330.

Baron, D. P. and T. R. Bowen (2015). Dynamic Coalitions. *Working paper*.

Baron, D. P. and J. A. Ferejohn (1989). Bargaining in Legislatures. *The American Political Science Review 83*(4), 1181–1206.

Battaglini, M. and S. Coate (2007, March). Inefficiency in legislative policymaking: A dynamic analysis. *American Economic Review 97*(1), 118–149.

Battaglini, M. and S. Coate (2008, March). A dynamic theory of public spending, taxation, and debt. *American Economic Review 98*(1), 201–36.

Battaglini, M., S. Nunnari, and T. S. Palfrey (2012). Legislative Bargaining and the Dynamics of Public Investment. *American Political Science Review 106*(02), 407–429.

Besley, T. and S. Coate (1997). An Economic Model of Representative Democracy. *The Quarterly Journal of Economics 112*(1), 85–114.

Bhaskar, V., G. J. Mailath, and S. Morris (2013). A foundation for markov equilibria in sequential games with finite social memory. *Review of Economic Studies 80*(3), 925–948.

Bowen, T. R., Y. Chen, and H. Eraslan (2014). Mandatory versus discretionary spending: The status quo effect.

Bowen, T. R., Y. Chen, H. Eraslan, and J. Zápal (2017). Efficiency of flexible budgetary institutions. *Journal of Economic Theory 167*(167), 148–176.

Chatterjee, S. and B. Eyigungor (2012, October). Maturity, Indebtedness, and Default Risk. *American Economic Review 102*(6), 2674–2699.

Chatterjee, S. and B. Eyigungor (2020). Policy Inertia, Election Uncertainty and Incumbency Disadvantage of Political Parties. *Review of Economic Studies*.

Diermeier, D., G. Egorov, and K. Sonin (2017). Political Economy of Redistribution. *Econometrica 85*(3), 851–870.

Duggan, J. and T. Kalandrakis (2012a). Dynamic legislative policy making. *Journal of Economic Theory 147*(5), 1653–1688.

Duggan, J. and T. Kalandrakis (2012b). Dynamic legislative policy making. *Journal of Economic Theory 147*(5), 1653 – 1688.

Dziuda, W. and A. Loeper (2016). Dynamic Collective Choice With Endogenous Status Quo. *Journal of Political Economy 124.4*, 1148–1186.

Dziuda, W. and A. Loeper (2018). Dynamic Pivotal Politics. *American Political Science Review 112.3*, 580–601.

Epple, D. and M. H. Riordan (1987, sep). Cooperation and punishment under repeated majority voting. *Public Choice 55*(1-2), 41–73.

Eyigungor, B. and S. Chatterjee (2019, January). Incumbency Disadvantage of Political Parties: The Role of Policy Inertia and Prospective Voting. Working Papers 19-7, Federal Reserve Bank of Philadelphia.

Gordon, G. (2019, September). Efficient Computation with Taste Shocks. Working Paper 19-15, Federal Reserve Bank of Richmond.

Grechyna, D. (2017). Mandatory Spending, Political Polarization, and Macroeconomic Volatility Mandatory Spending.

Halac, M. and P. Yared (2014). Fiscal Rules and Discretion Under Persistent Shocks. *Econometrica: Journal of the Econometric Society 82*(5).

Kalandrakis, A. (2004). A three-player dynamic majoritarian bargaining game. *Journal of Economic Theory 116*(2), 294–322.

Karakas, L. D. (2017). Institutional constraints and the inefficiency in public investments. *Journal of Public Economics 152*, 93–101.

Lizzeri, A. and N. Persico (2001, March). The provision of public goods under alternative electoral incentives. *American Economic Review 91*(1), 225–239.

Mailath, G. J. and L. Samuelson (2006). *Repeated Games and Reputations*, Volume 84.

Martin, F. (2009, October). A Positive Theory of Government Debt. *Review of Economic Dynamics 12*(4), 608–631.

McFadden, D. (1973). *Conditional logit analysis of qualitative choice behavior*.

Mihalache, G. (2019). Sovereign Default Resolution Through Maturity Extension. Technical report.

Osborne, M. J. and A. Slivinski (1996). A model of political competition with citizen-candidates. *Quarterly Journal of Economics 111*(1), 65–96.

Persson, T. and L. Svensson (1989). Why a stubborn conservative would run a deficit: Policy with time-inconsistent preferences. *The Quarterly Journal of Economics 104*(2), 325–345.

Persson, T. and G. E. Tabellini (2000). *Political Economics: Explaining Economic Policy*. MIT Press.

Piguillem, F. and A. Riboni (2011). Dynamic Bargaining over Redistribution in Legislatures. (April), 1–16.

Richter, M. (2014). Fully absorbing dynamic compromise. *Journal of Economic Theory 152*(1), 92–104.

Rubinstein, A. (1982). Perfect Equilibrium in a Bargaining Model. *Econometrica 50*(1), 97–109.

Rust, J. (1987, September). Optimal Replacement of GMC Bus Engines: An Empirical Model of Harold Zurcher. *Econometrica 55*(5), 999–1033.

Sanchez, J. M., M. Dvorkin, E. Yurdagul, and H. Sapriza (2018, June). Sovereign Debt Restructurings. Working Papers 2018-13, Federal Reserve Bank of St. Louis.

Yared, P. (2019, May). Rising government debt: Causes and solutions for a decades-old trend. *Journal of Economic Perspectives 33*(2), 115–40.

10 Appendix

10.1 Proof to Proposition 1.

Let $\bar{x}_g = \bar{x}_c = \bar{x}$. Given $\lambda \in [0,1]$ that denotes the Pareto-weight of type-B agents, the planner's Lagrangian is given by:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \{(1-\lambda)\ln(c_{A,t}) + \lambda \ln(c_{B,t}) + \ln(g_t) + \psi_t [Y - c_{A,t} - c_{B,t} - g_t] + \xi_t (c_{A,t} - \overline{x}) + \kappa_t (c_{B,t} - \overline{x}) + \mu_t (g_t - \overline{x})\}$$

The first order and Kuhn-Tucker conditions for this problem are $c_{A,t}, c_{B,t}, g_t \geq \overline{x}$, $\psi_t, \xi_t, \kappa_t, \mu_t \geq 0$

and

$$
\begin{aligned}
[c_{A,t}] \quad & \frac{1-\lambda}{c_{A,t}} - \psi_t + \xi_t = 0 \\
& \xi_t[c_{A,t} - \bar{x}] = 0 && (17) \\
[c_{B,t}] \quad & \frac{\lambda}{c_{B,t}} - \psi_t + \kappa_t = 0 \\
& \kappa_t[c_{B,t} - \bar{x}] = 0 && (18) \\
[g_t] \quad & \frac{1}{g_t} - \psi_t + \mu_t = 0 \\
& \mu_t[g_t - \bar{x}] = 0 && (19) \\
[RC] \quad & [Y - c_{A,t} - c_{B,t} - g_t]\psi_t = 0 && (20)
\end{aligned}
$$

Let $\psi_t > 0$. From eq. (20), we have that $g_t = Y - \overline{x} - c_{A,t}$. There are several cases to consider:

- Let $c_{B,t} = \overline{x}$ and $c_{A,t}, g_t > \bar{x}$. This implies $\xi_t = \mu_t = 0$. From (17) and (19), we have that $c_{A,t} = (1-\lambda)g_t$. Going back to eq. (20), we have that $g_t = \frac{Y-\overline{x}}{2-\lambda}$, which implies $c_{A,t} = \frac{(1-\lambda)(Y-\overline{x})}{2-\lambda}$. From eq. (18), for these cases to hold, we require $\lambda \leq \lambda_1 = \frac{2\overline{x}}{Y}$.

- Let $c_{A,t} = \overline{x}$ and $c_{B,t}, g_t > \bar{x}$. This implies $\kappa_t = \mu_t = 0$. By (18) and (19), we have that $c_{B,t} = \frac{\lambda(Y-\overline{x})}{1+\lambda}$. Going back to eq. (20), we have that $g_t = \frac{Y-\overline{x}}{1+\lambda}$. By 17, for this cases to hold we require $\lambda \geq \lambda_2 = \frac{Y-2\overline{x}}{Y}$.

- Let $g_t = \overline{x}$ and $c_{A,t}, c_{B,t} > \bar{x}$. This implies $\kappa_t = \xi_t = 0$. By eqs. (17) and (18), we have that $c_{A,t} = \frac{(1-\lambda)(Y-\overline{x})}{1+2\lambda}$. Going back to eq. (20), we have that $c_B = \frac{3\lambda(Y-\overline{x})}{1+2\lambda}$. By eq. (19), for this cases to hold we require $\lambda \leq \frac{Y-2\overline{x}}{2\overline{x}}$. This would require $\overline{x} \geq \frac{Y}{4}$, which goes against our assumption that $\overline{x}$ is relatively small compared to Y. Therefore, this case can be disregarded.

- Let $c_{A,t}, c_{B,t}, g_t > \overline{x}$. By eqs. (17) and (19), we have that $c_{A,t} = (1-\lambda)g_t$. By (18) and (19), we have that $c_{B,t} = \lambda g_t$. By (20), we have that $g_t = \frac{Y}{2}$. Going back to eqs. (17) and (18), we have that $c_{A,t} = \frac{(1-\lambda)Y}{2}$ and $c_{B,t} = \frac{\lambda Y}{2}$. For this, we need $\lambda \in [\lambda_1, \lambda_2]$.

10.2 Proof to Proposition 3

Party's B Lagrangian for this problem at $t = 2$ is given by:

$$
\begin{aligned}
\mathcal{L} = \ln(c_{B,2}) + \ln(g_2) + \psi\left[Y - c_{B,2} - c_{A,2} - g_2\right] + \gamma\left[\ln(c_{A,2}) + \ln(g_2) - K_{A,2}(\boldsymbol{s}_2)\right] + \\
\xi\left(c_{B,2} - \overline{x}\right) + \kappa\left(c_{A,2} - \overline{x}\right) + \mu\left(g_2 - \overline{x}\right)
\end{aligned}
$$

Let s define the relevant status quo allocation to simplify notation. Under a mandatory spending rule $s = g_1$, and with entitlement rules, $s = c_{A,1}$. We can thus write $K_{A,2}(s) = \ln(\bar{x}) + \ln(s)$. The first order and Kuhn-Tucker conditions for this problem are $c_{B,2}, c_{A,2}, g_2 \geq \bar{x}$, $\psi, \gamma, \xi, \kappa, \mu \geq 0$ and

$$\begin{aligned}
[c_{B,2}] \quad & \frac{1}{c_{B,2}} - \psi + \xi = 0 \\
& \xi[c_{B,2} - \bar{x}] = 0 && (21) \\
[c_{A,2}] \quad & \frac{\gamma}{c_{A,2}} - \psi + \kappa = 0 \\
& \kappa[c_{A,2} - \bar{x}] = 0 && (22) \\
[g] \quad & \frac{(1+\gamma)}{g_2} - \psi + \mu = 0 \\
& \mu[g_2 - \bar{x}] = 0 && (23) \\
[RC] \quad & [Y - c_{B,2} - c_{A,2} - g_2]\psi = 0 && (24) \\
[IRC] \quad & [\ln(c_{A,2}) + \ln(g_2) - K_{A,2}(s_2)]\gamma = 0 && (25) \\
& && (26)
\end{aligned}$$

First assume $\psi = 0$. By (21) we have that $c_{B,2} < 0$, which contradicts the fact that we require $c_{B,2} \geq \bar{x}$. We conclude that $\psi > 0$ which implies that $Y - c_{B,2} - c_{A,2} - g_2 = 0$. There are several cases to consider:

- $\gamma = 0$: Since $\psi > 0$, eq. (22) implies that $c_{A,2} = \bar{x}$. Eqs. (21) and (23) imply that $c_{B,2} = g_2$. Combined with (24), this implies that $c_{B,2} = g_2 = \frac{Y-\bar{x}}{2}$. For the inequality (25) to hold, $s < s_L = \frac{Y-\bar{x}}{2}$ given $s = c_{A,1}$ in the case of entitlements and $s = g_1$ for mandatory spending over public goods.

- $\gamma > 0, c_{A,2} = \bar{x}, c_{B,2}, g_2 > \bar{x}$: Since $c_{A,2} = \bar{x}$, (25) directly implies that $g = s$. (24) implies that $c_{B,2} = Y - s - \bar{x}$. For inequality (22) to hold, we require $s < s_H = \frac{Y}{2}$ and $s \geq s_L$.

- $\gamma > 0$, $g_2 = \bar{x}$, and $c_{B,2}, c_{A,2} > \bar{x}$. This implies $\xi = \kappa = 0$. Since $g_2 = \bar{x}$, eq. (25) implies that $c_{A,2} = s$ and eq. (24) implies that $c_{B,2} = Y - \bar{x} - s$. By eq. (21), $\psi = \frac{1}{c_{B,2}}$. Combining this with eq.(22), we obtain $\gamma = \frac{c_{A,2}}{c_{B,2}}$. For the inequality (23) to hold, we require $\bar{x} > s_H = \frac{Y}{2}$, a contradiction to the assumption that $\bar{x}$ is relatively "small". This case can be disregarded.

- $\gamma > 0$, $c_{B,2} = \bar{x}$, and $c_{A,2}, g_2 > \bar{x}$: By similar construction than above, this case can also be disregarded.

- $\gamma > 0$, and $c_{B,2}, c_{A,2}, g_2 > \bar{x}$. This implies $\xi = \kappa = \mu = 0$. By eq. (21), $\psi = \frac{1}{c_{B,2}}$. Combining this with (22) implies $\gamma = \frac{c_{A,2}}{c_{B,2}}$. From eq. (23), this implies that $c_{B,2} = g - c_{A,2}$. Using eq. (24), this implies that $g_2 = \frac{Y}{2}$. Combining this with eq. (25) , we obtain $c_{A,2} = \frac{2s\bar{x}}{Y}$. Going back to eq.(23), this implies that $c_{B,2} = \frac{Y^2 - 4s\bar{x}}{2Y}$. Since $\bar{x} \leq c_{B,2} \leq 1$, $\bar{x} \leq c_{A,2} \leq 1$ and $\bar{x} \leq g_2 \leq 1$, we require that $s \geq s_H = \frac{Y}{2}$.

10.3 Proof to Proposition 4

Party's B Lagrangian for this problem at $t = 1$ is given by:

$$\mathcal{L} = \ln(c_{B,1}) + \ln(g_1) + \beta \left\{ pV_B(g_1, m) + (1-p)W_B(g_1, m) \right\} + \psi \left[Y - c_{B,1} - c_{A,1} - g_1 \right] + \xi \left(c_{B,1} - \bar{x} \right) + \kappa \left(c_{A,1} - \bar{x} \right) + \mu \left(g_1 - \bar{x} \right)$$

where V_B and W_B are defined in the main body of this paper. The first-order and Kuhn-Tucker conditions party B are

$$[c_{B,1}] \quad \frac{1}{c_{B,1}} - \psi + \xi = 0.$$
$$\xi[c_{B,1} - \bar{x}] = 0. \tag{27}$$
$$[c_{A,1}] \quad -\psi + \kappa = 0.$$
$$\kappa[c_{A,1} - \bar{x}] = 0. \tag{28}$$
$$[g_1] \quad \frac{1}{g_1} - \psi + \mu + \beta \left\{ p\frac{\partial V_B(g_1, m)}{\partial g_1} + (1-p)\frac{\partial W_B(g_1, m)}{\partial g_1} \right\} = 0.$$
$$\mu[g_1 - \bar{x}] = 0. \tag{29}$$
$$[RC] \quad [Y - c_{B,1} - c_{A,1} - g_1]\, \psi = 0 \tag{30}$$
$$\tag{31}$$

- First assume $\psi = 0$. By eq. (27) we have that $c_{B,1} < 0$, a contradiction. We conclude that $\psi > 0$ which implies that the resource constraint binds. Next, we have several cases to consider:
- Since $\psi > 0$, eq. (28) implies that $c_{A,1} = \overline{x}$. By eq.(30), $c_{B,1} = Y - \overline{x} - g_1$. Combining (27) and (29), we have:

$$\frac{1}{Y - \overline{x} - g_1} = \frac{1}{g_1} + \beta \left\{ p\frac{\partial V_B(g_1, m)}{\partial g_1} + (1-p)\frac{\partial W_B(g_1, m)}{\partial g_1} \right\} \tag{32}$$

Let's first assume $g_1 < \frac{Y - \overline{x}}{2}$. This implies that $\frac{\partial V_B(g_1,m)}{\partial g_1} = \frac{\partial W_B(g_1,m)}{\partial g_1} = 0$ and so, $g_1 = \frac{Y-\overline{x}}{2}$, a contradiction. Now let's assume $\frac{Y-\overline{x}}{2} \leq g_1 \leq \frac{Y}{2}$. This implies that $\frac{\partial V_B(g_1,m)}{\partial g_1} = \frac{1}{g_1} - \frac{1}{Y-\overline{x}-g_1}$ and $\frac{\partial W_B(g_1,m)}{\partial g_1} = \frac{1}{g_1}$. By (32), we have that $g_1 = \frac{(1+\beta)(Y-\overline{x})}{2+\beta(1+p)}$. For this case to hold, we require that $\frac{Y-\overline{x}}{2} \leq g_1 \leq \frac{Y}{2}$, which implies $p \geq p^* = 1 - \frac{2\overline{x}(1+\beta)}{Y\beta}$. Finally, let's assume $g_1 \geq \frac{Y}{2}$. This implies that $\frac{\partial V_B(g_1,m)}{\partial g_1} = \frac{-4\overline{x}}{Y^2 - 4\overline{x}g_1}$ and $\frac{\partial W_B(g_1,m)}{\partial g_1} = \frac{1}{g_1}$. By (32), we have that g_1 is such that

$$\frac{1}{Y - \overline{x} - g_1} - \frac{1 + \beta(1-p)}{g_1} - \frac{4p\beta\overline{x}}{Y^2 - 4\overline{x}g_1} = 0. \tag{33}$$

Rearranging, we have:

$$(Y - \overline{x})Y^2\,(1 + \beta\,(1-p)) - \left[4\overline{x}(1+\beta)(Y - \overline{x}) + Y^2\,(2 + \beta(1-p))\right] g_1 + 4\overline{x}(2+\beta)g_1^2 = 0 \tag{34}$$

where $\delta(p) = (Y - \overline{x})Y^2\,(1 + \beta\,(1-p))$, $\alpha(p) = \left[4\overline{x}(1+\beta)(Y-\overline{x}) + Y^2\,(2+\beta(1-p))\right]$ and $\kappa(p) = 4\overline{x}(2+\beta)$. Therefore, $g_1 = \frac{\alpha(p) - \sqrt{\alpha(p)^2 - 4\delta(p)\kappa(p)}}{2\kappa(p)}$.

10.4 Proof to Proposition 5

Party's B Lagrangian for this problem at $t = 1$ is given by:

$$\mathcal{L} = \ln(c_{B,1}) + \ln(g_1) + \beta\left\{pV_B(c_{A,1}, m) + (1-p)W_B(c_{B,1}, e)\right\} + \psi\left[Y - c_{B,1} - c_{A,1} - g_1\right] + \xi\left(c_{B,1} - \overline{x}\right) + \kappa\left(c_{A,1} - \overline{x}\right) + \mu\left(g_1 - \overline{x}\right)$$

where V_B and W_B are defined as in the main body of this paper.

The first-order and Kuhn-Tucker conditions party B are

$$\begin{aligned}
[c_{B,1}] \quad & \frac{1}{c_{B,1}} + \beta(1-p)\frac{\partial W_B(c_{B,1},e)}{\partial c_{B,1}} - \psi = 0 \\
& \xi[c_{B,1} - \bar{x}] = 0 && (35) \\
[c_{A,1}] \quad & -\psi + \beta p \frac{\partial V_B(\boldsymbol{x_1},e)}{\partial c_{A,1}} = 0 \\
& \kappa[c_{A,1} - \bar{x}] = 0 && (36) \\
[g_1] \quad & \frac{1}{g_1} - \psi = 0 \\
& \mu[g_1 - \bar{x}] = 0 && (37) \\
[RC] \quad & [Y - c_{B,1} - c_{A,1} - g_1]\,\psi = 0. && (38)
\end{aligned}$$

First assume $\psi = 0$. By eq. (35) we have that $c_{B,1} < \bar{x}$, a contradiction. We conclude that $\psi > 0$ which implies that the resource constraint binds. Since $\psi > 0$, eq. (36) implies that $c_{A,1} = \overline{x}$. By eq. (38), $c_{B,1} = Y - \overline{x} - g_1$. Combining eqs. (35) and (37), we have:

$$\frac{1}{Y - \overline{x} - g_1} + \beta(1-p)\frac{\partial W_B(c_{B,1},e)}{\partial c_{B,1}} = \frac{1}{g_1} \tag{39}$$

Differently from the mandatory spending case, the solution now only depends on $W_B(.)$, which is a piecewise function. Let's first assume $c_{B,1} < \frac{Y-\overline{x}}{2}$. This implies that $\frac{\partial W_B(x_2)}{\partial g_1} = 0$ and so, $c_{B,1} = \frac{Y-\overline{x}}{2}$, a contradiction.

Now let's assume $c_{B,1} \geq \frac{Y-\overline{x}}{2}$. In this case $\frac{\partial W_B(c_{B,1},e)}{\partial c_{B,1}} = \frac{1}{c_{B,1}}$. By eq. (39), we have that $c_{B,1} = \frac{(1+\beta(1-p))(Y-\overline{x})}{2+\beta(1+p)}$.

11 Numerical Implementation

11.1 Model Under Symmetry

Value of proposer:

$$\begin{aligned}
V(\overline{c}_i,\overline{c}_j,\overline{g}) = \max_{c_i,c_j,g} \; & u(c_i,g) + \beta\left\{pV(c_i,c_j,g) + (1-p)W(c_j,c_i,g)\right\} \\
\text{s.t. } & u(c_j,g) + \beta\left\{(1-p)V(c_j,c_i,g) + pW(c_i,c_j,g)\right\} \geq K(\overline{c}_i,\overline{c}_j,\overline{g}) \\
& c_i \geq \overline{x}_c, \quad c_j \geq \overline{x}_c, \quad g \geq \overline{x}_g, \quad c_i + c_j + g \leq Y
\end{aligned} \tag{40}$$

Equilibrium payoff for agent not proposing:

$$\begin{aligned}
W(\overline{c}_i,\overline{c}_j,\overline{g}) = & u\left[\mathcal{C}_j(\overline{c}_i,\overline{c}_j,\overline{g}),\mathcal{G}(\overline{c}_i,\overline{c}_j,\overline{g})\right] \\
& + \beta(1-p)V\left[\mathcal{C}_j(\overline{c}_i,\overline{c}_j,\overline{g}),\mathcal{C}_i(\overline{c}_i,\overline{c}_j,\overline{g}),\mathcal{G}(\overline{c}_i,\overline{c}_j,\overline{g})\right] \\
& + \beta pW\left[\mathcal{C}_i(\overline{c}_i,\overline{c}_j,\overline{g}),\mathcal{C}_j(\overline{c}_i,\overline{c}_j,\overline{g}),\mathcal{G}(\overline{c}_i,\overline{c}_j,\overline{g})\right]
\end{aligned} \tag{41}$$

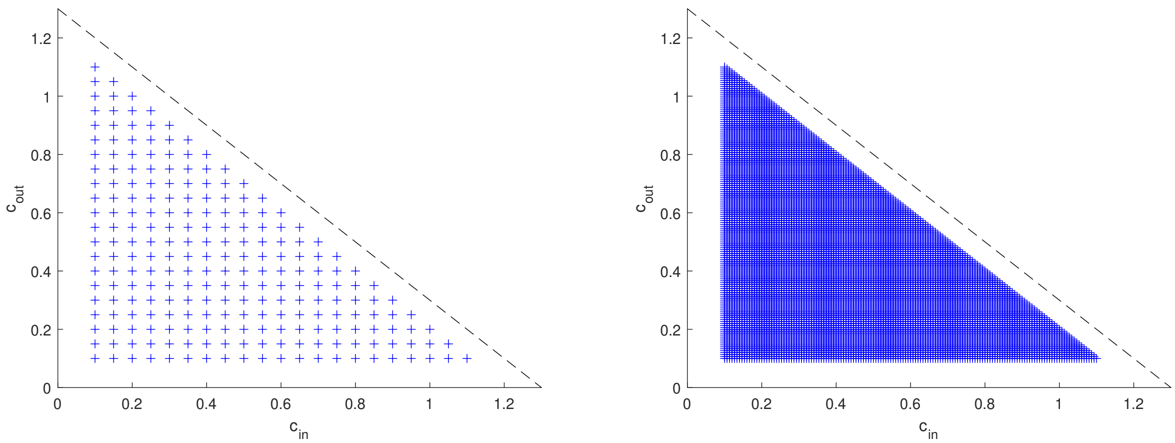
Reservation payoff under status quo:

$$K(\overline{c}_i,\overline{c}_j,\overline{g}) = \begin{cases} u(\overline{c}_j,\overline{x}_g) + \beta\left\{(1-p)V(\overline{c}_j,\overline{c}_i,\overline{g}) + pW(\overline{c}_i,\overline{c}_j,\overline{g})\right\} & \text{C Rule} \\ u(\overline{x}_c,\overline{g}) + \beta\left\{(1-p)V(\overline{c}_j,\overline{c}_i,\overline{g}) + pW(\overline{c}_i,\overline{c}_j,\overline{g})\right\} & \text{G Rule} \end{cases} \tag{42}$$

11.2 Triangular Grid

We solve the model "on the grid," so that both the status quo and chosen allocations must lie on our grid, using taste shocks methods to be detailed shortly. We construct a triangular grid so that all points respect resource feasibility.

We fix a grid for the level of private consumption, n elements equally spaced over $\left[\overline{x}_c, Y - \overline{x}_c - \overline{x}_g\right]$. Then, we consider the Cartesian product of two such vectors, discarding pairs that would violate $g = Y - c_i - c_j \geq \overline{x}_g$. We are left with $\tilde{n} = (n+1)n/2$ grid points. Note that we must restrict attention to odd values of n. Denote, for future reference, the set of all grid elements by $\mathbb{S}$.



(a) Example: $n = 21, \tilde{n} = 231$ **(b)** Actual: $n = 151, \tilde{n} = 11,476$

Figure 20: Triangular Grids

In Figure 20 we plot two such grids, first an example for illustration in Figure 20a, for $n = 21$, and the actual grid used in the computation, in Figure 20b, with $n = 151$ for a total of $11,476$ possible choices.

11.3 Computation with Taste Shocks

We perturb the proposer's problem by augmenting it with choice-specific taste shocks, distributed Gumbel (Extreme Value Type I), following Gordon (2019)[6], as commonly employed with discrete choice methods, e.g. Rust (1987). To simplify notation, let the status quo be given by $\overline{s} = \langle \overline{c}_i, \overline{c}_j, \overline{g} \rangle$ and any potential choice denoted by $s = \langle c_i, c_j, g \rangle$ and its associated proposer turn-over counterpart $f(s) = \langle c_j, c_i, g \rangle$.

Define the acceptance set as

$$A(\overline{s}) = \left\{ s \in \mathbb{S} \,\middle|\, u(c_j, g) + \beta \left[(1-p)V(s) + pW(f(s)) \right] \geq K(\overline{s}) \right\}. \tag{43}$$

We write the value to the proposer from proposing s', net of taste shocks, as

$$\mathcal{J}(\overline{s}, s) = \begin{cases} u(c_i, g) + \beta \left\{ pV(s) + (1-p)W(f(s)) \right\}, & \text{if } s \in A(\overline{s}) \\ -\infty, & \text{otherwise} \end{cases} \tag{44}$$

[6]Related, recent applications to fiscal policy and sovereign default include Sanchez et al. (2018), Mihalache (2019), and Arellano et al. (2019).

and record the greatest value over s by

$$\overline{\mathcal{J}}(\bar{s}) = \max_{s \in \mathbb{S}} \mathcal{J}(\bar{s}, s) \tag{45}$$

The value to the proposer, given realized iid taste shocks $\{\varepsilon_s\}_s$ is

$$\mathcal{V}(\bar{s}, \{\varepsilon_s\}_s) = \max_{s \in \mathbb{S}} \{\mathcal{J}(\bar{s}, s) + \rho\, \varepsilon_s\}. \tag{46}$$

Following the standard proofs for discrete choice methods, e.g. McFadden (1973), it can be shown that ex-ante, before taste shocks are realized, the probability of choosing a particular option $\hat{s}$ is given by

$$\Pr(s = \hat{s}|\bar{s}) = \frac{\exp\left[\mathcal{J}(\bar{s}, \hat{s})/\rho\right]}{\sum_z \exp\left[\mathcal{J}(\bar{s}, z)/\rho\right]} = \frac{\exp\left[(\mathcal{J}(\bar{s}, \hat{s}) - \overline{\mathcal{J}}(\bar{s}))/\rho\right]}{\sum_z \exp\left[(\mathcal{J}(\bar{s}, z) - \overline{\mathcal{J}}(\bar{s}))/\rho\right]} \tag{47}$$

and the expected value to the proposer, before observing the taste shocks, is

$$V(\bar{s}) = \mathbb{E}_{\{\varepsilon_s\}_s}\{\mathcal{V}(\bar{s}, \{\varepsilon_s\}_s)\} = \overline{\mathcal{J}}(\bar{s}) + \rho \log\left\{\sum_z \exp\left[(\mathcal{J}(\bar{s}, z) - \overline{\mathcal{J}}(\bar{s}))/\rho\right]\right\} \tag{48}$$

while the value of the agent receiving the proposal is

$$W(\bar{s}) = \sum_z \left\{\Pr(s = z|\bar{s})\left[u(c_j, g) + \beta\left((1-p)V(z) + pW(f(z))\right)\right]\right\}. \tag{49}$$

We remark briefly on key properties of the choice probabilities and of the solution. First, the probability of choosing s is strictly increasing in the value net of taste shocks for s, $\mathcal{J}(\bar{s}, s)$, so that better options are picked with higher probability. Second, given our use of the acceptance set, all s that would not be accepted are proposed to probability zero. Third, the mean level of the Gumbel tastes shocks is non-zero, yet this does not alter choice probabilities: what matters for the likelihood of choosing s over e.g. z is the difference between ε_s and ε_z, which is Logistic(0, 1), as well as values net of taste shocks. Finally, the parameter ρ scales the importance of the taste shocks for the proposal decision. If we take value $\rho \to 0$, tastes shocks no longer play a role and the underlying best option is picked with probability 1. In turn, for arbitrarily high ρ values, all members of the acceptance set would be proposed with equal probability.