The politics of sovereign default under financial integration

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Abstract

In this paper we study the role of portfolio diversification on optimal default of sovereign debt in a two-country model with large economies that are financially integrated. Relative to a closed economy, the integration of financial markets increases the incentives to default by a given country not only because part of the defaulted debt is owned by foreigners (the standard channel), but also because the endogenous macroeconomic cost of a default is smaller when the defaulting country is financially integrated. We show that increases in external liquidity (e.g. in the stock of debt issued by a foreign country) may trigger a domestic default, and that the associated macroeconomic costs are borne by both domestic and foreign creditors. Because of this, creditor countries may find it beneficial to bail-out defaulting ones. Despite of inducing moral hazard, bailouts can be Pareto improving ex-ante relative to a world in which the creditor countries commit never to renegotiate debt repayments.

1 Introduction

The last thirty years have been characterized by a rapid integration of financial markets. As a result, governments were able to 'export' public debt to non-residents. The top panel of Figure 1 illustrates the increase in the share of sovereign debt held abroad for a set of countries between 1997 and 2010.

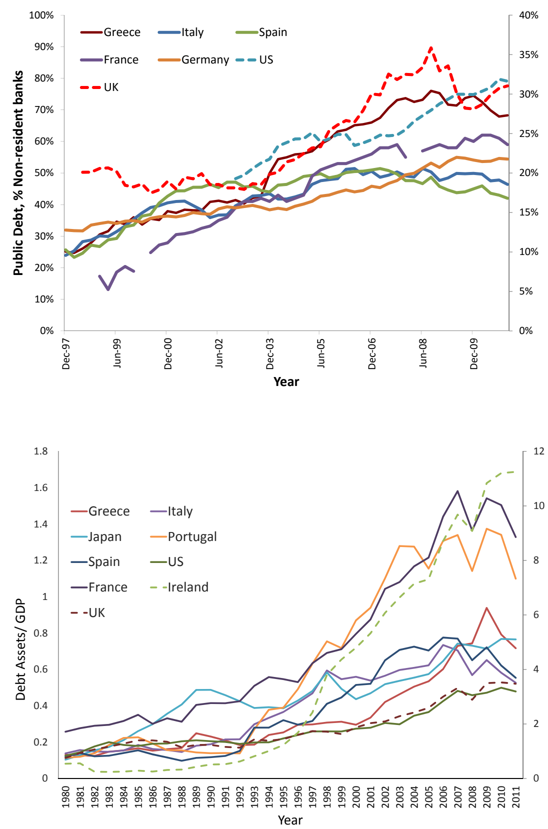


Figure 1: Share of Public Debt Held Abroad (top) and 'External Debt Assets as a Percentage of GDP' (bottom)

During the same period, domestic residents have also 'imported' foreign public debt. This is evident from the large increase in the cross-country ownership of external financial assets (including sovereign debt) as a percentage of GDP, depicted at the bottom panel of Figure 1. One of the main consequences of financial integration is a radical change in the portfolio composition of domestic residents, allowing for greater international diversification.

This, in turn, generates important financial linkages among countries. The objective of this paper is to investigate how the international diversification of portfolios affects: (i) the countries' incentives to default on their sovereign debt, (ii) the international transmission of macroeconomic costs that arise as a consequence of default (e.g. spillover effects), and (iii) the incentives of creditor countries to bail-out defaulting ones.

Of course, if a larger share of sovereign debt is held by foreigners, the incentive of the government to default may increase since default redistributes wealth from foreign residents to domestic residents. This mechanism is well recognized in the literature although there some studies challenge its relevance (see Broner, Martin and Ventura (2010)). In this paper, however, we explore a different mechanism through which financial globalization increases the incentive of a country to default. We show that the macroeconomic cost of default declines when financial markets become more globalized.

The majority of studies examine the financial and macroeconomic dynamics that take place within a defaulting country. For example, a sequence of negative productivity or fiscal shocks induces the country to borrow more and, if the economic situation continues to deteriorate, it becomes optimal or necessary for the country to default. Sometimes, however, the mechanism that leads a country to default, may not originate domestically. In particular, how much debt is issued by *other* countries may also be a relevant determinant of default incentives. Why are the negative macroeconomic consequences of default smaller when the country is financially integrated? The central mechanism is the disruption of financial markets created by default. When a government defaults on its debt, the holders of government debt incur capital losses. To the extent that financial wealth is important for economic decisions for certain agents, this has a negative effect on aggregate economic activity. When financial markets are integrated, however, domestic residents hold a smaller share of their wealth in domestic assets and a larger share in foreign assets. This implies that, when the domestic government defaults, the wealth losses of domestic residents are smaller and this generates a smaller macroeconomic contraction. Being the macroeconomic cost smaller, the government has more incentives to default.

To study this mechanism, we consider a two-period/two-country model where the domestic country h is a 'risky' country and the foreign country f is a 'safe' country. The issuance of debt and its repayment are chosen optimally by the governments of both countries. Using this model we consider an exogenous change in the debt of the foreign country and study how this affects the incentive of the domestic country to default. As the debt of country f increases, residents in country h acquire (hold) more foreign debt. This implies that the holding of safe, nondefaultable debt increases in country h. Then, if the domestic government defaults, domestic agents face a proportionally smaller loss in their financial wealth, which in turn implies that the negative macroeconomic consequences of default are smaller. This reduces the macroeconomic cost of default and increases the incentive of the government to default.

A related implication is that this mechanism creates the conditions for greater instability also in the foreign country even if its debt is safe (non-defaultable). This is because the higher incentive to default in the home country implies higher potential wealth losses for the residents of country f since they hold part of the debt of country h in their portfolios. Thus, the foreign country will also experience a macroeconomic cost if h defaults (in addition to the direct capital loss). This helps us understand why safe countries have a vested interest in avoiding default of 'risky' countries. It also illustrates a potential inefficiency in government policies: when a government chooses to default, it does not take into account the 'macroeconomic' cost that other countries would incur.

The international macroeconomic spillover from default generates interesting implications regarding the strategic interaction between large economies. Because the safe country internalizes how its own supply of debt could potentially trigger a default in the other country, it has a strategic incentive to manipulate the other country default decision by limiting the issuance of its own debt (for example, by implementing austerity measures). Assuming that both countries choose their levels of debt in the first period simultaneously and non-cooperatively, we find that there is a multiplicity of equilibria within an endogenously determined bounded set. At one extreme, there are equilibria characterized by a large proportion of risky debt, low defaults and low risk premia. At the other extreme, we observe equilibria with a high level of risky debt, large defaults, and high risk premia. All these equilibria involve sub-optimally low worldwide levels of debt relative to the autarkic case. The reason being that with financial integration, domestic debt provides lower benefits to the domestic economy, since of the debt is purchased by foreigners. Due to the fact that each country ignores the benefits to the other country (an externality), the aggregate level of liquidity is too low.

Because of the non-cooperative nature of the equilibrium and the macroeconomic splillover from default, when a country defaults ex-post, the other country may have an incentive to supply a bailout package in order to prevent a default. Thus we also study the second period bargaining problem between the two countries in the event of default. This will take the form of financial transfers or concessions that the two countries negotiate upon in order to guarantee full debt repayment. Under those states in which the risky country has an incentive to default, a bailout package that transfers resources from the safe country to the risky country can be Pareto improving relative to a world in which the safe country commits never to renegotiate repayments. This is because preventing the risk country from defaulting eliminates the macroeconomic cost of default in both countries. Moreover, with bailouts the worldwide level of debt is higher and closer to that attained under autarky.

2 Literature review

This paper builds on a large literature on public debt determination under incomplete markets. The main role of government debt in our paper is to partially complete the assets market when agents are subject to uninsurable idiosyncratic risk. The mechanism is similar to that in Aiyagari and McGrattan (1998), Azzimonti, de Francisco, and Quadrini (2014, AFQ henceforth), Golosov and Sargent (2012), and Floden (2001), who study heterogeneous agents models without default. Closest to our paper is the work by Azzimonti, de Francisco, and Quadrini (2014), in which debt also acts a self-insurance mechanism affecting consumption dispersion. There are, however, three main departures from that paper. First, our economy is subject to both idiosyncratic and aggregate uncertainty, whereas AFQ only considers the first type of uncertainty. Second, the stock of public debt affects labor markets and hence the level of production. Finally, and more importantly, debt can be partially defaulted on in this model. Because of this, our paper is also related to a growing literature on external sovereign default based on the influential work of Eaton and Gersovitz (1981) (e.g. Aguiar and Amador (2013), Aguiar and Gopinath (2006), Arellano (2008), Cuadra, Sanchez, and Sapriza (2010), Pouzo and Presno (2014), Yue (2010), among others).[1]

Because defaults cause redistribution, and affects agents asymmetrically, our work is related to the literature on political economy and sovereign default. Alesina and Tabellini (1990), Aghion and Bolton (1990), D'Erasmo

[1]See Aguiar and Amador (2014) or Tomz and Wright (2012) for recent reviews of this literature.

and Mendoza (2013, 2016), Dovis, Golosov, and Shourideh (2014) focused on the consequences of default in closed economies, whereas Amador (2003), Aguiar, Amador, Farhi, and Gopinath (2013), Guembel and Sussman (2009), Hatchondo, Martinez, and Sapriza (2009), Mendoza and Yue (2012), and Tabellini (1991) concentrated on external defaults instead.

The paper is also related to a sub-strand of this literature that focuses on the consequences of default on domestic agents and the role of secondary markets in cases where debt provides liquidity (see Guembel and Sussman (2009), Broner, Martin, and Ventura (2010), Broner and Ventura (2011), Gennaioli, Martin, and Rossi (2014), Basu (2009), Brutti (2011), and Di Casola and Sichlimiris (2014)). As in these studies, the government cannot discriminate across any of its creditors (local vs foreign) when it defaults. Extending the work of Gennaioli, Martin, and Rossi (2014), a recent set of papers studies the interaction between sovereign debt and domestic financial institutions (e.g. Sosa-Padilla (2012), Bocola (2014), and Perez (2015)). Like in our paper, the cost of default is endogenous in their work, as it disrupts the financing of productive firms and creates a recession. Because they focus on domestic debt, the effects of increases in the supply of debt by safe countries is ignored, which is at the core of our paper.

The multiplicity of equilibria that arises under no renegotiation results from strategic manipulation of default decisions by the safe country in our model. This is different from Cole and Kehoe (2000)'s self fulfilling equilibria arising from lack of coordination between lenders, from Arellano and Yue (2014) in which multiplicity arises due to lack of coordination between borrowers, and from Cooper and Nikolov (2016) where multiplicity arises from the game between domestic banks and the government.

As in Arellano and Bai (2008), a default in the domestic country affects other countries through changes in the interest rate.[2] In their paper, this results from borrowers being risk-averse, whereas in our case it arises from strategic interaction between the governments of these countries. The difference arises because they restrict attention to small open economies, whereas we consider large open economies which are not price-takers.

[2]See also Borri and Verdelhan (2009), Park (2013), and Lizarazo (2013) for similar environments and Pouzo and Presno (2011) for a setup with lenders with uncertainty aversion

3 The model

We analyze a two-period economy composed of two large countries: home and foreign. The only difference between the two countries is that the foreign country can credibly commit to repay its public debt (so it will be referred to as the 'safe' country) whereas the home country may default (and hence we will refer to it as the 'risky' country). In all other dimensions, the two countries are completely symmetric. To understand how default decisions affect the economy, we first describe the politico-equilibrium under autarky. The case with integrated financial markets is described in the next section. This presentation sequence makes clear how financial integration affects the costs of default through portfolio diversification.

In each country there are two types of agents: a measure 1 of workers and a measure 1 of entrepreneurs. The assumption that the number of workers is the same as the number of entrepreneurs is without loss of generality: the equilibrium is unaffected by the relative size of the two groups of agents. In the first period, workers are endowed with wealth e_1 and entrepreneurs with wealth a_1. We can think of e_1 and a_1 as the wealth of workers and entrepreneurs accumulated up to the end of period 1. Given their wealth, agents make consumption/saving decisions and move to the second period. In the terminal period 2, entrepreneurs produce with the input of labor hired from workers. Therefore, production takes place only in period 2.

Workers value consumption and leisure as follows

$$U(c_1) + \beta U\Big(\varphi(c_2, h_2)\Big) = \ln(c_1) + \beta \ln\left(c_2 - \alpha \frac{h_2^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}\right),$$

where c_1 and c_2 denote consumption in period 1 and 2, respectively, and h_2 is the labor supplied in period 2. The function $\varphi(.,.)$ denotes, in compact form, consumption net of the dis-utility of working. Workers receive lump-sum transfers T_t from the government each period and are excluded from financial markets (e.g., they are hand-to-mouth). Their budget constraints are therefore,

$$\begin{aligned} c_1 &= e_1 + T_1, \\ c_2 &= e_2 + w_2 h_2 + T_2, \end{aligned}$$

where w_2 denotes the wage rate. The stark assumption of market segmentation simplifies the exposition, but it is not necessary. Our results would hold in an environment in which workers had access to financial markets but were subject to a borrowing limit.[3]

The utility of entrepreneurs is

$$u(d_1) + \beta u(d_2) = \ln(d_1) + \beta \ln(d_2),$$

where d_1 and d_2 denote their consumption levels in periods 1 and 2, respectively. Entrepreneurs produce output in the second period using a linear technology

$$y_2 = A(z_2, \varepsilon_2) l_2,$$

where l_2 is the input of labor, z_2 is an aggregate productivity shock, and ε_2 is an idiosyncratic productivity shock. We assume that entrepreneurs observe the state of the economy z_2 before hiring workers, but do not know whether their project was successful until production takes place. Hence, they choose l_2 under uncertainty. The importance of this timing assumption will become clearer when we describe the labor market equilibrium.

Entrepreneurs have access to financial instruments, but markets are incomplete. In particular, we assume that while they can trade one-period government bonds, there is no market for contingent claims. Therefore, the idiosyncratic risk cannot be perfectly insured. The budget constraints for entrepreneurs in the first and second periods are,

$$\begin{aligned} d_1 &= a_1 - \frac{b_1}{R_1}, \\ d_2 &= \Big[A(z_2, \varepsilon_2) - w_2\Big] l_2 + \delta_2 b_1, \end{aligned}$$

where b_1 represents the government bonds purchased in period 1 and $1/R_1$ the equilibrium price for these bonds.

The government issues B_1 bonds in the first period and distributes all revenues to workers. Thus, the per-worker transfers are equal to

$$T_1 = \frac{B_1}{R_1}. \tag{1}$$

Effectively, the government borrows on behalf of workers. Notice that, since B_1 is not restricted to be positive, the government could choose to

[3] See Azzimonti, de Francisco, and Quadrini 2015 for a discussion of such an environment in a model without default.

save. In practice, we will focus on parameter values for which B_1 is positive but, theoretically, it could be possible for the government to save instead of borrowing.

In period 2 the government repays debt by taxing workers (or making transfers to them if $B_1 < 0$). However, the government could also choose to partially default by repaying only a fraction $\delta_2 \in [0, 1]$ of debt.[4] Denoting by $\tilde{B}_2 = \delta_2 B_1$ the chosen repayment, the lump-sum taxes paid by workers are

$$T_2 = -\tilde{B}_2. \tag{2}$$

The government's welfare function is the weighted sum of the utility of workers and entrepreneurs,

$$(1-\Psi)\Big[U(c_1) + \beta \mathbb{E}_z U\Big(\varphi(c_2, h_2)\Big)\Big] + \Psi\Big[u(d_1) + \beta \mathbb{E}_{z,\varepsilon} u(d_2)\Big],$$

where Ψ denotes the relative weight assigned to entrepreneurs in period 1. The expectation in the second term of the welfare function is with respect to both the aggregate and idiosyncratic shocks.

Uncertainty and timing. There are two sources of uncertainty: (i) The aggregate productivity shock z_2; and (ii) The idiosyncratic productivity shock ε_2. Aggregate and idiosyncratic shocks are both realized in period 2. However, while the aggregate shock z_2 is revealed before agents and government make any decisions in period 2, the idiosyncratic shock is revealed at the end of the period after the repayment decision of the government and after the hiring decisions of entrepreneurs. Following is the detailed description of timing.

Period 1:

1. The government chooses debt B_1.
2. Entrepreneurs choose savings b_1.
3. The interest rate R_1 clears the market for government bonds.

[4]See Arellano, Mateos-Planas and Rios-Rull, who document that countries always default on only part of their debt.

Period 2:

1. Aggregate productivity z_2 becomes public knowledge.

2. Given debt B_1, the government chooses the repayment $\tilde{B}_2 = \delta_2 B_1$.

3. Entrepreneurs choose the input of labor l_2 before knowing the idiosyncratic productivity ε_2 and workers choose the supply of labor h_2. The wage w_2 clears the labor market.

4. The idiosyncratic productivity ε_2 is realized, production and consumption take place.

3.1 Equilibrium given policy

We start by characterizing the competitive equilibrium given government policies. To simplify notation, from now on we abstract from time subscripts unless it is necessary to avoid ambiguities. For example, B without subscript denotes the debt issued by the government in period 1 and $\tilde{B}$ denotes the repayment of the debt in period 2. If the government repays the debt in full then $\tilde{B} = B$. The variable h denotes the labor supplied by a worker in period 2. Also in this case we can abstract from time subscripts because workers supply labor only in period 2. Along the same line, we omit the time subscript in the shocks since they are only realized in period 2.

While government borrowing is chosen in period 1, the repayment is chosen in period 2 after the observation of the aggregate shock. Denoting by $\mathbf{s} = (z, B)$ the states in period 2, the debt repayment is denoted by $\tilde{B}(\mathbf{s})$. To use a compact notation we denote the government policies by $\pi = (B, \tilde{B}(\mathbf{s}))$. Individual decisions and the equilibrium wage in period 2 are also functions of the states $\mathbf{s}$.

The problems solved by workers is

$$U\Big(c_1(\pi)\Big) + \beta \mathbb{E}_z U\bigg(\varphi\Big(c_2(\pi, z), h(\pi, z)\Big)\bigg) \tag{3}$$

subject to

$$c_1(\pi) = e_1 + \frac{B}{R(\pi)}$$

$$c_2(\pi, z) = e_2 + w(\pi, z)h(\pi, z) - \tilde{B}(\mathbf{s}).$$

Entrepreneurs solve the problem

$$u\Big(d_1(\pi)\Big) + \beta \mathbb{E}_{z,\varepsilon} u\Big(d_2(\pi,z,\varepsilon)\Big) \tag{4}$$

subject to

$$d_1(\pi) = a - \frac{b(\pi)}{R(\pi)}$$

$$d_2(\pi,z,\varepsilon) = \Big[A(z,\varepsilon) - w(\pi,z)\Big] l(\pi,z) + \delta(z) b(\pi).$$

Definition 1 *A competitive equilibrium given policy $\pi = \{B, \tilde{B}(\mathbf{s})\}$ is defined by price functions $R(\pi)$ and $w(\pi,z)$, decision functions for workers, $c_1(\pi)$, $h(\pi,z)$, $c_2(\pi,z)$, and entrepreneurs, $b(\pi)$, $d_1(\pi)$, $l(\pi,z)$, $d_2(\pi,z,\varepsilon)$, such that workers solve problem (3), entrepreneurs solve problem (4), asset and labor markets clear, that is, $b(\pi) = B$ and $l(\pi,z) = h(\pi,z)$.*

While the decisions of workers reduce to the choice of labor in period 2, the decisions of entrepreneurs are more complex. Because of the concavity of the utility function, the saving and hiring decisions of entrepreneurs take into account the risk associated with production. The following lemma characterizes the optimal entrepreneurs' decisions.

Lemma 2 *Let $\phi(\pi,z)$ satisfy the condition $\mathbb{E}_\varepsilon \frac{A(z,\varepsilon)-w(\pi,z)}{1+[A(z,\varepsilon)-w(\pi,z)]\phi(\pi,z)} = 0$. The entrepreneurs's decisions take the form*

$$\begin{aligned}
b(\pi) &= \frac{a\beta}{1+\beta} R(\pi), \\
d_1(\pi) &= \frac{a}{1+\beta}, \\
l(\pi,z) &= \phi(\pi,z)\delta(z)b(\pi), \\
d_2(\pi,z,\varepsilon) &= \Big[1 + \Big(A(z,\varepsilon) - w(\pi,z)\Big)\phi(\pi,z)\Big]\delta(z)b(\pi).
\end{aligned}$$

Proof. See Appendix A ■

The competitive equilibrium given government policies $\pi = (B, \tilde{B}(\mathbf{s}))$ can be computed recursively as we show in Appendix B.

For the analysis that follows it would be convenient to characterize the sensitivity of some key equilibrium variables to changes in the debt $\tilde{B}(\mathbf{s})$. This is done in the following lemma.

Lemma 3 *Suppose that $\phi(\pi, z) > 0$ for all $\tilde{B}(\mathbf{s}) \geq 0$. Then,*

1. *The factor $\phi(\pi, z)$ is strictly decreasing in $\tilde{B}(\mathbf{s})$;*

2. *Wages $w(\pi, z)$ and employment $l(\pi, z)$ are increasing in $\tilde{B}(\mathbf{s})$.*

Proof. See Appendix C. ■

We have established that $w(\pi, z)$ and $l(\pi, z)$ are increasing functions of the debt repayment $\tilde{B}(\mathbf{s})$. This implies that if in the second period the government decides to default, that is, $\tilde{B} < B$, both employment and wages will decline. Therefore, default generates a macroeconomic contraction.

The central mechanism through which default generates a macroeconomic contraction is by destroying the financial wealth of entrepreneurs. This has two effects. The first effect is to redistribute wealth from entrepreneurs (who hold government debt) to workers (who pay taxes to repay the debt). The assumption that only workers pay taxes is not essential. The mechanism would still operate if taxes were equally paid by workers and entrepreneurs. What matters is that taxes are not proportional to the holding of public debt so that default implies that agents who hold public debt (the entrepreneurs) experience a net loss while agents who do not hold public debt (the workers) experience a net gain. The second effect, which is a consequence of the first, is to generate a macroeconomic contraction: since entrepreneurs end up with lower wealth, they will hire fewer workers.

Although the redistribution effect is beneficial for workers, the recessionary effect has negative consequences for workers (because of the lower demand for labor which reduces wages). Therefore, from the perspective of workers, government default implies a trade-off: the benefit is the lower payment of taxes and the cost is the reduction in labor income. From the perspective of entrepreneurs, instead, government default implies only a cost. These considerations will be key for understanding optimal government policy.

3.2 Government's optimal policy (t=2)

To characterize the government's problem we proceed backward. We first consider the problem solved in period 2 and then, given the optimal policy in the second period, we solve the government's problem in period 1. In doing so we are effectively characterizing the optimal policy without commitment (time-consistent). Later, however, we will compare it to the commitment policy in which the government chooses the repayment in period 1 and commits not to change it in period 2.

Government problem in period 2. In the second period, the government chooses debt repayment $\tilde{B}$ given its stock of debt B and the realization of aggregate productivity z. We assume that the government makes a take-it-or-leave it offer to domestic creditors, as entrepreneurs have no bargaining power (e.g. they are small creditors). This amounts to choosing $\tilde{B} \leq B$ in order to maximize the weighted utility of domestic citizens,

$$\max_{\tilde{B} \leq B} \quad \left\{ (1-\Psi) U\Big(\varphi\Big(c_2(\pi, z), h(\pi, z) \Big) \Big) + \Psi \mathbb{E}_{\varepsilon} u\Big(d_2(\pi, z, \varepsilon) \Big) \right\}. \tag{5}$$

Consider first the relaxed problem where the repayment is not subject to the constraint $\tilde{B} \leq B$ (or $\delta \leq 1$). Assuming that the objective function (5) is strictly concave in $\tilde{B}$, there will be a unique solution to the government problem. The first order condition, derived in Appendix D, takes the form

$$(1-\Psi) U'\Big(\varphi\Big(c_2(\pi, z), h(\pi, z) \Big) \Big) = \Psi \mathbb{E}_{\varepsilon} u'\Big(d_2(\pi, z, \varepsilon) \Big), \tag{6}$$

where the prime denotes derivatives. The government equalizes the marginal utility of consumption for workers (net of the working dis-utility) to the expected marginal utility of consumption for entrepreneurs. Using this condition we derive the following result.

Proposition 4 *Let $e_2 = 0$ and $A(z, \varepsilon) = z + \varepsilon$. The optimal repayment $\tilde{B}(\mathbf{s})$ is strictly increasing in the aggregate shock z.*

Proof. See Appendix E ■

This result shows that the incentive to default, that is, the incentive to repay a lower value of debt, is higher when the country is in recession. To understand this result we have to consider the two effects of default as described earlier. The first effect is the redistribution of wealth from entrepreneurs to workers. The second is the macroeconomic recession: as $\tilde{B}$ declines, the financial wealth of entrepreneurs declines and this reduces the demand for labor. This is also damaging for workers. However, since their consumption is lower when productivity declines, the marginal utility of consumption is higher for workers. From the prospective of the government, this increases the benefit of redistributing wealth toward workers and, therefore, the incentive to default (the first effect). Also, since labor is less productive, the loss of output from reducing the input of labor is smaller (second effect).

Denote by $\hat{B}(z)$ the unconstrained optimal repayment of the public debt. This is the solution to the government problem (5) without the constraint $\tilde{B} \leq B$ and satisfies the first order condition (6). Figure 2 plots the government indirect utility for two levels of aggregate productivity. The graph is constructed using a specific parametrization of the model. Since the graph is only meant to provide a numerical example, we will postpone the discussion of the parameter values. As we can see, when productivity is high the government prefers a higher repayment, that is, $\hat{B}(z_H) > \hat{B}(z_L)$.

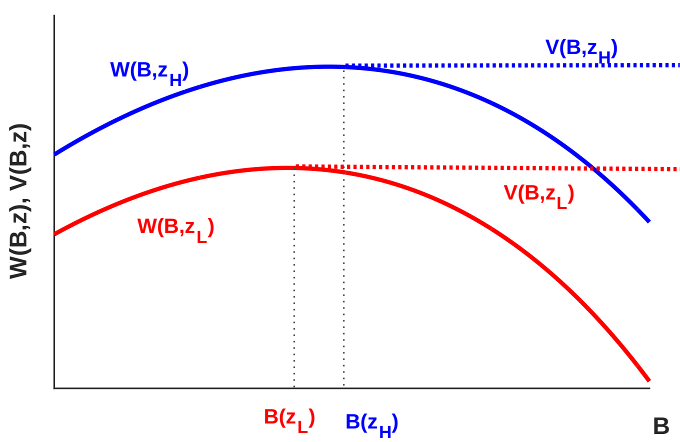


Figure 2: Second period welfare as a function of B. The continuous line denoted by $W(B, z)$ is the government welfare when the government repays the whole debt B. The dotted line denoted by $V(B, z)$ is the optimized government welfare. Below the optimal repayment, the continuous and dotted lines overlap. Above the optimal repayment, the optimized welfare becomes constant because the government defaults on the debt.

After characterizing the unconstrained optimal repayment, we can now characterize the constrained optimal policy which is subject to the constraint

$\tilde{B}(\mathbf{s}) \leq B$. This is given by

$$\mathcal{B}(B,z) = \begin{cases} B, & \text{if } B \leq \hat{B}(z) \\ \\ \hat{B}(z), & \text{if } B > \hat{B}(z). \end{cases}$$

In the second period the government can choose to repay the full value of the debt or negotiate a lower repayment. If the preferred repayment $\hat{B}(z)$ is larger than B, the government will repay the debt in full, that is, $\tilde{B}(\mathbf{s}) = B$. However, if $\hat{B}(z) < B$, the government will default and repay $\hat{B}(z)$. The fraction repayed is simply equal to $\delta(B,z) = \hat{B}(z)/B$.

Government problem in period 1. Because of the particular specification of preferences (log-utility), government policies do not affect consumption in period 1. In fact, Lemma 2 shows that the consumption of entrepreneurs in period 1 is equal to $d_1 = a/(1+\beta)$ and, therefore, is independent of B. In equilibrium, a higher B will be associated with a proportional increase in the interest rate $R(\pi)$ so that entrepreneurial savings $B/R(\pi)$ remain unchanged.

The consumption of workers is $c_1 = e_1 - B/R(\pi)$. But Lemma 2 shows that $B/R(\pi) = a\beta/(1+\beta)$. Therefore, c_1 is also independent of B. We can then characterize the government problem in period 1 ignoring the flow of utility in period 1 (since this is independent of government policies) and write it as

$$\max_{B} \quad \mathbb{E}_z \left\{ (1-\Psi) U\Big(\varphi\Big(c_2(\hat{\pi},z), h(\hat{\pi},z)\Big)\Big) + \Psi \mathbb{E}_\varepsilon u\Big(d_2(\hat{\pi},z,\varepsilon)\Big) \right\},$$

where the policy vector $\hat{\pi} = (B, \mathcal{B}(B,z))$ now contains the optimal repayment policy chosen by the government in period 2 (as characterized above). Therefore, by choosing the debt today the government affects the optimal repayment in period 2.

The objective is a weighted sum of the dotted lines in Figure 2, where the weights are given by the probabilities of different realizations of aggregate productivity z. Because $V(B,z_L)$ is constant for $B > \hat{B}(z_L)$ whereas $V(B,z_H)$ is increasing, the objective function $\mathbb{E}_z V(B,z)$ is increasing up to $\hat{B}(z_H)$. This implies that the optimal value of debt chosen by the first period government is

$$B^* = \hat{B}(z_H).$$

The government in the first period is constrained by default decisions of the second period government. By choosing the largest possible value of sustainable debt $\hat{B}(z_H)$, it will have the option of reducing it in bad times. Notice that during booms debt is repaid in full $\delta(B^*, z_H) = 1$, whereas partial defaults are observed in recessions, $\delta(B^*, z_L) = \hat{B}(z_L)/\hat{B}(z_H)$.

The size of default depends on the relative weights given by the government to workers and entrepreneurs, as well as on the degree of aggregate uncertainty (that is, the distance between z_H and z_L). The choice of the first period debt, together with the option to partially default in the second period, attempts to replicate allocations that would be achievable with state-contingent debt (an instrument that, by assumption, is not available to the government).

Remark. By solving the government's problem backward, we have effectively characterized the time consistent problem. This raises the question of whether government commitment matters for the equilibrium. With commitment the government chooses both B and $\tilde{B}(\mathbf{s})$ in period 1. Notice that the repayment choice is contingent on the realization of the aggregate productivity in period 2. It turns out that in autarky the commitment problem is time-consistent. In other words, the government has no incentive to change $\tilde{B}(\mathbf{s})$ chosen in period 1 after the observation of z even if it is allowed to do so. As we will see, this result does not apply to the environment with financial integration.

4 Financial Integration with no renegotiation

We now extend the model considering two countries that are financially integrated and, therefore, they can trade sovereign bonds issued by each of the two countries. Labor, however, is immobile. We refer to the first country as 'home' country and to the second as 'foreign' country.

We denote by B^h the debt issued by the home country and by B^{hh} and B^{hf} the home debt held, respectively, by entrepreneurs in the home and foreign countries. In equilibrium $B^h = B^{hh} + B^{hf}$. Similarly, the debt issued by the foreign country is denoted by B^f, in part held by entrepreneurs in the home country, B^{fh}, and in part by entrepreneurs in the foreign country, B^{ff}. Therefore, the first superscript denotes the nationality of the government that issued the debt and the second subscript denotes the nationality of the holders (entrepreneurs) of that debt.

The two countries are homogeneous except in the commitment to repay the debt. While the home country government does not commit to repay the debt, meaning that it could default in period 2, the government of the foreign country always commits to repay. This implies that, while for the home country $\tilde{B}^h$ may differ from B^h, for the foreign country $\tilde{B}^f = B^f$. Equivalently, $\delta^h \leq 1$ while $\delta^f = 1$. The prices for home and foreign bonds are, respectively, $1/R^h$ and $1/R^f$. Even though financial markets are perfectly integrated, the prices for home and foreign bonds could differ because of the different probability to default.

We first consider the case in which there is no renegotiation over debt repayments, analogous to the analysis in the previous sections where the home country makes a take-it-or-leave it offer to its creditors (which are now composed of domestic and foreign entrepreneurs). In the next section, we consider the possibility of renegotiation between the domestic and foreign governments.

4.1 Equilibrium for given policies

Let's first define the set of aggregate states in period 2 which are given by $\mathbf{s} = (z^h, z^f, B^{hh}, B^{hf}, B^{fh}, B^{ff})$. Since the two countries are financially integrated, the states include variables of both countries. The policy variables are $\pi = (B^h, B^f, \tilde{B}^h(\mathbf{s}))$.

Definition 5 *With financially integrated markets, a competitive equilibrium for given policy π is defined by prices $\{R^i, w^i(\pi, \mathbf{s})\}_{i\in\{h,f\}}$, decision functions for workers $\{c_1^i(\pi), h^i(\pi, \mathbf{s}), c_2^i(\pi, \mathbf{s})\}_{i\in\{h,f\}}$, decision functions for entrepreneurs $\{b^{hi}(\pi), b^{fi}(\pi), d_1^i(\pi), l^i(\pi, \mathbf{s}), d_2^i(\pi, \mathbf{s}, \varepsilon)\}_{i\in\{h,f\}}$, such that*

1. *Workers in country i maximize*

$$U\Big(c_1^i(\pi)\Big) + \beta \mathbb{E}_{\mathbf{s}} U\bigg(\varphi\Big(c_2^i(\pi, \mathbf{s}), h^i(\pi, \mathbf{s})\Big)\bigg)$$

$$\textit{subject to}$$

$$c_1^i(\pi) = e_1 + \frac{B^i}{R^i(\pi)}$$

$$c_2^i(\pi, \mathbf{s}) = e_2 + w^i(\pi, \mathbf{s})h^i(\pi, \mathbf{s}) - \tilde{B}^i(\mathbf{s});$$

2. *Entrepreneurs in country i maximize*

$$u\Big(d_1^i(\pi)\Big) + \beta\mathbb{E}_{\mathbf{s},\varepsilon}\Big(d_2^i(\pi,\mathbf{s},\varepsilon)\Big)$$

subject to

$$d_1^i(\pi) = a - \frac{b^{hi}(\pi)}{R^h(\pi)} - \frac{b^{fi}(\pi)}{R^f(\pi)}$$

$$d_2^i(\pi,\mathbf{s},\varepsilon) = \Big[A(z^i,\varepsilon) - w^i(\pi,\mathbf{s})\Big] l^i(\pi,\mathbf{s}) + \delta^h(\pi,\mathbf{s}) b^{hi}(\pi) + b^{fi}(\pi).$$

3. *Asset and labor markets clear, that is, for* $i \in \{h, f\}$

$$\begin{aligned} B^i &= b^{ih}(\pi) + b^{if}(\pi), \\ l^i(\pi,\mathbf{s}) &= h^i(\pi,\mathbf{s}). \end{aligned}$$

An important difference between the autarky equilibrium and the equilibrium with integrated financial markets is that in the latter entrepreneurs hold a portfolio of assets issued by both home and foreign governments. This has three implications. First, since part of the pubic debt is held by foreigners, a government may have higher incentives to default. Second, the default of one country (let's say the government of the home country) affects employment and output in both countries. In other words, the macroeconomic consequences of sovereign default are *exported* to other countries (spillover effects). Third, the holding of foreign assets in the portfolio of home entrepreneurs (financial diversification) reduces the macroeconomic cost of default for the home country. This also implies that when the foreign country issues more debt, entrepreneurs in the home country hold a larger share of foreign assets and this reduces the macroeconomic cost of defaulting (and, therefore, the incentive to default). While the first implication is common to any model of sovereign default, the second and third implications are special features of this model.

We can now provide a characterization of the competitive equilibrium. Since labor is immobile, the optimal choices of workers under financial integration are the same as in the closed economy. Thus, the labor supply is still given by

$$h^i(\pi,\mathbf{s}) = \left(\frac{w^i(\pi,\mathbf{s})}{\alpha}\right)^{\nu}.$$

The equilibrium wage rate, however, could differ in the two countries depending on the realization of aggregate shocks, which in turn implies differences in employment.

As in the closed economy, entrepreneurs' decisions can be characterized in closed form as summarized by the following lemma.

Lemma 6 *Let* $\phi^i(\pi, \mathbf{s})$ *satisfy the condition* $\mathbb{E}_\varepsilon \frac{A(z^i,\varepsilon)-w^i(\pi,\mathbf{s})}{1+[A(z^i,\varepsilon)-w^i(\pi,\mathbf{s})]\phi^i(\pi,\mathbf{s})} = 0$. *The entrepreneur's policies in country i are*

$$
\begin{aligned}
d_1^i(\pi) &= a\Big[1-\theta^h(\pi)-\theta^f(\pi)\Big] \\
b^{hi}(\pi) &= \theta^h(\pi)R^h(\pi)a \\
b^{fi}(\pi) &= \theta^f(\pi)R^f(\pi)a \\
h^i(\pi,\mathbf{s}) &= \phi^i(\pi,\mathbf{s})\Big[\delta^h(\pi,\mathbf{s})b^{hi}(\pi)+b^{fi}(\pi)\Big] \\
d_2^i(\pi,\mathbf{s},\varepsilon) &= \Big[1+\Big(A(z^i,\varepsilon)-w^i(\pi,\mathbf{s})\Big)\phi^i(\pi,\mathbf{s})\Big]\left[\delta(\pi,\mathbf{s})b^{hi}(\pi)+b^{fi}(\pi)\right]
\end{aligned}
$$

where $\theta^h(\pi)$ *and* $\theta^f(\pi)$ *solve*

$$
\begin{aligned}
1+\beta &= \beta\mathbb{E}_{\mathbf{s}}\left[\frac{1}{\theta^h(\pi)\delta(\pi,\mathbf{s})\frac{R^h(\pi)}{R^f(\pi)}+\theta^f(\pi)}\right] \\
\theta^h(\pi) &= \frac{\beta}{1+\beta}-\theta^f(\pi)
\end{aligned}
$$

Proof. See Appendix F ■

Entrepreneurs split their initial wealth a between current consumption d_1^i and financial assets. Since the fraction saved, $\theta^h(\pi)+\theta^f(\pi)$, does not depend on policies, consumption in the first period is exactly the same as in a closed economy. The main difference with the autarky equilibrium is that entrepreneurs hold both home and foreign bonds. Because the foreign country commits to repay the debt whereas the home country does not, the returns on the two bonds are different. The fraction of savings allocated by entrepreneurs to home and foreign bonds, θ^h and θ^f, respectively, is independent of their residence. Thus, entrepreneurs in home and foreign countries choose the same portfolio composition. This results from the assumption that the two countries are identical in preferences and technology (including the distribution of the idiosyncratic shock). The second period consumption, on the other hand, may differ due to different realizations of the shocks z^h and z^f.

The cross-country diversification of portfolios implies that the sovereign default of one country creates macroeconomic costs for both countries. In fact, because entrepreneurs hold bonds issued by both countries, they all experience a financial loss once the government of one country defaults. This causes a contraction in their demand for labor, which in turn reduces employment $h^i(\pi, \mathbf{s})$ and wages $w^i(\pi, \mathbf{s})$. Thus, with financially integrated markets, even if only one country defaults on its sovereign debt, the negative macroeconomic consequences are transmitted to other countries.

Another implication of Lemma 6 is that the consumption of entrepreneurs in period 1 is independent of the borrowing decisions of the governments of the two countries. More specifically, $d_1^h = d_1^f = a/(1+\beta)$. This implies that the worldwide consumption of workers in the first period is independent of policies since $c_1^h + c_1^f + d_1^h + d_1^f = 2a + 2e_1$. In other words, the total worldwide consumption of workers and entrepreneurs must be equal to their initial wealth. However, even if total worldwide consumption of workers is independent of policies, their distribution among home and foreign workers depends on policies. This is an important difference between the autarky regime and the regime with integrated financial markets: while in autarky the first period welfare is independent of the debt chosen by the government, with financial integration the borrowing decision of a country impacts on the first period welfare of both countries. As we will discuss later, this new feature makes the optimal government policies time-inconsistent when financial markets are integrated.

4.2 Optimal policies

As for the analysis of the autarky regime we characterize the optimal government policies backward starting with the problem solved in period 2. Then, taking as given the optimal government strategy in period 2, we will solve for the optimal policies in period 1.

4.2.1 Optimal policies in period 2

Since the debt issued by the foreign country is always repaid, the foreign government does not make any decision in period 2. The government of the home country, instead, will choose the optimal repayment of the debt.

The problem solved by the home government consists of the choice of the repayment $\tilde{B}^h$ to maximize the weighted sum of utilities for workers and entrepreneurs, analogous to problem (5). Appendix G shows that, using the equilibrium conditions, the optimization problem of the home government can be written as

$$\max_{\tilde{B}^h \leq B^h} \quad \Bigg\{(1-\Psi)\ln\left(\tilde{\nu} w^h\left(\pi, z^h\right)^{1+\nu} - \tilde{B}^h\right) + \Psi \ln\left(\frac{\tilde{B}^h + B^f}{2}\right) + \\ \Psi E_\varepsilon \ln\left(\left[A(z^h,\varepsilon) - w^h\left(\pi, z^h\right)\right]\phi^h\left(\pi, z^h\right) + 1\right)\Bigg\}. \quad (7)$$

The derivation provided in the appendix uses the equilibrium property for which entrepreneurs in home and foreign countries hold the same portfolio of home and foreign financial assets (see Lemma 6). This implies that the policies π together with the realization of domestic productivity z^h are sufficient to characterize the equilibrium. Thus, the arguments of all relevant functions are now π and z^h.

This equilibrium property is captured by the term $(\tilde{B}^h + B^f)/2$ in the welfare function. This is the post-default wealth of entrepreneurs which is equal to the per-capita worldwide sovereign debt issued by the two countries. Because of the portfolio diversification, the welfare function of the home country depends also on the foreign debt B^f. Notice that the aggregate productivity in the foreign country z^f is irrelevant for the default decision of the home government. The only channel through which the foreign country affects the home country is through the foreign debt B^f.

To characterize the optimal repayment policy of the home country, consider first the unconstrained optimization, that is, ignore the constraint $\tilde{B}^h \leq B^h$. Assuming that the objective is strictly concave, there will be a unique solution. The first order condition, derived in Appendix G, takes the form

$$\Psi E_\varepsilon u'\Big(d_2^h(\pi, z^h, \varepsilon)\Big) = (1-\Psi) U'\Big(\varphi_2^h(\pi, z^h)\Big)\Omega(\pi, z^h),$$

where $\Omega(\pi, z^h)$ is a term that is equal or bigger than 1. The solution, denoted by $\hat{B}(B^f, z^h)$, depends on the public debt of the foreign country and on aggregate productivity.

Inspecting the first order condition we can see that, if the term $\Omega(\pi, z^h)$ were equal to 1, then the government would equalize the marginal utilities of the two types of agents, re-scaled by the weights Ψ and $1-\Psi$. This was the optimality condition in the autarky equilibrium which is very intuitive. With financial integration, however, $\Omega(\pi, z^h) \geq 1$. Therefore, it is as if the government of the home country assigns a higher weight to workers when financial markets are integrated.[5] This implies that, keeping everything else equal, the incentive to default is higher compared to autarky.

There are two reinforcing effects that increase the home country incentive to default. The first effect arises from the redistribution of wealth from foreigners to domestic agents. Because some of the domestic debt is held by foreigners, default redistributes wealth not only from domestic entrepreneurs to domestic workers but also from foreign entrepreneurs to domestic workers. Recall that the portfolio of entrepreneurs is now diversified, with holdings of $(\tilde{B}^h + B^f)/2$. The redistribution from foreigners to domestic agents has been widely studied in the literature and it is not the focus of this paper. The second effect, which is the novel channel emphasized in this paper, results from the fact that default generates lower financial losses for home entrepreneurs because they are internationally diversified. This implies that, compared to the autarky regime, the macroeconomic impact of default on the home country is smaller. This mechanism, which is novel in the sovereign default literature, increases the government's incentive to default.

External supply of debt and default. The solution to the constrained problem, that is, problem (7) subject to the constraint $\tilde{B}^h \leq B^h$, the takes the form

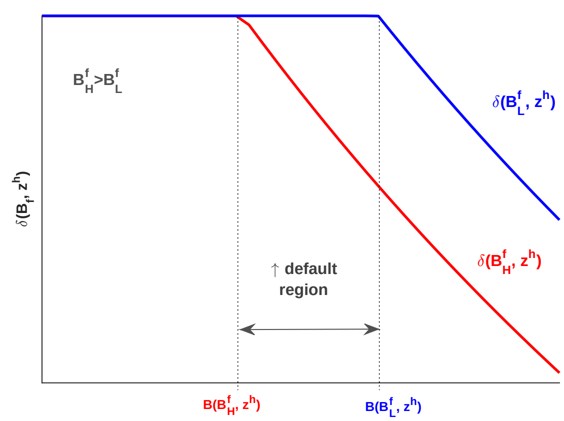
$$\mathcal{B}(B^h, B^f, z^h) = \begin{cases} B^h, & \text{if } B^h \leq \hat{B}(B^f, z^h) \\ \hat{B}(z^h, B^f), & \text{if } B^h > \hat{B}(B^f, z^h) \end{cases}. \tag{8}$$

The analysis conducted so far suggests that changes in the supply of foreign bonds may affect the default decision of the home government. Inspection of condition (8) shows that the external supply of debt B^f does not affect the optimal repayment when the home debt B^h is low. However, it becomes relevant when the outstanding debt B^h is higher than a threshold,

[5] In the appendix we show that $\Omega(\pi, z^h) = \frac{1 - \frac{\partial w_2}{\partial F} h_2}{\frac{1}{2} - \frac{\partial w_2}{\partial F} h_2} \geq 1$, where $F = \frac{\tilde{B}^h(z) + \tilde{B}^f}{2}$.

since $\mathcal{B}(B^h, B^f, z^h) = \hat{B}(z^h, B^f)$ in this case. Intuitively, when the foreign country increases B^f, part of the foreign debt is held by home entrepreneurs. Hence, a lower fraction of financial wealth held by home entrepreneurs is at risk if the home government defaults. This implies that the macroeconomic costs of default become smaller when B^f rises. On the other hand, the net redistributive benefits of default remain unchanged (keeping constant the first period government policies). Because the macroeconomic cost of default is smaller while the redistributive effects do not change, an increase in foreign debt results in higher incentives to default for the home country.

Figure 4.2.1 illustrates this mechanism. The figure plots the optimal debt repayment from the home government for two alternative values of foreign debt, $B^f_H > B^f_L$ (and a given realization of aggregate uncertainty z^H). As B^f increases, the repayment schedule moves to the left. In other words, $\hat{B}(B^f, z^h)$ goes down. The effects of a higher supply of B^f on prices and the risk premium are discussed in Appendix H.



4.2.2 Optimal policies in period 1

After characterizing the optimal government policy in period 2, we can now move to period 1 when the governments of the home and foreign countries choose, respectively, B^h and B^f. We assume that the two countries choose their debt simultaneously and without cooperation through a Nash strategic game. In doing so they take as given the optimal policy of the home government in period 2, that is, $\mathcal{B}(B^h, B^f, z^h)$ as characterized above.

To simplify notation, define $B = (B^h, B^f)$ and $z = (z^h, z^f)$. Thus B and z without superscripts denote, respectively, the vector of debts issued in period 1 and the vector of aggregate productivities observed at the beginning of period 2. Using this notation, the second period indirect utility of country i is equal to

$$\begin{aligned} V^i(B, z; \mathcal{B}) \quad = \quad & \Bigg\{ (1 - \Psi) \ln \left(\tilde{\nu} w^i\,(\pi, z)^{1+\nu} - \tilde{B}^i \right) + \Psi \ln \left(\frac{\mathcal{B}(B, z^h) + B^f}{2} \right) \; + \\ & \Psi E_\varepsilon \ln \left(\Big[A(z^i, \varepsilon) - w^i\,(\pi, z) \Big] \phi^i\,(\pi, z) + 1 \right) \Bigg\}, \quad (9) \end{aligned}$$

where $\pi = \{B, \mathcal{B}(B, z^h)\}$, $\tilde{B}^f = B^f$ and $\tilde{B}^h = \mathcal{B}(B, z^h)$.

Recall that under autarky, first period consumption of workers and entrepreneurs is independent of the level of debt chosen by the government in period 1. Because of this, optimal debt is determined by simply maximizing the expected second period indirect utility. This is no longer the case when financial markets are integrated. The reason being that the choice of debt in the first period affects the consumption of workers through its effect on the interest rate. The consumption of entrepreneurs, instead, remains unaffected by the choice of B^i as in the autarky regime. Thus, the objective of the government in country i can be written as

$$\max_{B^i} \Bigg\{ (1 - \Psi) U(c_1^i(\pi)) + \beta \mathbb{E}_z\, V^i(B, z; \mathcal{B}) \Bigg\},$$

where we have omitted the utility of entrepreneurs in period 1 which is unaffected by policies.

The optimal solution, denoted by $B^i = g^i(B^j)$, defines the best response function of country $i \in \{h, f\}$ to the debt issued by the other country (identified by the index $j \neq i$). It turns out that the optimal debt for the home country is the maximum debt that the country would repay in the next period, given the debt issued by the foreign government. In the previous subsection we denoted the unconstrained optimal repayment by the function $\hat{B}(B^f, z^h)$. Thus, the optimal debt issued in period 1 by the home country is equal to

$$g^h(B^f) = \max_{z^h} \hat{B}(B^f, z^h).$$

This property has a simple intuition. Since the home government has the option to default in period 2 but will not repay more than the face value of the debt, issuing more debt in period 1 imposes less stringent constraints in period 2. Effectively, B^h imposes an upper bound to the feasible repayment of public debt but it does not affect the lower bound. Thus, the government will choose B^h as the maximum repayment that it will ever choose in period 2. In this way the optimal repayment will not be constrained. Of course, the government could choose an even higher value. This, however, would not result in higher welfare for the government: higher debt levels would be defaulted upon, which would increase the current price of the debt $1/R^h$.

A Nash equilibrium for the policy game played by the two governments is then defined by a pair $(\bar{B}^h, \bar{B}^f)$ that satisfies the conditions

$$\begin{aligned} \bar{B}^h &= g^h(\bar{B}^f), \\ \bar{B}^f &= g^f(\bar{B}^h). \end{aligned}$$

Unfortunately, we are unable to derive an analytical characterization of the Nash equilibria. Therefore, we will characterize them numerically.

Numerical characterization For the numerical characterization, we use the following parameter values: $\beta = 0.9825$, $\nu = \alpha = 1$, $a = e_1 = e_2 = 1$, ε is uniformly distributed over the interval $[0.9, 1.1]$, $z \in \{0.95, 1.05\}$, and $\Psi = 0.5$. For this parametrization we will show that the equilibrium is not unique but there is a continuum of Nash equilibria (multiplicity).

The top panel in Figure 3 plots the response functions of the two countries as a function of the other country's debt. The response function of the home country is always downward sloping. This is because as B^f increases, part of which is purchased by domestic entrepreneurs, there is less need for liquidity in country h. The costs of default go down, as described in the previous section, and thus the maximum level of sustainable debt $\hat{B}(z_H, B^f)$ declines.

The response function for the foreign country, on the other hand, has a peculiar shape. When B^h is relatively small (e.g. smaller than point B in the graph), the home country never defaults in period 2. Thus, B^h and B^f are perfect substitutes so that the foreign country's response to increases in B^h simply reflects a lower need to provide liquidity to foreign entrepreneurs. When B^h lies between points B and A in the graph, default may occur in equilibrium, that is, the home country will repay a smaller fraction of debt. Since the default decision of the home country depends on B^f, the foreign country will take this into consideration when it chooses the optimal debt. So now the foreign government may choose to reduce more its own debt when

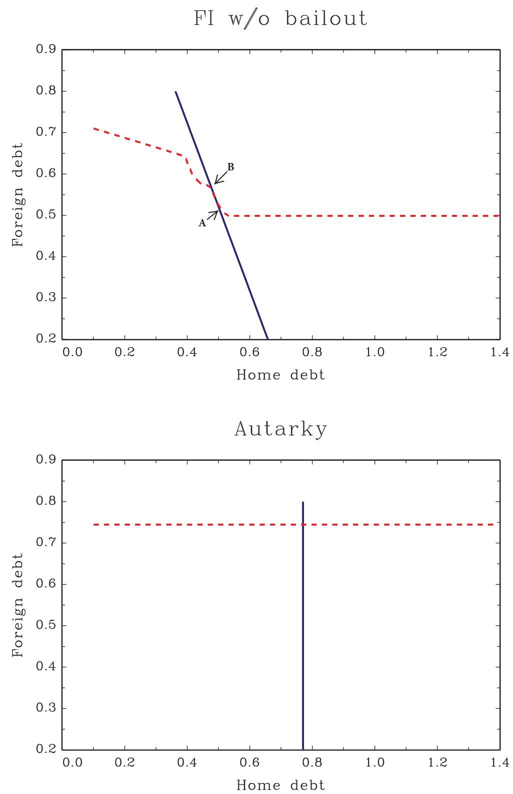


Figure 3: Response functions

B^h increases in order to prevent the home country from defaulting in period 2. The reaction function is therefore steeper in this range. Once B^h has reached a certain level (point A in the graph), it is no longer beneficial to reduce B^f to prevent default and the optimal B^f becomes independent of B^h. In this region higher values of B^h do not affect allocations since the higher debt issued by the home country will not be repaid in period 2. The two reaction functions overlap between points B and A indicating the existence of a continuum of equilibria in this range.

The bottom panel of Figure 3 depicts the solution under autarky for the same parameters. Clearly, when countries are not financially integrated, their optimal choices are independent of each other and therefore their best responses are completely inelastic. By comparing the two panels we see that equilibrium worldwide supply of debt, $B^h + B^f$, is larger when the economies are closed than in any of the potential equilibria under financial integration. This happens because the government fully internalizes the effects of debt as a source of liquidity in their domestic economy, but ignores it's effects in the foreign economy. Under financial integration, part of the debt issued by country i is purchased by entrepreneurs residing in country j. Since the government in country i does not take their welfare into consideration, it finds it optimal to issue less debt than under autarky. This negative externality results in inefficiently low levels of debt when countries are financially integrated (i.e. too little liquidity).[6]

Welfare ranking of equilibria In this subsection we want to study the main characteristics of the equilibria that arise under financial integration with no renegotiation (e.g. those between points B and A in Figure 3). To analyze this, we investigate the behavior of key allocations and prices across different Nash equilibria. That is, we consider how financial assets, interest rates, defaults, and welfare change as we move from B (e.g. equilibria with high B^f and low B^h) to A (e.g. equilibria with low B^f and high B^h).

First, as we move from B to A, there is a reduction in overall liquidity, that is, the stock of worldwide debt $B^h + B^f$ declines. In other words, as the debt of the home country B^h increases, the foreign country reduces B^f more than the increase in B^h. This is illustrated in the first panel of Figure 4.

In addition, equilibria with a relatively high ratio of defaultable debt are characterized by *higher* repayment rates in period 2. This is shown in the second panel of Figure 4, which plots the repayment rate $\delta(B^h, B^f, z^h)$ for the various equilibria. The size of defaults is given by the difference between the two curves in the figure. The foreign country's reduction in B^f in response to higher B^h ensures that $\delta(B, z_H) = 1$. That is, when the domestic country faces a good realization of the shock z_H, debt is fully repaid. However, the home country defaults when bad shocks are realized, as can be seen by the fact that $\delta(B, z_L) < 1$. At one extreme (point B), defaults are large due to the fact that the financial disruption in the domestic country is small when entrepreneurs have a large amount of safe debt in their portfolio. At the other extreme (point A), most of the domestic entrepreneur's portfolio

[6]Because we have not modeled the potential beneficial effects of financial integration, a move from autarky to financially integrated markets results in a welfare loss for both countries in our model. The objective of this paper is not to analyze the trade-off between having an open or a closed economy, but rather to discuss the implications of external forces on the incentives for a given country to default.

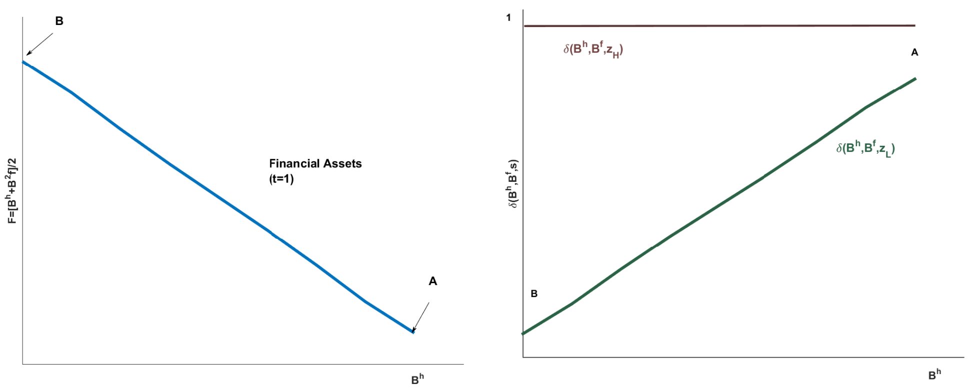


Figure 4: Worldwide financial assets issued in period 1 (first panel) and its repayment in period 2 (second panel) for different Nash equilibria.

is composed of risky debt. A large default in such case is too costly for the home country, which explains why we see higher repayment rates when a bad shock is realized.

High B^h equilibria are therefore associated with lower interest rates for the risky country, and lower spreads, as seen in Figure 5. The risk-free rate is smaller in equilibria with high B^h because the associated B^f is smaller.

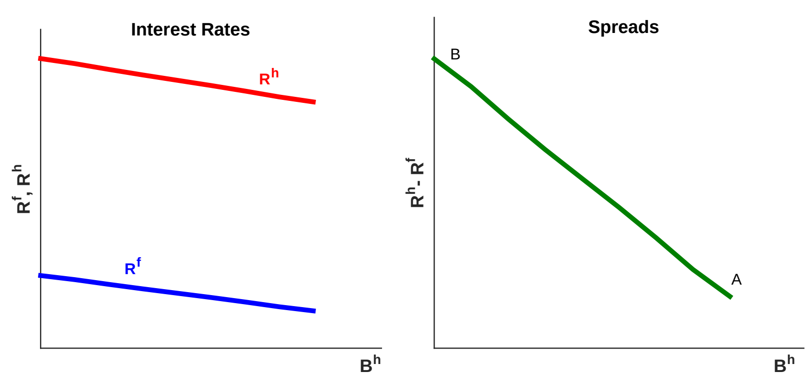


Figure 5: Interest rate and risk premia for different Nash equilibria

The two countries rank the different Nash equilibria quite differently. Figure 6 depicts the first period welfare of the two countries for the various equilibria. The domestic country prefers equilibria where liquidity is provided mostly by foreign non-defaultable debt (e.g. point B). In B there are more financial assets in the economy and the risky country retains the option value to default in case of a bad realization of its aggregate productivity shock z^h. Given that the costs of these defaults are shared with foreign investors, equilibria closer to B are characterized by lower repayment rates. The foreign country, on the other hand, is better off in equilibria in which repayment rates are higher. The only instrument for the foreign government to achieve this is by reducing its own level of debt. Therefore, even though its own citizens are highly exposed to risk, expected defaults are smaller in equilibrium. We expect, then, the safe country to favor 'austerity' (e.g. low levels of debt and transfers) in the first period.

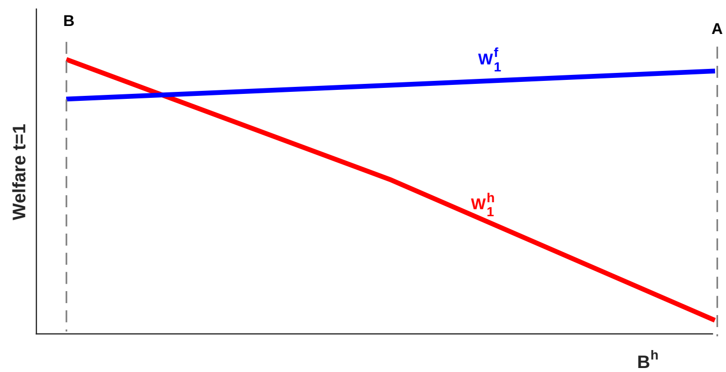


Figure 6: Country's first period welfare for different Nash equilibria

Because the welfare levels of the two countries move in opposite direction, the various equilibria cannot be Pareto ranked.

5 Financial integration with bailouts

So far we have characterized the equilibrium under the assumption that the foreign country commits not to renegotiate the debt if the home country defaults. In many cases of sovereign default, debt is restructured at the aggregate level (i.e. between governments) and with it there is some form of direct or indirect subsidies. For example, with rescue packages the defaulting countries are able to borrow at rates that are smaller than the rates at which they could borrow in the market. Thus, in this section we allow for renegotiation and debt restructuring. This seems a natural assumption since both countries could gain from renegotiating.

5.1 Second period: Renegotiation and debt restructuring

We start the analysis at the outset of the second period. After observing aggregate productivity, $z = (z^h, z^f)$, and given the stocks of home and foreign debt, $B = (B^h, B^f)$, the home country may have an incentive to default by only partially repaying its debt. In such case, the two countries negotiate over two variables: debt repayment—which we denote by P—and a transfer from the foreign government to the home government—which we denote by τ and refer to as the 'bailout' amount.

To characterize the negotiation outcome we use Nash bargaining, assuming that if no agreement is reached, countries obtain the payoffs associated with the no-renegotiation equilibrium described in the previous section. Let's first define some key functions. Given the negotiated policies P and τ, the value for the home country in period 2 is

$$\begin{aligned}\overline{V}^h(\tau, P, B^f, z) &= \Bigg\{(1-\Psi)\ln\left(\tilde{\nu}w^h(\pi,z)^{1+\nu} - P + \tau\right) + \Psi\ln\left(\frac{P+B^f}{2}\right) + \\ &\quad \Psi E_\varepsilon \ln\left(\Big[A(z^h,\varepsilon) - w^h(\pi,z)\Big]\phi^h(\pi,z) + 1\right)\Bigg\},\end{aligned} \tag{10}$$

where the policy vector is $\pi = (B^h, B^f, P)$. This equation is analogous to expression 9, but where repayment is now P instead of $\tilde{B}^h = \mathcal{B}(B, z^h)$ and where the domestic workers receive a transfer equal to the bailout τ.

For the foreign country the value of renegotiating is

$$\begin{aligned}\overline{V}^f(\tau, P, B^f, z) &= \Bigg\{(1-\Psi)\ln\left(\tilde{\nu}w^f(\pi,z)^{1+\nu} - B^f - \tau\right) + \Psi\ln\left(\frac{P+B^f}{2}\right) + \\ &\quad \Psi E_\varepsilon \ln\left(\Big[A(z^f,\varepsilon) - w^f(\pi,z)\Big]\phi^f(\pi,z) + 1\right)\Bigg\},\end{aligned} \tag{11}$$

where domestic repayment is again P but where the bailout τ represents a cost for foreign workers.

The above equations illustrate how renegotiation affects the welfare of the two countries. For both countries, a higher repayment P implies positive effects coming from lower macroeconomic distortions (higher wages for workers and profits for entrepreneurs captured by the first and third terms of equations (10) and (11)) and higher repayment to entrepreneurs (captured by the second term). A higher repayment P also implies a cost for the home country due to the higher taxes that workers need to pay (see first term in equation (10)) while there is no cost for the foreign country. The transfer τ, instead, is a benefit for the home country since it reduces the tax burden of workers but it is a cost for the foreign country since foreign workers will pay for it. Effectively, foreign workers help home workers to repay the debt of the home country.

The above analysis illustrates why the foreign country may gain on net from subsidizing the repayment of the home debt: since a higher P increases the welfare of the foreign country, the foreign government may be willing to pay τ in order to induce a higher repayment from the home government. Since a higher repayment P has also some positive effects for the home country (in addition to the negative effects), the foreign government can convince the home government to repay by subsidizing only a fraction of the repayment. This will become clear in the numerical example below. Before doing so, however, we need to define the bargaining problem formally.

Denote by $\underline{V}^i(B,z)$ the threat value for country i. This is the value that country i would experience in period 2 if there is no agreement. In this case the home country would use the optimal repayment policy $\tilde{B}^h = \mathcal{B}(B, z^h)$ characterized earlier, and receive no bailout, $\tau = 0$. Thus the threat value is

$$\underline{V}^i(B,z) \equiv V^i(B,z;\mathcal{B}),$$

where $V^i(B,z;\mathcal{B})$ was defined in (9) and it represents the value for country i when the home country implements the optimal policy without renegotiation.

We can then write the bargaining problem as

$$\max_{\tau,\, P \leq B^h} \quad \Big[\overline{V}^f(P, B^f, z) - \underline{V}^f(B,z)\Big]^{\eta}\Big[\overline{V}^h(P, B^f, z) - \underline{V}^h(B,z)\Big]^{1-\eta},$$

where η is the relative bargaining power for the foreign country. As it is standard, the bargaining problem maximizes the weighted product of the net renegotiation surpluses of the negotiating parties. Let the solution to this problem be denoted by $\tau = \mathcal{T}(B, z)$ and $P = \mathcal{P}(B, z)$.

Suppose for a moment that the foreign country had all the bargaining power, that is, $\eta = 1$. Then the problem above is equivalent to solving

$$\max_{\tau,\, P \leq B^h} \overline{V}^f(P, B^f, z) \quad \text{s.t}$$

$$\overline{V}^h(P, B^f, z) - \underline{V}^h(B, z) \geq 0.$$

In other words, the safe country must find a package $\{P, \tau\}$ which maximizes the utility of its residents subject to a participation constraint where the utility of the home country must be greater or equal to the one they would receive in the no-renegotiation case. Clearly, when the home country is in a boom $z^h = z_H$, it has no incentives to default. In such case there is full repayment $P = B^h$ and no bailouts $\tau = 0$, regardless of the realization of the aggregate shock in the foreign country. This is no the case when $z^h = z_L$, as the home country may want to default in a recession. Letting λ denote the lagrange multiplier in the participation constraint, we find that the first order condition (assuming an interior solution $P < B^h$) satisfies

$$\Psi \mathbb{E}_\epsilon MU^{Ef}\left[-\frac{\partial w^f}{\partial P}h^f + \frac{1}{2}\right] + (1-\Psi)MU^{Wf}\frac{\partial w^f}{\partial P}h^f +$$

$$\lambda\left\{\Psi \mathbb{E}_\epsilon MU^{E}\left[-\frac{\partial w}{\partial P}h + \frac{1}{2}\right] + (1-\Psi)MU^{W}\left[\frac{\partial w}{\partial P}h - 1\right]\right\} = 0,$$

where $\lambda = \frac{MU^{wf}}{MU^{w}}$ denotes the shadow price of a dollar transfer from foreign workers to home workers and MU^W and MU^E are the marginal utilities of workers and entrepreneurs, respectively. By increasing the transfer, at the relative cost λ, the foreign government can relax the participation constraint of the home country and induce a higher repayment. The term in parenthesis is equal to zero in the no renegotiation equilibrium, because the home country only considers costs and benefits to its residents. Under renegotiation, the foreign country will choose repayment rates incorporating costs and benefits from both the foreign (first row) and home (second row) residents instead. Because this partly internalizes the externality of higher liquidity on production of both countries, we should expect the total amount of financial assets to be larger ex-post.

5.2 First period: Financial liquidity and bailouts

We can now move backward to the first period. Each country chooses B^i in order to maximize

$$\max_{B^i} \left\{ (1-\Psi)U(c_1^i(\pi^R)) + \beta \mathbb{E}_z \overline{V}^i(\mathcal{P}(B,z), B^f, z) \right\},$$

with policy $\pi^R = \{\mathcal{P}(B,z), B\}$. Note that this problem is analogous to the one studied in the previous section, with the difference that second period decisions are chosen under renegotiation. That is, given that in the second period repayment equals $\mathcal{P}(B,z)$. As before, we assume that first period decisions are taken simultaneously and non cooperatively by the home and safe countries. As before, we will focus on a Nash equilibrium in first period policies. Because further theoretical characterization is not possible, we turn to the numerical example.

Baseline parameters are identical to the ones described in Section 4.2.2 and the bargaining weight is $\eta = 0.2$. That is, we analyze a case where the foreign country does not have the whole bargaining power. The best response functions are depicted in the Figure 7 (top panel).

Relative to the case under no renegotiation, we observe three main differences. First, the Nash equilibrium is unique. This arises due to the fact that the foreign country can now affect the decision of the domestic country through renegotiation in the second period using a transfer. The only instrument to achieve this in the previous case was through reductions in its own level of debt. Second, the debt level of the home country is much larger whereas that of the foreign country is significantly lower. This can be seen by comparing the intersection of impulse responses in the top and bottom panels of Figure 7, the latter representing the case under no renegotiation. Knowing that it will be bailed out with certainty by the foreign country, the home country has incentives to significantly increase borrowing in the first period. The prospect of the bailout reduces the macroeconomic costs of a default, as the home country can secure higher consumption for its workers without having to sacrifice production were a recession to happen. Renegotiation thus make defaults cheaper. Finally, we see that when bailouts are allowed, the worldwide debt level is higher than under no renegotiation and closer to the one attained under autarky. With no renegotiation, each country ignores the benefits that higher liquidity brings to the other country (a positive externality). During the renegotiation process countries must take

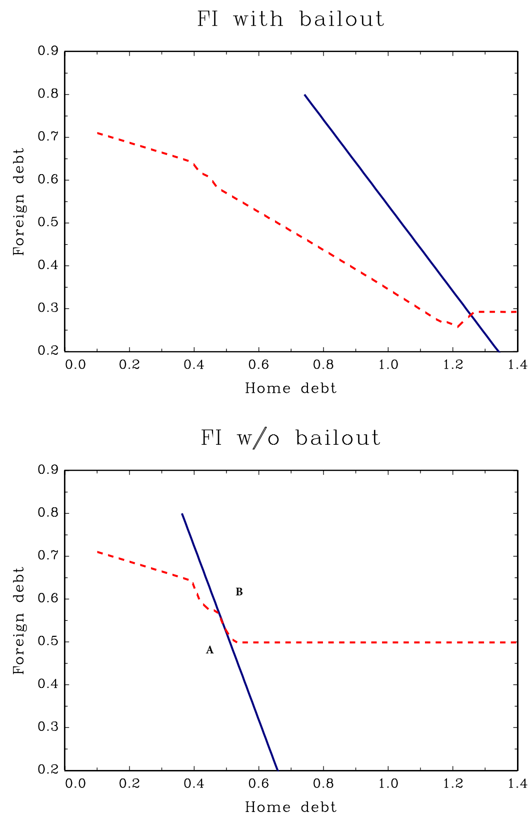


Figure 7: Impulse response functions of the safe (dotted lines) and foreign (solid line) countries under renegotiation (top panel) and no renegotiation (bottom panel)

this into account, as captured by the fact that both, the foreign and domestic second period indirect utilities, are part of the objective function being maximized under Nash bargaining. This results in lower defaults and hence higher overall debt levels in the renegotiation equilibrium.

5.3 To bailout or not to bailout?

Given that the bailout costs are borne by the foreign country, it is reasonable to ask whether the safe country would be better off by committing never to renegotiate with the risky country. That is, whether it would achieve a higher welfare ex-ante. Clearly, welfare ex-post (i.e. in the second period) is higher under renegotiation, since the safe country can always choose not to bail out the risky country and revert to the threat value of no-renegotiation. In the first period, however, there is a cost as higher bailouts imply less restrain from the risky country when choosing B^h and potentially higher bailout costs in the future.

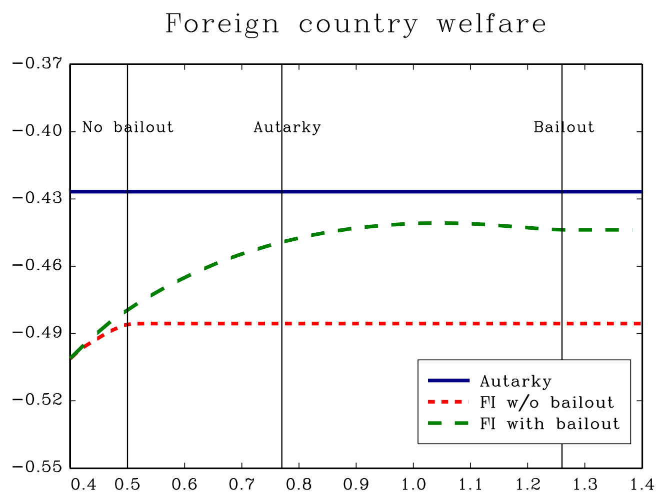
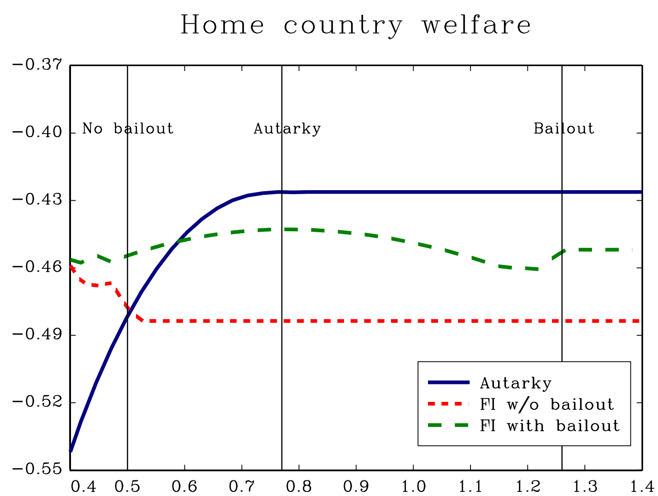


Figure 5.3 shows the first period utility of the foreign country as a function of B^h and given that B^f is chosen optimally. The solid line represents this value under autarky, which as expected, is independent of the level of risky debt. The dotted line represents the case under financial integration with no renegotiation, whereas the dashed line welfare with renegotiation. The vertical lines indicate the equilibrium values in the three scenarios, indicating that the foreign country would be definitely better off if it were allowed to renegotiate by compering across the two equilibria with financial integration. Moreover, we can see that the level of welfare attained with bailouts is larger than that under no renegotiation for every possible value of B^h considered in the plot.

Figure 5.3 shows first period welfare for the home country as a function of its own debt (and given the optimal response of the foreign country). The home country is too, better off with the possibility of bailouts than under no renegotiation.

This implies that the possibility to renegotiate debt can make *both* countries better off from an ex-ante perspective. This holds even in this example where most of the bargaining power is in the hands of the home country.



The underlying intuition goes back to the role of debt as a source of liquidity. Because a higher level of liquidity can be attained worldwide in the renegotiation case, and hence more production, welfare is higher.

6 Conclusion

In this paper we have shown that the default of a country on its sovereign debt could be induced by excessive borrowing from other countries if financial markets are integrated. The integration of financial markets increases the incentive to default not only because part of the defaulted debt is owned by foreigners (as widely emphasized in the literature) but also because the 'endogenous' macroeconomic cost of default is smaller when the defaulting country is financially integrated.

In our model government debt is held by producers for insurance purposes. When financial markets are integrated, producers also hold foreign government debt. Therefore, when the domestic government defaults, producers are only partially affected by default with smaller consequences for aggregate production in the domestic country. Furthermore, higher is the debt issued by the foreign country and higher is the incentive of the home country to default since domestic producers are more insured by holding the foreign debt. This implies that the sovereign default of a country could be externally induced by the excessive borrowing of foreign countries. From this perspective, the recent debt problems experienced by some European countries can be the result (at least in part) of the increased debt in 'safe' industrialized countries since the early 1980s.

7 Appendix

A Proof of Lemma 2

The FOC of the entrepreneur maximize the problem stated in Condition 1 of Definition 1 are

$$l(\pi, z): \qquad E_{\varepsilon}\left[\frac{A(z,\varepsilon) - w(\pi, z)}{d_2(\pi, z, \varepsilon)}\right] = 0 \tag{12}$$

and

$$b: \qquad \frac{1}{d_1(\pi)} = \beta E_{z,\varepsilon}\left[\frac{\delta(z) R(\pi)}{d_2(\pi, z, \varepsilon)}\right] \tag{13}$$

Guess policy functions to be

$$l(\pi, z) = \phi(\pi, z) b \delta(z)$$

$$b = \gamma R(\pi) a$$

where $\{\phi(\pi, z)\}_z$ and γ are unknown parameters. These imply that $d_1 = a(1-\gamma)$ and $d_2(\pi, z, \varepsilon) = b\delta(z)[1 + \phi(\pi, z)(A(z,\varepsilon) - w(\pi, z))]$. Using eq. (13) and the two guesses, we obtain

$$\beta \frac{1-\gamma}{\gamma} E_{\varepsilon,z}\left[\frac{1}{1 + \phi(\pi, z)[A(z,\varepsilon) - w(\pi, z)]}\right] = 1 \tag{14}$$

Replacing the guesses in eq. (12) we get

$$E_{\varepsilon}\left[\frac{A(z,\varepsilon) - w(\pi, z)}{1 + \phi(\pi, z)[A(z,\varepsilon) - w(\pi, z)]}\right] = 0$$

This equation implicitly defines $\phi(\pi, z)$. Multiply each side of it by $\phi(\pi, z)$, then subtract 1 from both sides and reorganize to obtain

$$E_{\varepsilon}\left[\frac{1}{1 + \phi(\pi, z)[A(z,\varepsilon) - w(\pi, z)]}\right] = 1$$

Taking expectations with respect to s, yields

$$E_{\varepsilon,z}\left[\frac{1}{1+\phi(\pi,z)[A(z,\varepsilon)-w(\pi,z)]}\right]=1$$

Replacing the last expression into eq. (14) and simplifying yields $\gamma=\frac{\beta}{1+\beta}$. Replacing this back into the guess for b, we obtain $b=\frac{\beta}{1-\beta}R(\pi)a$. The expressions for d_1 and $d_2(\pi,z,\varepsilon)$ in the Lemma are obtained by replacing the results into the entrepreneurs' budget constraints. QED

B Competitive equilibrium for given policies

The interest rate is given by

$$R(\pi)=\frac{(1-\beta)B}{\beta a}. \tag{15}$$

The wage rate $w(\pi,z)$ and the labor demand factor $\phi(\pi,z)$ are implicitly determined by the equations

$$w(\pi,z)=\alpha\left(\phi(\pi,z)\tilde{B}(z)\right)^{\frac{1}{\nu}} \tag{16}$$

$$\mathbb{E}_{\varepsilon}\left\{\frac{A(z,\varepsilon)-w(\pi,z)}{1+[A(z,\varepsilon)-w(\pi,z)]\phi(\pi,z)}\right\}=0. \tag{17}$$

Aggregate labor is given by

$$l(\pi,z)=\phi(\pi,z)\tilde{B}=h(\pi,z). \tag{18}$$

Finally, consumption is determined by the equations

$$c_1 = e_1+\frac{\beta a}{(1+\beta)} \tag{19}$$

$$\bar{c}_2(\pi,z) = \left(\frac{\alpha\nu}{1+\nu}\right)h(\pi,z)^{1+\frac{1}{\nu}}+\left(\frac{\alpha^{\nu}}{1+\nu}\right)w(\pi,z)^{1+\nu}+e_2-\tilde{B}(z) \tag{20}$$

$$d_1 = \frac{a}{1+\beta} \tag{21}$$

$$d_2(\pi,z,\varepsilon) = \left[1+\left(A(z,\varepsilon)-w(\pi,z)\right)\phi(\pi,z)\right]\tilde{B}(z) \tag{22}$$

C Proof of Lemma 3

Replace eq. (16) into eq. (17) to obtain

$$E_\varepsilon \frac{A(z,\varepsilon) - \alpha\left(\phi\tilde{B}\right)^{1/\nu}}{1 + \left[A(z,\varepsilon) - \alpha\left(\phi\tilde{B}\right)^{1/\nu}\right]\phi} = 0,$$

where dependence on the aggregate state z has been omitted to ease readability. This can be written more compactly as the following implicit function

$$F(\phi,\tilde{B}) \equiv E_\varepsilon \left\{ \left[A(z,\varepsilon) - \alpha\left(\phi\tilde{B}\right)^{1/\nu}\right]^{-1} + \phi \right\}^{-1} = 0,$$

Using the implicit function theorem,

$$\frac{\partial\phi}{\partial\tilde{B}} = -\frac{\partial F/\partial\tilde{B}}{\partial F/\partial\phi} = -\frac{E_\varepsilon G^{-2}(A(z,\varepsilon)-w)^{-2}\frac{w}{\nu\tilde{B}}}{E_\varepsilon G^{-2}\left[(A(z,\varepsilon)-w)^{-2}\frac{w}{\nu\phi}+1\right]} < 0$$

since $G \equiv \left[A(z,\varepsilon) - \alpha\left(\phi\tilde{B}\right)^{1/\nu}\right]^{-1} + \phi > 0$. This establishes the first result.

Differentiate eq.(16) to obtain $\frac{\partial w(\pi,z)}{\partial\tilde{B}}$. After some algebraic manipulations,

$$\frac{\partial w}{\partial\tilde{B}} = \frac{1}{\nu}\frac{w}{\tilde{B}}\left[\frac{\tilde{B}}{\phi}\frac{\partial\phi}{\partial\tilde{B}} + 1\right]$$

where $\frac{\tilde{B}}{\phi}\frac{\partial\phi}{\partial\tilde{B}} \leq 0$ is the elasticity of the entrepreneurs' labor share ϕ with respect to $\tilde{B}$. We will show that wages and increasing in $\tilde{B}$ by contradiction. Suppose $\frac{\partial w}{\partial\tilde{B}} < 0$. Since $\tilde{B} \geq 0$ (by assumption), it must be the case that

$$\frac{\tilde{B}}{\phi}\frac{\partial\phi}{\partial\tilde{B}} < -1$$

Alternatively,

$$\frac{E_\varepsilon G^{-2}(A(z,\varepsilon)-w)^{-2}\frac{w}{\nu}}{E_\varepsilon G^{-2}\left[(A(z,\varepsilon)-w)^{-2}\frac{w}{\nu}+\phi\right]} > 1$$

But this would imply that $E_\varepsilon G^{-2}\phi < 0$, a contradiction.

Finally, using the fact that $H = l = \left(\frac{w}{\alpha}\right)^\nu$, we can show that

$$\frac{\partial H}{\partial \tilde{B}} = \frac{\nu H}{w}\frac{\partial w}{\partial \tilde{B}} \geq 0.$$

QED

D Optimality condition under autarky

The first order condition of the relaxed problem (that is, ignoring the constraint $\tilde{B} \leq B$) is

$$\Psi \mathbb{E}_\varepsilon \frac{\partial U}{\partial d_2}\left[-\frac{\partial w}{\partial \tilde{B}}h + 1 + (A - w)\frac{\partial h}{\partial \tilde{B}}\right] + \tag{23}$$

$$(1-\Psi)\frac{\partial U}{\partial c_2}\left[\frac{\partial w}{\partial \tilde{B}} - 1 + w\frac{\partial h}{\partial \tilde{B}}\right] + \frac{\partial U}{\partial h2}\frac{\partial h}{\partial \tilde{B}} = 0.$$

From the optimality condition of agents, we know that

$$\mathbb{E}_\varepsilon \frac{\partial U}{\partial d_2}(A - w) = 0$$

and

$$\frac{\partial U}{\partial c_2}w + \frac{\partial U}{\partial h2} = 0$$

which allows us to simplify terms involving $\frac{\partial h}{\partial \tilde{B}}$. Replacing these in eq. (D) and simplifying, we obtain

$$\left(1 - \frac{\partial w}{\partial \tilde{B}}h\right)\left(\Psi \mathbb{E}_\varepsilon \frac{\partial U}{\partial d_2} - (1-\Psi)\frac{\partial U}{\partial c_2}\right) = 0.$$

The result follows from the fact that the first term in parenthesis is nonzero. QED

E Proof of Proposition 4

Replacing c_2 and d_2 in eq. (6), defining $\psi = \frac{1-\Psi}{\Psi}$, and rearranging, we obtain

$$\mathbb{E}_\varepsilon \frac{1}{[1 + (A(z,\varepsilon) - w(\pi, z))\,\phi(\pi, z)]} = \psi \frac{\tilde{B}}{\tilde{\nu} w(\pi, z)^{1+\nu} - \tilde{B}}.$$

The left hand side is equal to 1 from the optimality condition of entrepreneurs. Hence,

$$\tilde{\nu} w(\pi, z)^{1+\nu} - \tilde{B} = \psi \tilde{B}. \;=>\; w(\pi, z) = \left[\frac{\tilde{B}(1+\psi)}{\tilde{\nu}}\right]^{\frac{1}{1+\nu}}.$$

Equating this to eq. (17) and simplifying delivers

$$\phi(\tilde{B}) = \frac{\phi_0}{\tilde{B}^{\frac{1}{1+\nu}}} \qquad \text{where} \qquad \phi_0 = \frac{1}{\alpha}\left[\frac{1+\psi}{\tilde{\nu}}\right]^{\frac{\nu}{1+\nu}}. \tag{24}$$

Wages become

$$w(\tilde{B}) = \omega_0 \tilde{B}^{\frac{1}{1+\nu}} \qquad \text{where} \qquad \omega_0 = (\alpha\phi_0)^{\frac{1}{\nu}} \tag{25}$$

Replacing eq. (24) and (25) into eq. (17), we obtain an implicit function $\tilde{B}(z)$

$$F(\tilde{B}, z) = \mathbb{E}_\varepsilon \left\{\frac{1}{1 + [A(z,\varepsilon) - w(\tilde{B})]\phi(\tilde{B})}\right\} - 1 = 0 \tag{26}$$

We can obtain $\frac{\partial \tilde{B}}{\partial z}$ using the implicit function theorem:

$$\frac{\partial \tilde{B}}{\partial z} = -\frac{\frac{\partial F(\tilde{B},z)}{\partial z}}{\frac{\partial F(\tilde{B},z)}{\partial \tilde{B}}},$$

where

$$\frac{\partial F(\tilde{B}, z)}{\partial z} = -\mathbb{E}_\varepsilon \left[1 + [A(z,\varepsilon) - w]\phi\right]^{-1} \phi < 0 \tag{27}$$

using that $A(z, \varepsilon) = z + \varepsilon$ to replace $\frac{\partial A}{\partial z} = 1$, and

$$\frac{F(\tilde{B}, z)}{\partial \tilde{B}} = -\mathbb{E}_\varepsilon \left[1 + [A(z,\varepsilon) - w]\phi\right]^{-1} \left[-\phi \frac{\partial w}{\partial \tilde{B}} + [A(z,\varepsilon) - w]\frac{\partial \phi}{\partial \tilde{B}}\right]$$

Using eqs. (24) and (25), we obtain

$$\frac{\partial w}{\partial \tilde{B}} = \frac{1}{1+\nu}\frac{w}{\tilde{B}} \quad \text{and} \quad \frac{\partial \phi}{\partial \tilde{B}} = -\frac{1}{1+\nu}\frac{\phi}{\tilde{B}}$$

Replacing these equations in $\frac{F(\tilde{B},z)}{\partial \tilde{B}}$ above, we have

$$\frac{F(\tilde{B}, z)}{\partial \tilde{B}} = \mathbb{E}_\varepsilon \left[1 + [A(z,\varepsilon) - w]\phi\right]^{-1} \frac{A(z,\varepsilon)\phi}{(1+\nu)\tilde{B}} > 0 \tag{28}$$

Using eq. (27) and eq. (28) in eq. (26) establishes the result.

QED

F Proof of Lemma 6

The entrepreneurs' maximization problem is

$$\max_{x_i} \ln d_1^i + \beta E_{s,\varepsilon} \ln d_2^i(\pi, s, \varepsilon)$$

$$d_1^i = a - \frac{b^{hi}}{R^h} - \frac{b^{fi}}{R^f}$$

$$d_2^i(\pi, s, \varepsilon) = (A(z_i, \varepsilon) - w^i(\pi, s))l^i(\pi, s) + b^{hi}\delta(\pi, s) + b^{fi},$$

where $x_i = \{d_1^i, d_2^i, l^i, b^{fi}, b^{hi}\}$ is their set of choices.

Their FOC are

$$\frac{1}{d_1^i}\frac{1}{R^h} = \beta E_{s,\pi,\varepsilon} \frac{\delta(\pi, z)}{d_2^i(\pi, s, \varepsilon)} \quad (29)$$

$$\frac{1}{d_1^i}\frac{1}{R^f} = \beta E_{s,\pi,\varepsilon} \frac{1}{d_2^i(\pi, s, \varepsilon)} \quad (30)$$

$$E_\varepsilon \frac{A(z_i, \varepsilon) - w^i(\pi, z)}{d_2^i(\pi, s, \varepsilon)} = 0 \quad (31)$$

From eqs. (29) and (30) we obtain

$$R^h = \eta R^f \qquad \text{where} \qquad \eta = \frac{E_{s,\pi,\varepsilon} \frac{1}{d_2^i(\pi,s,\varepsilon)}}{E_{s,\pi,\varepsilon} \frac{\delta(\pi,z)}{d_2^i(\pi,s,\varepsilon)}} \geq 1.$$

Hence, η is capturing the risk premium paid by defaultable debt B^h relative to the risk free debt B^f.

Guess the following

$$b^{hi} = \theta^{hi} R^h a$$

$$b^{fi} = \theta^{fi} R^f a$$

$$l_i = \phi^i [b^{hi}\delta + b^{fi}]$$

Under that guess (and abstracting from arguments to simplify notation)

$$d_1^i = a(1 - \theta^{hi} - \theta^{fi})$$

$$d_2^i = \left([A(z^i,\varepsilon) - w^i]\phi^i + 1\right)[b^{hi}\delta + b^{fi}],$$

Moreover,

$$b^{hi}\delta + b^{fi} = a(\theta^{hi}R^h\delta + \theta^{fi}R^f)$$

Replacing the equations above in eq. (31) and using the fact that $b^{hi}\delta + b^{fi}$ is independent of ε we obtain

$$E_\varepsilon \frac{A(z^i,\varepsilon) - w^i(\pi,z)}{[A(z^i,\varepsilon) - w^i(\pi,z)]\phi^i(\pi,z) + 1} = 0$$

Multiplying by $\phi^i(\pi,z)$ and subtracting 1 from both sides, we get

$$E_\varepsilon \frac{1}{[A(z^i,\varepsilon) - w^i(\pi,z)]\phi^i(\pi,z) + 1} = 1 \tag{32}$$

Replacing the guesses in eq. (30)

$$\frac{1}{1-\theta^{fi}-\theta^{hi}} = \beta E_{s,\pi}\left\{\frac{R^f}{\theta^{hi}R^h\delta + \theta^{fi}R^f} E_\varepsilon\left[\frac{1}{[A(z^i,\varepsilon) - w^i]\phi^i + 1}\right]\right\}$$

From eq. 32, we know that for each $\{s,\pi\}$, the term involving E_ε is equal to 1. Using the fact that $R^h = \eta R^f$,

$$\frac{1}{1-\theta^{fi}-\theta^{hi}} = \beta E_{s,\pi}\left[\frac{1}{\theta^{hi}\eta\delta + \theta^{fi}}\right] \tag{33}$$

Replace the guesses into eq. (29), and follow the same steps to obtain

$$\frac{1}{1-\theta^{fi}-\theta^{hi}} = \beta E_{s,\pi}\left[\frac{\delta\eta}{\theta^{hi}\eta\delta + \theta^{fi}}\right] \tag{34}$$

Multiply both sides of eq. (33) by θ^{fi}, and both sides of eq. (34) by θ^{hi}, and add the resulting expressions. This delivers, after some algebra,

$$\theta^{hi} = \frac{\beta - (1+\beta)\theta^{fi}}{1+\beta} \tag{35}$$

We can replace eq. (35) into eq. (33) and obtain

$$E_{s,\pi}\left[\frac{1}{\frac{\beta-(1+\beta)\theta^{fi}}{1+\beta}\eta\delta + \theta^{fi}}\right] = \frac{1+\beta}{\beta} \tag{36}$$

This determines θ^{fi} as a function of η. Notice that since η and δ are not country specific, the equation implies that $\theta^{fh} = \theta^{ff} \equiv \theta^f$. From eq. (35), we get that $\theta^{hh} = \theta^{hf} \equiv \theta^h$ as well. Hence, *the portfolio allocation is the same in both countries.*

QED

G Proof of Proposition ??

Condition 1 Let $\tilde{B}^h(z) = B^h\delta(z)$. Since all entrepreneurs are identical in $t = 1$,

$$b^{hh} + b^{hf} = B^h$$

Using the results from Lemma 6 and noticing that $\theta^h(\pi)$ is country-independent, we get

$$R^h(\pi) = \frac{B^h}{2a\theta^h(\pi)} \qquad \text{and} \qquad R^f(\pi) = \frac{B^f}{2a\theta^f(\pi)}.$$

We can use $R^h = \eta(\pi)R^f$ to get

$$\eta(\pi) = \frac{B^h}{B^f}\frac{(1+\beta)\theta^f(\pi)}{\beta - (1+\beta)\theta^f(\pi)}$$

Replace this into eq. (36) and simplify to obtain

$$\theta^f(\pi) = \frac{\beta}{1+\beta}E_z\left[\frac{1}{1+\frac{\tilde{B}^h(z)}{B^f}}\right]. \tag{37}$$

Replacing this into eq. (35), we obtain $\theta^h(\pi)$.

Condition 2 From the workers' optimality condition

$$l^i(\pi, z) = \left[\frac{w^i(\pi, z)}{\alpha}\right]^\nu$$

The aggregate demand for labor satisfies

$$h^i(\pi, s) = \phi^i(\pi, s)\left[\delta(z)b^{hi}(\pi) + b^{fi}(\pi)\right]$$

In equilibrium, $l^i(\pi, s) = h^i(\pi, s)$. Using Condition 1 and the results from Lemma 6,

$$b^{hi}(\pi) = \theta^h R^h(\pi) a \Rightarrow b^h(\pi) = \frac{B^h}{2},$$

and

$$b^{fi}(\pi) = \theta^f R^f(\pi) a \Rightarrow b^{fi}(\pi) = \frac{B^f}{2},$$

Hence, the financial wealth of both home and foreign entrepreneurs is $(B^h + B^f)/2$. This implies that B^h and B^f are sufficient state variables for the portfolio holdings of entrepreneurs. The wage rate and, therefore, the factor that determines the demand of labor depend on country i's productivity z^i and wealth of home entrepreneurs after government default $(\tilde{B}^h + B^f)/2$. Therefore, we will denote the wage as $w^h(\pi, z)$ and the labor demand factor as $\phi^h(\pi, z)$. Replacing $b^{hi}(\pi)$ and $b^{fi}(\pi)$ in the labor market equilibrium condition, delivers two equations determining w^i and ϕ^i,

$$w^i(\pi, z) = \alpha \left(h^i(\pi, z)\right)^{1/\nu}$$

$$E_\varepsilon \frac{A(z^i, \varepsilon) - w^i(\pi, z)}{[A(z^i, \varepsilon) - w^i(\pi, z)]\phi^i(\pi, z) + 1} = 0,$$

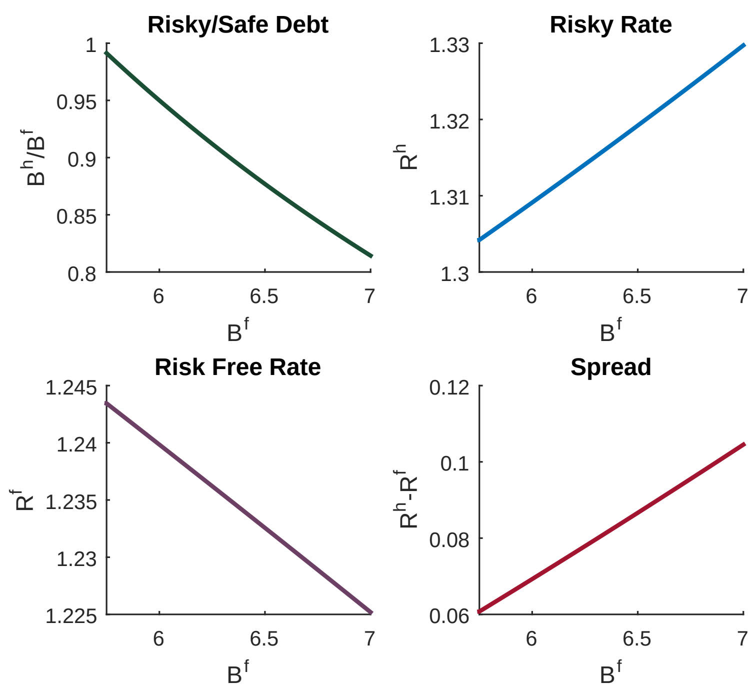
with $h^i(\pi, z) = \phi^i(\pi, z) \left[\frac{\tilde{B}^h(z) + B^f}{2}\right]$.

Conditions 3 and 4 : Replace the results above in entrepreneurs' and workers' budget constraints.

QED

H External liquidity and default

To further understand the role played by foreign debt on the incentive to default, we now consider the following thought experiment. Suppose that the foreign debt B^f increases but the debt of the home country issued in period 1, B^h, does not changed. We then ask how this change in foreign debt affects prices and allocations. We show the results numerically by assigning the following parameter values: $\beta = 0.9825$, $\nu = \alpha = 1$, $a = e_1 = e_2 = 10$, ε is uniformly distributed over the interval $[0.9, 1.1]$, $z \in \{0.9, 1.1\}$, and $\Psi = 0.7$. The upper left panel of Figure H depicts the ratio between risky and safe debt, B^h/B^f as a function external liquidity B^f. Since B^h is assumed to be constant, the ratio declines as B^f increases. From the previous analysis we know that as B^f increases, the incentive to default for the home country rises. This reduces the benefits for entrepreneurs from holding the debt issued by the home country and, therefore, it reduces the demand for B^h in period 1 ($\theta^h(\pi)$ declines). Obviously, risky debt becomes less attractive when agents understand that the default risk has gone up. Given that the supply of B^h remains unchanged in our thought experiment, its price $1/R^h$ must decline. In fact, the upper right panel of Figure H shows that R^h increases with B^f and ,therefore, the price of the risky debt decreases with the supply of the foreign debt.



While the price of the risky debt increases, the price of the safe foreign debt increases. In fact, the lower left panel of the Figure H shows that R^f decreases with B^f, which has a simple intuition. Entrepreneurs in both countries have an incentive to substitute home debt B^h with foreign debt B^f when the home debt becomes riskier: 'flight to quality'. This implies that the share of savings allocated to foreign debt, θ^f, increases. It turns out that the increase in the demand for safe assets dominates the increase in the supply leading to a negative relationship between the risk free rate R^f and the stock of risk free debt B^f. Therefore, even though the foreign country increases the supply of debt, the effect on the incentive to default of the home country makes B^f more valuable for entrepreneurs and they are willing to pay a higher price to have it in their portfolio.

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