



Distortionary taxes and public investment when government promises are not enforceable

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ABSTRACT

We characterize the optimal financing of productive public capital and compute the welfare loss from being unable to commit to the Ramsey policy. Because this calculation ultimately relies on numerical approximations, we contrast alternative approaches. While perturbation and linear quadratic methods deliver accurate steady states, the latter can yield misleading policy implications during transitions. We find that moving from a regime with commitment to one with discretion implies only a small welfare loss. Although Markov-perfect consumption falls noticeably short of its Ramsey counterpart in steady-state, consumption under discretion is higher in the short-run which largely offsets this long-run loss.

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1. Introduction

The notion that governments cannot always commit to a sequence of actions is a subject of increasing interest for economists in general and policymakers in particular. To this point, the literature on time-consistent fiscal policy has confined itself to simple environments where taxes are used to finance a flow of public goods or services that are rapidly exhausted. In contrast, the benefits of government spending have been mainly documented for durable public goods that can be accumulated over time.¹ This fact is ignored in recent studies because introducing public capital (an additional state variable) significantly complicates the characterization of the optimal discretionary policy. This paper, therefore, tackles the problem of understanding how the absence of government commitment affects the provision of public infrastructure, as well as the implied welfare effects over an economy's transition to its long-run equilibrium. We solve for Markov-perfect equilibria and provide a quantitative assessment of the value of commitment, which we define as the welfare loss incurred when governments cannot commit to the sequence of actions that produce second-best allocations. In doing so, we evaluate the performance of different numerical methods used in approximating time-consistent policy.

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¹ See Fernald (1999), or Haughwout (2002), for example.

Previous work on optimal public investment, including Glomm and Ravikumar (1994, 1997), or Turnovsky (1997), characterize optimal policy under full commitment only. More recently, several papers have analyzed optimal fiscal policy absent commitment, but in environments where public goods cannot be accumulated. These include, among others, Klein et al. (2008), who analyze the trade off between providing a consumable public goods and its financing, Hassler et al. (2007), who study time-consistent redistribution under repeated voting, and Azzimonti et al. (2006), who explore the distortionary effects of income taxes on the evolution of wealth inequality. In contrast to these papers, our analysis focuses on the provision of a durable public goods that expands the production frontier. Thus, we contribute to the literature on public investment and discretionary policy in mainly three ways.

First, at a theoretical level, we show that governments following a Markov-perfect policy choose a tax rate such that they trade off marginal inefficiencies arising in private savings with those arising in the provision of public infrastructure over two consecutive periods only. The derivation of the government Euler equation (GEE) with two state variables is substantially more involved than those developed in the previous work but remains analytically tractable. More importantly, we show that this derivation allows for the application of numerical methods that efficiently and accurately describe transition dynamics.

Second, in computing the Markov-perfect policy problem, we compare numerical solutions obtained using GEE-based perturbation methods recently suggested in Krusell et al. (2002), with those that emerge under a global method (GM) that does not require derivation of the GEE. We further gauge the more common linear quadratic (LQ) approximation approach developed in Klein and Rios-Rull (2003), as well as Svensson and Woodford (2004), against this global method. We know of no other papers in the literature that compare these numerical methods for a single problem. While both the perturbation and LQ approaches deliver accurate steady-state allocations, we find that the approximation errors associated with the latter can yield misleading policy recommendations in response to changes in the state variables. In contrast, an application of the perturbation method is able to generate decision and policy rules that differ minimally from those delivered by the global method.

Finally, our analysis indicates that while Markov-perfect and Ramsey policies can lead to considerably different allocations in the long-run, moving from an economy with government commitment to one with discretion implies only a small welfare loss. This finding stems from the greater emphasis that Markov governments place on short-run gains relative to a Ramsey planner. In particular, although the economy with discretion achieves noticeably lower long-run consumption relative to the regime with commitment, the tax policy chosen under discretion implies higher consumption in the transition that largely offsets this long-run loss. Ultimately, the absence of government commitment results in lower tax rates and, therefore, less public infrastructure being developed. Because a lower level of infrastructure reduces the marginal product of private capital, the economy operating under discretion gives rise to lower private investment and lower consumption in the long-run despite its lower taxes.

This paper is organized as follows. Section 2 describes the basic economic environment. In Section 3, we define the competitive equilibrium given a stationary policy rule. Section 4 characterizes the Markov-perfect equilibrium that yields the optimal policy. Section 5 contrasts numerical solution methods, and we calculate the value of government commitment in Section 6. Section 7 offers some concluding remarks.

2. Environment

Consider an economy populated by infinitely many households whose preferences are given by

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $0 < \beta < 1$ is a subjective discount rate, and households' period utility, $u(c_t)$, satisfies $u_c > 0$, $u_{cc} < 0$, and the usual Inada conditions. The size of the population is normalized to one.

A single consumption good is produced using the technology

$$y_t = F(k_t, l_t, k_{gt}),$$

where k_t and k_{gt} denote the date t stocks of aggregate capital in the private and public sectors, respectively. Labor input is denoted by l_t , and we assume that F exhibits constant returns to scale with respect to private capital and labor. We denote the public capital elasticity of output, $F_{k_g}(k_g/y)$, by $\theta \in (0, 1)$, and assume that $F_{kk_g} > 0$. These assumptions follow along the lines of earlier work, notably by Glomm and Ravikumar (1994, 1997). Since leisure is non-valued, we assume that agents supply labor inelastically and set $l_t = 1 \forall t$.² To simplify notation, we define $f(k_t, k_{gt}) \equiv F(k_t, 1, k_{gt})$.

Both types of capital can be accumulated over time and evolve according to

$$k_{t+1} = i_{kt} + (1 - \delta_k)k_t \tag{1}$$

² This assumption is made for simplicity, and helps keep the derivation of the GEE below somewhat concise despite the additional state variable. See Klein et al. (2008) for an extension with endogenous labor in a setting without public capital.

and

$$k_{gt+1} = i_{k_g t} + (1 - \delta_{k_g})k_{gt}, \quad (2)$$

where i_k and i_{k_g} denote private and public investments, respectively. Private capital depreciates at rate δ_k , and public capital at rate δ_{k_g} , where $\delta_i \in (0, 1)$.

The government taxes households to finance public infrastructure as well as conspicuous expenditures that do not provide direct utility to households. Following Cassou and Lansing (1998), we assume that non-productive expenditures are proportional to output, $g_t/y_t = \phi$ with $\phi \in (0, 1)$, and that the ratio ϕ is exogenous. As such, this specification allows us to introduce expenditures that are quantitatively important in calibration exercises but whose endogeneity we do not explicitly model. Our analysis in this paper will focus exclusively on productive capital expenditures.

2.1. The first-best solution

It is useful to first characterize efficient allocations in this environment. These allocations will then be used as a benchmark with which to compare the decentralized solutions that emerge under two key policy frictions: distortionary taxation and the fact that the government cannot credibly promise to abide by a sequence of future tax rates.

Pareto-optimal allocations are found by solving the problem of a benevolent planner who chooses sequences of consumption, private capital, and public capital so as to maximize households' lifetime utility

$$\begin{aligned} \max_{\{c_t, k_{t+1}, k_{gt+1}\}} \quad & \sum_{t=0}^{\infty} \beta^t u(c_t) \\ \text{s.t.} \quad & c_t + k_{t+1} + k_{gt+1} + g_t = f(k_t, k_{gt}) + (1 - \delta_k)k_t + (1 - \delta_{k_g})k_{gt}, \end{aligned} \quad (P^*)$$

where $g_t = \phi f(k_t, k_{gt})$. The first-order necessary conditions imply that³

$$u_{ct} = \beta [f_{kt+1}(1 - \phi) + 1 - \delta_k] u_{ct+1} \quad (3)$$

and that

$$u_{ct} = \beta [f_{k_{gt+1}}(1 - \phi) + 1 - \delta_{k_g}] u_{ct+1}. \quad (4)$$

Therefore, in the absence of frictions, the standard result obtains such that when $\delta_k = \delta_{k_g}$, it is optimal to invest in k and k_g up to the point where their marginal products are equalized at each date,

$$f_{kt+1} = f_{k_{gt+1}}.$$

We refer to the optimal levels of private and public capital as the “unconstrained optimum”. It is straightforward to show how to obtain these allocations in a decentralized competitive equilibrium when the government has access to lump sum taxation. In the absence of such an instrument, however, a distortion emerges that creates a wedge, or gap, in conditions (3) and (4). In addition, to the degree that policies other than lump sum taxes are used, such policies will generally be time inconsistent.⁴ In general, we define a wedge in the efficient private investment decision by

$$\Delta_{kt} = -u_{ct} + \beta u_{ct+1} [f_{kt+1}(1 - \phi) + 1 - \delta_k], \quad (5)$$

where $\Delta_{kt} = 0 \forall t$ under first-best allocations. Similarly, we define a wedge in public investment by

$$\Delta_{k_{gt}} = -u_{ct} + \beta u_{ct+1} [f_{k_{gt+1}}(1 - \phi) + 1 - \delta_{k_g}]. \quad (6)$$

3. The decentralized economy given policy

Because lump sum taxes are almost never observed, we focus on a decentralized economy where the government uses distortionary income taxes to finance its public capital. In that setting, a new government coming into office typically has an incentive to disregard promises made by its predecessors. Hence, setting taxes once and for all at time zero results in policy announcements that are not credible, since, in each subsequent period, policymakers take the states they inherit as given and do not account for the impact of distortionary taxes on the previous investment decisions. In other words, absent a commitment technology, the Ramsey policy is time inconsistent. Our analysis centers on the determination of optimal time-consistent tax rates, and contrasts the findings with those that obtain under full commitment. In particular, we study Markov-perfect equilibria whereby sequential governments choose policy optimally based on the state they inherit when taking office.⁵

³ We use u_{ct} to indicate $u_c(c_t)$, f_{kt+1} to indicate $f_k(k_{t+1}, k_{gt+1})$, etc. to simplify notation when the context is clear.

⁴ See Kydland and Prescott (1977) for an early formal treatment of time-inconsistent policy.

⁵ An alternative approach finds the set of all possible sustainable equilibria, and characterizes the problem using reputational mechanisms that rely on trigger strategies involving reversion to the worst possible equilibrium (Chari and Kehoe, 1993). Such mechanisms, however, are generally not renegotiation proof.

Throughout the paper, we assume that the government balances its budget every period. While the presence or absence of government debt can matter importantly for the types of policies that emerge as optimal, we maintain the balanced budget assumption in this paper for mainly two reasons. First, we wish to contrast our findings regarding the lack of government commitment with the previous studies of optimal public investment carried out under full commitment. We shall show that under full commitment, our framework nests findings in Glomm and Ravikumar (1994) among others. Second, although the absence of debt limits our findings, its inclusion implies adding an additional state variable to the model that would significantly complicate the analysis.⁶

Suppose that the tax rate households face at any date stems from a stationary policy rule that depends only on the states of the economy at that date, $\Psi(k, k_g)$. Since the government maintains a balanced budget, public investment satisfies $\dot{k}_g = \Psi(k, k_g)y - g_t$. We can then express new outlays of public capital, k'_g , as

$$k'_g = [\Psi(k, k_g) - \phi]f(k, k_g) + (1 - \delta_{k_g})k_g. \tag{7}$$

Throughout the analysis, we denote next period's value of a given variable x by x' .

3.1. The recursive competitive equilibrium

In order to address how governments might choose discretionary policy optimally in our environment, we first need to describe how households and firms make decisions given that a tax policy, $\Psi(k, k_g)$, is in place.

Firms: There exists a large number of homogeneous small size firms that act competitively, where k_g acts as a common externality with respect to each firm's production. We denote by r and w the rental price of private capital and the wage. Taking these prices as given, each firm maximizes profits and solves

$$\max_{k,l} \Pi = f(k, k_g) - rk - wl.$$

The corresponding optimality conditions yield $r = f_k(k, k_g)$ and $w = f_l(k, k_g) - f_k(k, k_g)k$.

Households: At each date, households decide how much to consume and save, as well as how much capital to rent to firms. Taking the policy rule $\Psi(k, k_g)$ as given, households maximize their lifetime utility subject to their budget constraint and the law of motion describing the accumulation of private capital. Because the policy rule Ψ is stationary (i.e. it does not depend on time as a separate argument), we can write the household problem recursively as follows:

$$\bar{V}(\bar{k}, k, k_g) = \max_{c, \bar{k}} \{u(c) + \beta \bar{V}(\bar{k}', k', k'_g)\} \tag{P^C}$$

s.t.

$$c + \bar{k}' - (1 - \delta_k)\bar{k} = (1 - \Psi(k, k_g))[wl + r\bar{k}], \tag{8}$$

$$k' = \mathcal{H}(k, k_g), \tag{9}$$

$$k'_g = \mathcal{H}_g(k, k_g), \tag{10}$$

where \bar{k} denotes individual capital.

The solution to the representative household's dynamic optimization problem yields the familiar Euler equation (henceforth EE),

$$u_c = \beta u'_c [(1 - \Psi(k', k'_g))r' + 1 - \delta_k]. \tag{EE}$$

Having described the behavior of households and firms, we can define the recursive competitive equilibrium given taxes:

Definition 1. Given the policy rule $\Psi(k, k_g)$, a recursive competitive equilibrium is a set of functions, $\bar{V}(\bar{k}, k, k_g)$, $w(k, k_g)$, $r(k, k_g)$, $\mathcal{H}(\bar{k}, k, k_g)$, $\mathcal{H}(k, k_g)$, and $\mathcal{H}_g(k, k_g)$, such that

1. $\mathcal{H}(\bar{k}, k, k_g)$ solves Eq. (EE), and $\bar{V}(\bar{k}, k, k_g)$ solves (P^C).
2. Prices reflect competitive factor markets, $r(k, k_g) = f_k$, and $w(k, k_g) = f_l(k, k_g) - f_k(k, k_g)k$.
3. Consistency is satisfied $\mathcal{H}(\bar{k}, k, k_g) = \mathcal{H}(k, k_g)$.
4. The government budget constraint holds, $\mathcal{H}_g(k, k_g) = (\Psi(k, k_g) - \phi)[w(k, k_g)l + r(k, k_g)k] + (1 - \delta_{k_g})k_g$.

In (EE), taxes distort private incentives to consume and save, but also induce higher future returns to private investment through the development of public infrastructure. Specifically, the return to private investment, r' , depends on k'_g through the marginal product of private capital. A question then immediately arises as to where to set the tax rate, or equivalently, the share of public investment in output.

⁶ See Ortigueira and Pereira (2007) for an analysis of Markov-perfect optimal income taxation and public debt policy in an economy with private capital.

A benevolent planner considers the effects of policy on agents' welfare. Because households are identical $\bar{k} = k/k$ and we can restrict our attention to the representative agent's policy functions. We denote the planner's value function by $V(k, k_g)$, where $\bar{V}(k, k, k_g) = V(k, k_g)$.

4. Description of the Markov-perfect problem

We define a stationary Markov equilibrium following Klein et al. (2008) where, in our setup, the tax rate depends on the stocks of public and private capital. Unlike Klein et al. (2008), however, and more generally models with a single state variable, the derivation of the government Euler equation—which characterizes the solution—is substantially more involved with two states. We show that such a derivation remains analytically tractable. More importantly, it also turns out to be helpful for the application of numerical methods that efficiently and accurately describe transition dynamics.⁷

A Markov-perfect equilibrium can be described as a sequence of successive governments, each choosing a single tax rate based on the state it inherits when taking office. In making this choice, each government correctly anticipates the optimal decision rule, $\Psi(k, k_g)$, adopted by its successors. In order for $\Psi(k, k_g)$ to be subgame perfect, no government must ever have an incentive to deviate from this rule. Joint deviations are not feasible since, by assumption, a new government chooses policy every period and, consequently, cannot enter binding contracts with future governments. It is sufficient, therefore, to analyze the problem of a government that potentially wishes to “deviate” in the current period by setting a tax rate $\tau \neq \Psi(k, k_g)$, under the assumption that $\Psi(k, k_g)$ is forever followed in the future. Thus, we now describe how an arbitrary deviation perturbs equilibrium allocations, and we allow the government to choose the best possible deviation τ . In a Markov-perfect equilibrium, it must be the case that the optimal τ coincides with the policy rule $\Psi(k, k_g)$. Put another way, $\Psi(k, k_g)$ describes a time-consistent policy if and only if it coincides with the deviation that a government would optimally choose.

From the government budget constraint, we can define the evolution of public capital as a function of current states and an arbitrary tax rate, $G(k, k_g, \tau)$, such that

$$G(k, k_g, \tau) = (\tau - \phi)f(k, k_g) + (1 - \delta_{k_g})k_g. \quad (11)$$

The representative household's budget constraint is given by

$$c = (1 - \tau)[w(k, k_g)l + r(k, k_g)k] + (1 - \delta_k)k - k',$$

where prices are written explicitly as a function of current states. Because of the representative household assumption, this constraint can also be expressed as an economy-wide resource constraint that implicitly defines a consumption function, \mathcal{C} ,

$$\mathcal{C}(k, k_g, \tau, k') \equiv (1 - \tau)f(k, k_g) + (1 - \delta_k)k - k'. \quad (12)$$

Let us define the evolution of private capital under the one-period deviation as $k' = H(k, k_g, \tau)$, where H solves households' optimal consumption–savings decision when the current tax rate is given by τ , and all future tax rates obey $\Psi(k, k_g)$. Since households take policy as given, Eq. (EE) then becomes

$$u_c[\mathcal{C}(k, k_g, \tau, k')] = \beta u_c[\mathcal{C}(k', k'_g, \Psi(k', k'_g), k'')][f'_k(1 - \Psi(k', k'_g)) + 1 - \delta_k], \quad (13)$$

where $k'_g = G(k, k_g, \tau)$ and $k'' = \mathcal{H}(k', k'_g)$.

Although each government only chooses taxes for the period in which it is in office, this choice also affects individual and aggregate behavior in subsequent periods. As savings and investment in public capital adjust following a change in the current tax rate, so do future taxes as a by-product. Put another way, since Ψ is followed tomorrow onwards, variations in τ that cause k' and k'_g to change will also cause the rate $\tau' = \Psi(k', k'_g)$ to change, thus affecting future savings and investment (k'' and k''_g). Hence, by affecting future states with current policy decisions, the current policymaker possesses some leverage over future governments through Ψ .

Formally, the Markov problem describing optimal discretionary policy at any date is

$$\max_{\tau} u[\mathcal{C}(k, k_g, \tau, H(k, k_g, \tau))] + \beta V(H(k, k_g, \tau), G(k, k_g, \tau)), \quad (P^M)$$

where $V(k, k_g)$ is determined by

$$V(k, k_g) = u[\mathcal{C}] + \beta V(\mathcal{H}(k, k_g), \mathcal{H}_{k_g}(k, k_g)). \quad (14)$$

$H(k, k_g, \tau)$ satisfies condition (13), and $\mathcal{C} \equiv (1 - \Psi(k, k_g))f(k, k_g) + (1 - \delta_k)k - \mathcal{H}(k, k_g)$.

A Markov-perfect equilibrium is found when, for any pair $\{k, k_g\}$, $\tau = \Psi(k, k_g)$. Formally,

$$\Psi(k, k_g) \in \operatorname{argmax}_{\tau} u[\mathcal{C}(k, k_g, \tau, H(k, k_g, \tau))] + \beta V(H(k, k_g, \tau), G(k, k_g, \tau)).$$

⁷ In related work, Azzimonti et al. (2006) study a politico-economic equilibrium, also with multiple state variables, but where taxes are determined by majority voting.

When the government optimally chooses not to deviate from the rule Ψ in setting τ , the resulting policy functions must be consistent with the recursive competitive equilibrium described in Definition 1. Hence, $H(k, k_g, \Psi(k, k_g)) \equiv \mathcal{H}(k, k_g)$ and $G(k, k_g, \Psi(k, k_g)) \equiv \mathcal{G}(k, k_g)$.

4.1. The government Euler equation

Assuming that the policy functions are differentiable, the first-order condition associated with problem (P^M) is

$$u_c \mathcal{C}_\tau + \beta[V'_k H_\tau + V'_{k_g} G_\tau] = 0. \tag{15}$$

The expressions for V_k and V_{k_g} (where V is given in Eq. (14)) involve \mathcal{H}_k and \mathcal{H}_{k_g} . Since households' Euler equation (EE) depends on the policy rule $\Psi(k, k_g)$, the calculation of \mathcal{H}_k and \mathcal{H}_{k_g} —using the implicit function theorem—in turn involves derivatives of the policy function, Ψ_k and Ψ_{k_g} . This feature of the problem imposes an additional burden on solving for the Markov-perfect equilibrium since, to arrive at the unknown policy Ψ (or even just the tax rate in the steady state), one already needs to take into account how this policy changes with the states of the economy.

We saw in Section 2.1 that in the presence of lump sum taxes, the government would set all distortions to zero so that first-best allocations could be attained. In contrast, in our setting, distortionary taxes induce wedges in the intertemporal conditions describing the efficient provision of private and public capital. The following proposition states that the optimal discretionary policy is such that it sets a linear weighted sum of these distortions to zero.

Proposition 1. *Let $\Delta_k = -u_c + \beta u'_c (f'_k(1 - \phi) + 1 - \delta_k)$ and $\Delta_{k_g} = -u_c + \beta u'_c (f'_{k_g}(1 - \phi) + 1 - \delta_{k_g})$. Then, the government's first order condition (15) may be re-written as an Euler equation as follows:*

$$H_\tau \Delta_k + G_\tau \Delta_{k_g} + \beta \{ \tilde{H}'_\tau \Delta'_k + \tilde{G}'_\tau \Delta'_{k_g} \} = 0, \tag{16}$$

where $\tilde{H}'_\tau = \xi B'_{k_g}$ and $\tilde{G}'_\tau = -\xi B'_k$, with $B_i = G_i - G_\tau(H_i/H_\tau)$, $i = k, k_g$ and $\xi = -H_\tau(B'_k H_\tau + B'_{k_g} G_\tau / B'_k H'_\tau + B'_{k_g} G'_\tau)$.

Proof. See Appendix A.

The derivation of the government Euler equation in terms of a weighted sum of deviations from efficient intertemporal decisions is somewhat involved, but perhaps most intuitive in that form. Furthermore, it is worth noting that although the economy is dynamic, so that there are potentially an infinite number of distortions, only those in the current and subsequent period matter directly. The recursive nature of the problem together with the envelope theorem ensure that other wedges are handled optimally. Why is it that two inefficiency wedges in the current and subsequent period matter in Proposition 1? In Klein et al. (2008), or Azzimonti et al. (2006), the government attempts to manipulate one intertemporal distortion using one policy instrument, income taxes. As a result, the GEE in Klein et al. (2008), for instance, involves trading off one intertemporal wedge, the one that distorts savings, with one intratemporal wedge, the one between the marginal utilities of private and public consumption. In our setting, however, the government attempts to manipulate two intertemporal margins, those that determine private capital, Δ_k , and public capital, Δ_{k_g} , but still having access to only one instrument, income taxes. Thus, relative to the earlier frameworks with a single state variable, the optimal policy with public investment dictates trading off two intertemporal wedges over time.⁸

The first term in Eq. (16) depicts the increase in the inefficiency of private savings induced by a marginal increase in distortionary taxes. This inefficiency is captured by the intertemporal savings distortion that arises with distortionary taxes, if future taxes satisfy $\tau' > \phi$ then $\Delta_k > 0$, scaled by the reduction in household savings that takes place when the tax rate increases, $H_\tau < 0$.⁹ Similarly, the second term $G_\tau \Delta_{k_g}$ in Eq. (16) summarizes how changes in current taxes affect the inefficiency of public capital provision. In particular, this effect is characterized by the wedge Δ_{k_g} , present whenever $\tau' \neq \phi$, scaled by the rise in public investment implied by a marginal increase in τ , $G_\tau > 0$.¹⁰ The third and fourth terms in the weighted sum of distortions in Eq. (16) may be interpreted in an analogous way and account for how current policy affects next period's wedges through its effect on future levels of private and public capital. In sum, therefore, governments following a Markov-perfect policy choose a tax rate such that they trade off marginal inefficiencies that arise in private savings, $H_\tau \Delta_k$, with those that arise in the provision of public infrastructure, $G_\tau \Delta_{k_g}$, across two consecutive periods.

It is important to recognize that, depending on the nature of preferences and technology, the commitment problem on the part of the government does not always bind. For the special case where households' period utility is logarithmic, $u(c) = \log c$, technology is Cobb–Douglas, $y = k_g^\theta k^\alpha l^{1-\alpha}$, capital depreciates fully within the period, $\delta_k = \delta_{k_g} = 1$, and there are no unproductive expenditures, $\phi = 0$, optimal discretionary income tax rates are independent of the states, $\Psi(k, k_g) = \beta \theta \forall (k, k_g)$.¹¹ Moreover, private investment is a constant fraction of output in this case and, therefore, independent of future

⁸ In general, what affects the number of intertemporal trade offs in the GEE involves both the number of state variables and the number of independent financing instruments.

⁹ To see this, simply substitute households' first-order condition (EE) into the definition of Δ_k to obtain $\Delta_k = \beta u'_c f'_k (\tau' - \phi)$. Thus, $\Delta_k > 0$ when $\tau' > \phi$, which implies a level savings that is lower than optimal.

¹⁰ Recall that in the first best economy, the optimal level of optimal public capital is determined by the condition $-u_c + \beta u'_c (f'_{k_g}(1 - \phi) + 1 - \delta_{k_g}) = 0$. With distortionary taxes such that if $\tau' \neq \phi$ this equality no longer necessarily holds and $\Delta_{k_g} \neq 0$.

¹¹ See Appendix B.

tax rates. As a result, the Ramsey plan is not subject to a time consistency problem and, in fact, coincides with the discretionary solution. We return to this point in Section 6.

5. Numerical solutions for a calibrated economy

In this section, we carry out numerical simulations of our economy with public investment. The parameters we use are mostly standard and selected along the lines of other studies in quantitative general equilibrium theory. Following Klein et al. (2008), we assume that utility is logarithmic, $u(c) = \log c$, and that households discount the future at rate $\beta = 0.96$, where a time period represents a year. Turnovsky (2004) estimates depreciation rates for private and public capital such that $\delta_k = 0.05$ and $\delta_{k_g} = 0.035$, respectively. The share of private capital in output in the U.S. is approximately 36 percent which implies $\alpha = 0.36$. Government spending on goods and services, net of public investment, averaged approximately 17 percent of output over the period 1960–2006 so that we set $\phi = 0.17$. The one non-standard parameter relates to the elasticity of output with respect to public capital. Estimates of θ vary significantly across studies. Glomm and Ravikumar (1997) cite values ranging from as low as 0.03 (Eberts, 1986) to as high as 0.39 (Aschauer, 1989). Munnell (1990) also estimates a value of 0.39 while Fernald (1999) finds $\theta = 0.35$. We set θ to 0.25 as our benchmark case to approximate a consensus view. However, because of the uncertainty around this elasticity, we also discuss results for alternative calibrations in Section 6, as well as this parameter's implications for the measured stock of public capital in our model.

5.1. Overview of alternative numerical methods

Numerical work on optimal time-consistent policy has generally adopted one of three alternative methods:

- (i) a global approximation method (henceforth GM),
- (ii) a linear quadratic approximation method, and
- (iii) a perturbation method.

Global approximation methods that solve for equilibria on a discretized state space are simplest, in the sense that they do not necessarily require derivation of the GEE, and accurate. A significant disadvantage of this method, however, relates to its potentially lengthy computational time which grows exponentially with the number of state variables. As an alternative, LQ approximations allow for rapid computations of steady states and transition dynamics. They are widespread in the literature and easily handle multiple state variables. Examples of this approach are found, for instance, in Krusell and Rios-Rull (1999), and Benigno and Woodford (2006), in the context of fiscal policy, but are more prevalent in the monetary policy literature as illustrated in Dotsey and Hornstein (2003), Svensson and Woodford (2004), and King and Wolman (2004) among many others. Developments related to the perturbation method are more recent and, to this point, have been used mostly to approximate steady-state equilibria (see Klein et al., 2008 or Azzimonti et al., 2006). In this paper, we use the perturbation method to also approximate transitions to the steady state. To our knowledge, our work offers the first comparisons of relative accuracy between different approaches in a time-consistent policy problem.

Surprisingly, we find that all methods deliver accurate estimates of the steady-state allocations. Transition dynamics obtained using either an LQ approximation or a perturbation method with linear policy functions, however, can prove significantly inaccurate. In particular, transition paths for taxes and public investment computed from an LQ approximation are noticeably different from those suggested by the global method solution. On a more practical level, the strict parametric form imposed on the savings and income tax rules under the LQ approach can lead to misleading policy recommendations.

The *perturbation method* essentially consists of approximating decision rules with n -th order polynomials. In our environment, conditional on Ψ , the government's budget constraint (7) immediately gives us rule G . Consequently, we need only approximate the savings rule H and the tax policy Ψ . Given the assumed functional forms for H and Ψ , calculating the derivatives that appear in the GEE is now straightforward. In essence, this method recasts the problem in terms of finding unknown coefficients. For comparison with the LQ approach, we use the perturbation method with two different approximations of the policy functions, one that assumes a linear functional form for H and Ψ , referred to as "method PL", and one that assumes a quadratic form for H , referred to as "method PQ".

Under method PL, we specify H as follows:

$$H^{PL}(k, k_g, \tau) = a_0^k + a_1^k k + a_2^k k_g + a_3^k \tau. \quad (17)$$

Next period's public capital satisfies the government's budget constraint,

$$G^{PL}(k, k_g, \tau) = (\tau - \phi)k^\alpha k_g^\theta + (1 - \delta_{k_g})k_g \quad (18)$$

and taxes are linear in the state variables,

$$\Psi^{PL}(k, k_g) = a_0^\tau + a_1^\tau k + a_2^\tau k_g. \quad (19)$$

There are seven unknown "a" parameters and two unknown steady-state values (i.e. k^M and k_g^M). The corresponding system of nine equations needed to solve for these parameters consists of the Euler equation, the GEE, the derivatives of the

Table 1
Markov-perfect steady states.

Method	Policy ^a , τ	k	k_g
Global method	26.041	14.281	12.701
Perturbation method (quadratic decision rules)	26.041	14.280	12.700
Perturbation method (linear decision rules)	26.045	14.283	12.709
Linear quadratic method	26.007	14.258	12.627

^a The tax rate is expressed in percentage terms.

Euler equation with respect to τ , k , and k_g , the derivatives of the GEE with respect to k and k_g , as well as Eqs. (17) and (18) evaluated at the steady state. See Klein et al. (2008) for additional details.¹²

Under the second approximation used with the perturbation approach, method PQ, Ψ and g follow Eqs. (18) and (19), but H is quadratic in its arguments. As a result, the number of unknown “ a ” parameters rises to 15. The required additional equations in this case are given by the second derivatives of the Euler equation.

The LQ method re-states problem (P^M) as one with a quadratic objective subject to a set of linear constraints. The end result is one where all decision rules are linear in the state variables, including the decision rule for G in contrast to Eq. (18) under method PL. Thus, the fundamental difference between the LQ and the perturbation approach is that the latter solves the original problem while only approximating the decision rules but not necessarily the objective function or constraints.

The relative accuracy of these methods are gauged against a *global method* carried out as follows. Given arbitrary initial guesses for equilibrium taxes, $\Psi^n(k, k_g)$, savings, $\mathcal{H}^n(k, k_g)$, and the value function, $V^n(k, k_g)$, on a discretized state space, households’ savings for given tax rates, $H(k, k_g, \tau; \Psi^n)$, are obtained by solving the Euler equation in (13) at each grid point. Similarly, the evolution of public capital, $G(k, k_g, \tau; \Psi^n)$, can be directly calculated from the government’s budget constraint (7). This process immediately yields private consumption from Eq. (12), $\mathcal{C}(k, k_g, \tau; \Psi^n)$, which can then be used to compute households’ indirect utility for arbitrary tax rates on a grid. One can then directly solve the government’s problem (P^M) without deriving the GEE. The solution to this problem yields an updated optimal tax function, $\Psi^{n+1}(k, k_g)$, and in turn a new savings function, $\mathcal{H}^{n+1}(k, k_g)$, and value function, $V^{n+1}(k, k_g)$. By iterating on this procedure, we search for an equilibrium where the policy and value functions have converged. In particular, $\|\Psi^n - \Psi^{n+1}\| < \varepsilon$ suggests that a policymaker solving problem (P^M) will not deviate from the rule expected to be followed in the future. Because $H(k, k_g, \tau; \Psi^n)$ and $G(k, k_g, \tau; \Psi^n)$ may imply values of k' and k'_g , respectively, that are not on the discretized state space, we approximate the policy rule, $\Psi^n(k, k_g)$, and welfare function, $V^n(k, k_g)$, at each iteration with quadratic polynomials so that they may be evaluated at arbitrary points.¹³ Additional points are added to the grids for k , k_g , and τ , until solutions no longer differ across different discretizations of the state space.

5.2. Comparison between methods

Table 1 summarizes the steady-state tax rates and capital allocations under optimal discretionary policy using the four methods we have just described. Interestingly, all methods yield steady-state solutions that are very close. The perturbation method with quadratic decision rules (PQ) yields solutions that are closest to those obtained with the global method. Note also that the bias for the perturbation method with linear rules (PL) is smaller, and goes in the opposite direction, of that associated with the LQ method.

We can further assess the performance of each method by examining how well it ultimately solves the Euler equation (13), and the government Euler equation (16), at points outside the steady state. To this end, let $v_{EE}(k_i, k_{g,j})$ denote the (demeaned) error associated with households’ Euler equation evaluated at given points k_i and $k_{g,j}$ when using the solutions for \mathcal{H} , \mathcal{H}_g , and Ψ from each of the four methods we have described. Similarly, let $v_{GEE}(k_i, k_{g,j})$ denote the error that arises from evaluating the government Euler equation using these solutions. Because these errors vary over two dimensions, k and k_g , we first present results where we integrate over each dimension one at a time.

Fig. 1, panel A, shows the root mean square error (RMSE) associated with the Euler equation for different values of k , averaged over k_g for each k , $\rho_{EE}(k) = [(1/n_{k_g}) \sum_{j=1}^{n_{k_g}} v_{EE}(k, k_{g,j})^2]^{1/2}$, where n_{k_g} denotes the number of grid points describing public capital. Conversely, Fig. 1, panel B, illustrates the RMSE computed from the Euler equation for different values of k_g , averaged over k for each k_g , $\rho_{EE}(k_g) = [(1/n_k) \sum_{i=1}^{n_k} v_{EE}(k_i, k_g)^2]^{1/2}$, where n_k denotes the number of grid points for private capital. It is immediately apparent from the figures that the linear quadratic and linear perturbation methods display the largest errors among the four numerical approaches, with corresponding RMSE statistics that tend to increase rapidly as we

¹² A technical Appendix with a detailed description of all methods used in this paper is available from the authors.

¹³ The results presented here rely on grids containing 100 points for τ , 40 points for k , and 40 points for k_g . Quadratic polynomial fits for $\Psi^n(k, k_g)$ and $V^n(k, k_g)$ were associated with R^2 statistics in excess 0.9999 at each iteration and for each function. Using Matlab version 7.1 on a PC with a 2GHz processor and 2GB of RAM resulted in a computing time of approximately two days. In contrast, the other three methods can be solved in less than a minute. That said, global methods do not necessarily require value function iteration. An alternative that would be quicker is to solve the GEE using a collocation method with Chebyshev polynomials.

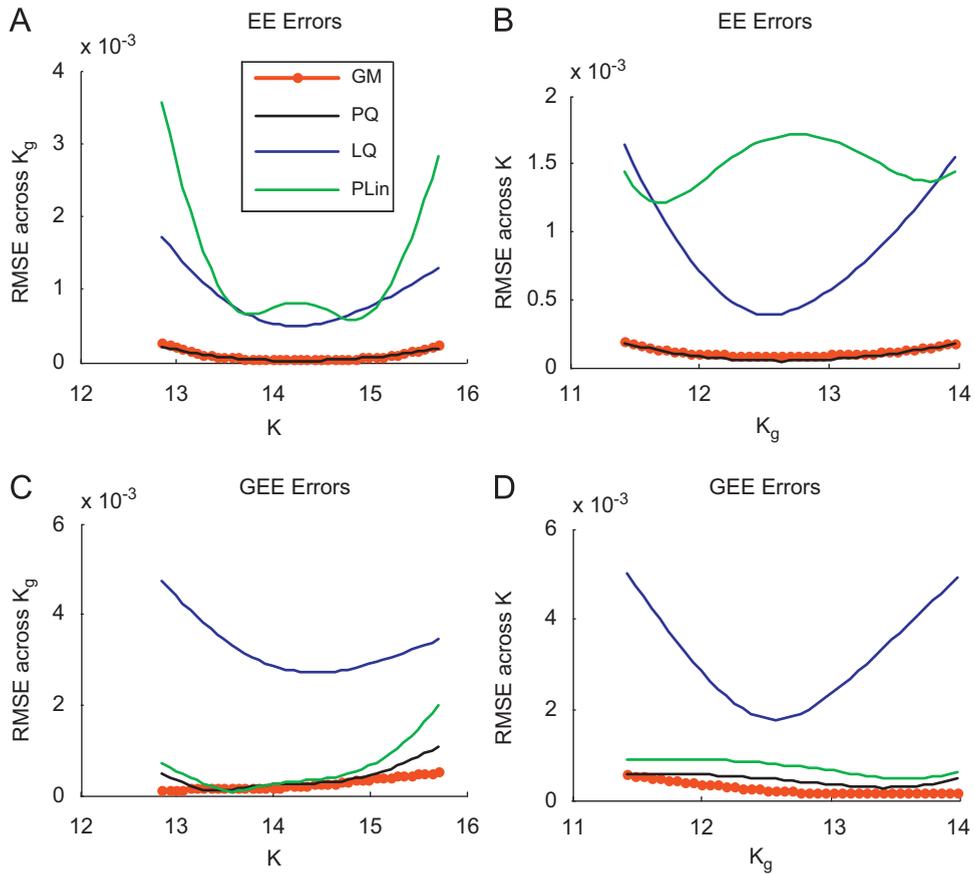


Fig. 1. Root mean square residuals under alternative methods. Panels A and B: EE errors, panels C and D: GEE errors.

Table 2
Accuracy comparisons between methods.

Method	Root mean square error ^a
Global method	4.06e–004
Perturbation method (quadratic decision rules)	5.56e–004
Perturbation method (linear decision rules)	4.20e–003
Linear quadratic method	2.24e–003

^a The RMSE is computed over grids whose bounds are set at ±10 percent of steady state values.

move away from the steady state presented in Table 1. In contrast, the perturbation method with quadratic decision rules performs just about as well as the global method, and even slightly better near the steady state in one case. Fig. 1, panel C, depicts the RMSE associated with the government Euler equation for different values of k , averaged over k_g , $\rho_{GEE}(k) = [(1/n_k) \sum_{j=1}^{n_{k_g}} v_{GEE}(k, k_{g,j})^2]^{1/2}$. Fig. 1, panel D, plots its counterpart, $\rho_{GEE}(k_g) = [(1/n_k) \sum_{i=1}^{n_k} v_{GEE}(k_i, k_g)^2]^{1/2}$. As before, methods that rely solely on linear approximations are least accurate, with the linear quadratic approximation yielding notably larger errors that increase steeply outside the steady state. Interestingly, with respect to the GEE, panels C and D suggest that the perturbation method with linear decision rules (PL) performs almost as well as when computed with quadratic decision rules (PQ). Here, the global method always yields the smallest errors.

In Table 2, we report the total RMSE, ρ , when both dimensions of the state space are taken into account and across both optimality conditions,

$$\rho = \left\{ \frac{1}{n_k n_{k_g}} \sum_{i=1}^{n_k} \sum_{j=1}^{n_{k_g}} v_{EE}(k_i, k_{g,j})^2 \right\}^{1/2} + \left\{ \frac{1}{n_k n_{k_g}} \sum_{i=1}^{n_k} \sum_{j=1}^{n_{k_g}} v_{GEE}(k_i, k_{g,j})^2 \right\}^{1/2} . \tag{20}$$

Table 3
Policy responses to changes in economic states.^a

Method	$\frac{\partial \Psi(k, k_g)}{\partial k}$	$\frac{\partial \Psi(k, k_g)}{\partial k_g}$
Global method	9.192	–9.761
Perturbation method (quadratic decision rules)	9.177	–10.467
Perturbation method (linear decision rules)	10.069	–10.360
Linear quadratic method	2.603	–10.996

^a Changes in the tax rate are expressed in percentage terms.

While the global method shows the smallest aggregate RMSE, the perturbation method with quadratic decision rules proves nearly as accurate. In contrast, the LQ approximation and the perturbation method with linear rules are associated with RMSE statistics that are 5–10 times larger.

To sum up, all approaches considered here yield similar steady-state tax rates and capital allocations.¹⁴ The accuracy of the long-run Markov-perfect equilibrium under the LQ method, however, may be specific to this environment where decisions rules tend to be “linear” close to the steady state. In addition, as discussed in Judd (1996, 1999), and Benigno and Woodford (2006), by ignoring higher order effects that may be important in providing a welfare ranking of different policies, the LQ approach produces linear decision rules that may poorly approximate optimal policies away from the steady state. In our framework, we find that a perturbation approach based on the GEE can perform nearly as well as a global method. While the global approach is conceptually simpler, deriving the GEE allows for the implementation of a numerical method that takes virtually no computational time.

5.3. Implications for transition dynamics

Table 3 presents what the different methods we have just reviewed imply for the response of optimal discretionary policy to changes in the states of the economy. Specifically, we compute $\partial \Psi(k, k_g)/\partial k$ and $(\partial \Psi(k, k_g))/\partial k_g$ evaluated at the steady state. In general, the Markov-perfect policy prescribes a tax rate rule that increases with private capital and decreases with public capital. This suggest that Markov governments purposefully create incentives for the private sector to invest when private capital falls below its steady state, $(\partial \Psi(k, k_g))/\partial k > 0$. In contrast, optimal discretionary tax rates rise when public infrastructures fall short of their long-run equilibrium, $(\partial \Psi(k, k_g))/\partial k_g < 0$. In this case, higher taxes are necessary to fund public investments.

Interestingly, the response of optimal discretionary tax rates is roughly of the same magnitude with respect to changes in either type of capital. Moreover, the perturbation methods yield policy responses that are quantitatively similar to those obtained with the global method. The linear quadratic approximation, however, considerably underestimates the response of optimal policy to an increase in private capital.

More generally, Fig. 2 illustrates the policy functions describing the evolution of public capital and Markov-perfect tax rates with respect to each state variable. The left-hand panels hold public capital at its steady state and show the policy functions with respect to private capital. Conversely, the right-hand panels hold private capital fixed at its steady state and illustrates how the policy variables move with public capital. Although the policy rules can differ off steady state, especially in the case of the LQ approximation when public capital is held fixed, these functions all intersect at the steady state values for public and private capital. These plots, therefore, are consistent with all numerical methods giving us accurate estimates of the long-run Markov-perfect equilibrium in Table 1.

The policy functions in Fig. 2 show relatively little difference between the Perturbation method with quadratic decision rules (PQ) and the global method. In contrast, the LQ approximation considerably overstates the optimal discretionary tax rate, and hence the evolution of public capital, when private capital falls below steady state (Fig. 2, panels A and C). In addition, Fig. 3 shows that LQ estimates of households’ decision rules notably understate optimal savings when private capital falls short of its long-run equilibrium.

To gain a sense of the quantitative implications from these different policy functions for transition dynamics, Fig. 4 depicts the evolution of both types of capital, tax rates, and consumption when the economy starts off with stocks of public and private capital that are initially 5 percent below their respective steady states. The most striking difference across methods concerns the evolution of tax rates (Fig. 4, panel D). Specifically, while the solution computed using the global method predicts a small decrease in tax rates of less than 1 percent initially, the LQ approximation prescribes a tax rate increase of almost 5 percent (from 26 to 31 percent). Moreover, the global method suggests a smooth gradual increase in tax rates throughout the transition to the steady state while the LQ method shows an abrupt decline in tax rate over the

¹⁴ Total RMSE statistics are on the order of 10^{-6} at the steady state for both the global method and the perturbation method with quadratic decision rules.

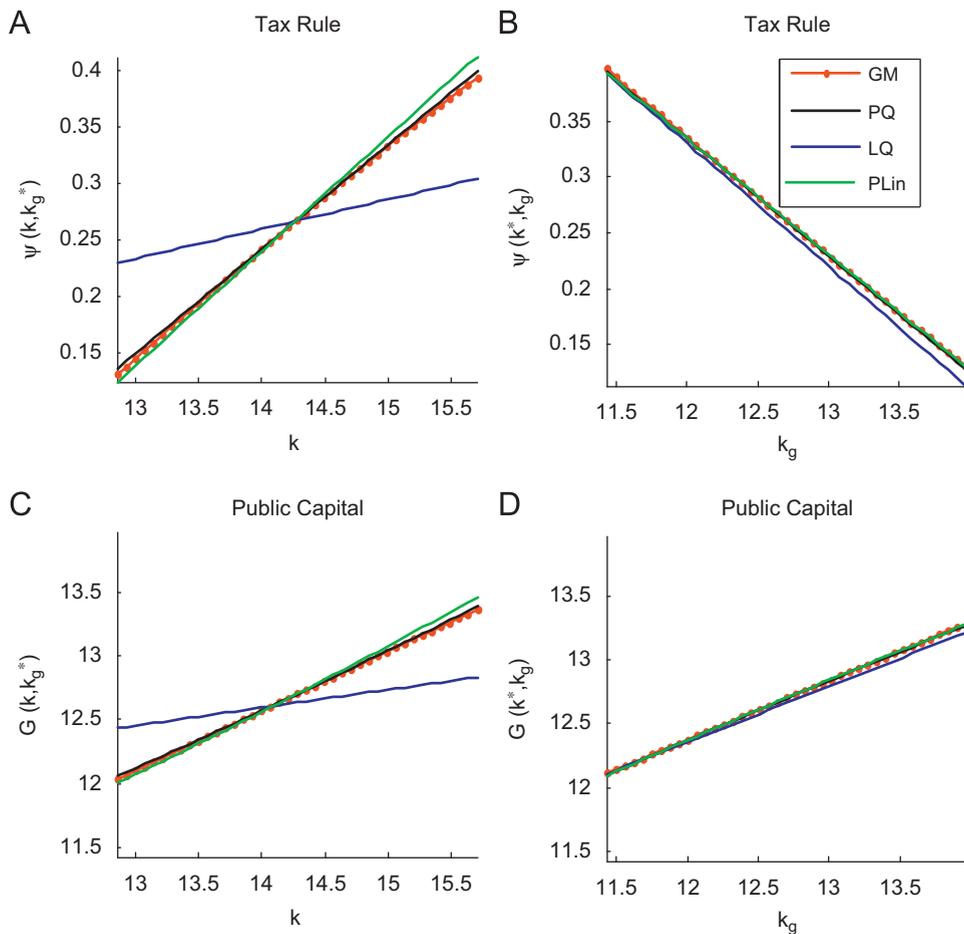


Fig. 2. Panels A and B: tax policy— $\Psi(k, k_g)$, panels C and D: public capital accumulation rules— $G(k, k_g)$ as functions of the state variables.

first five periods. As the economy moves towards its long-run equilibrium, these differences in tax rates imply a noticeably smoother consumption profile under the global method than when computed with the LQ approximation (Fig. 4, panel C).

Our example suggests that the choice of numerical methods in problems of optimal discretionary policy should depend, in part, on the goal of the investigation. To the degree that one is mostly interested in long-run equilibrium properties of the Markov-perfect problem, the LQ method is relatively accurate, straightforward, fast, and can accommodate a large number of state variables without additional complications. That said, our findings regarding the accuracy of the long-run equilibrium may not necessarily extend to problems whose decision rules are markedly non-linear around the steady state. Furthermore, if the focus is on transition paths, then the perturbation method seems a preferable choice. The perturbation method, however, can present its own challenge in that the number of parameters characterizing the policy functions increases exponentially with the degree of the polynomials that describe those functions and the number of state variables. This introduces a trade-off between the gains from using a larger polynomial and the error introduced by the algorithm used to solve for its parameters. For the model at hand, with two state variables, method PQ generates decision and policy rules that are very close to those obtained with the global method (Figs. 2 and 3).

6. The value of government commitment

Having described the optimal public investment problem under discretion, we briefly present the more conventional Ramsey setting to assess the importance of a commitment technology. We show that the special case of an optimal constant policy, $\tau_t = \beta\theta \forall t \geq 0$, discussed in Section 4 also emerges as a solution to the full commitment problem under the same functional form assumptions. In this sense, our framework nests previous results in the literature, notably by Glomm and Ravikumar (1997). While this finding implies that the commitment assumption is innocuous in their setting, that is not the case in a more general setting. The objective of this section, therefore, is to compare allocations under the two institutional extremes of full commitment and no commitment.

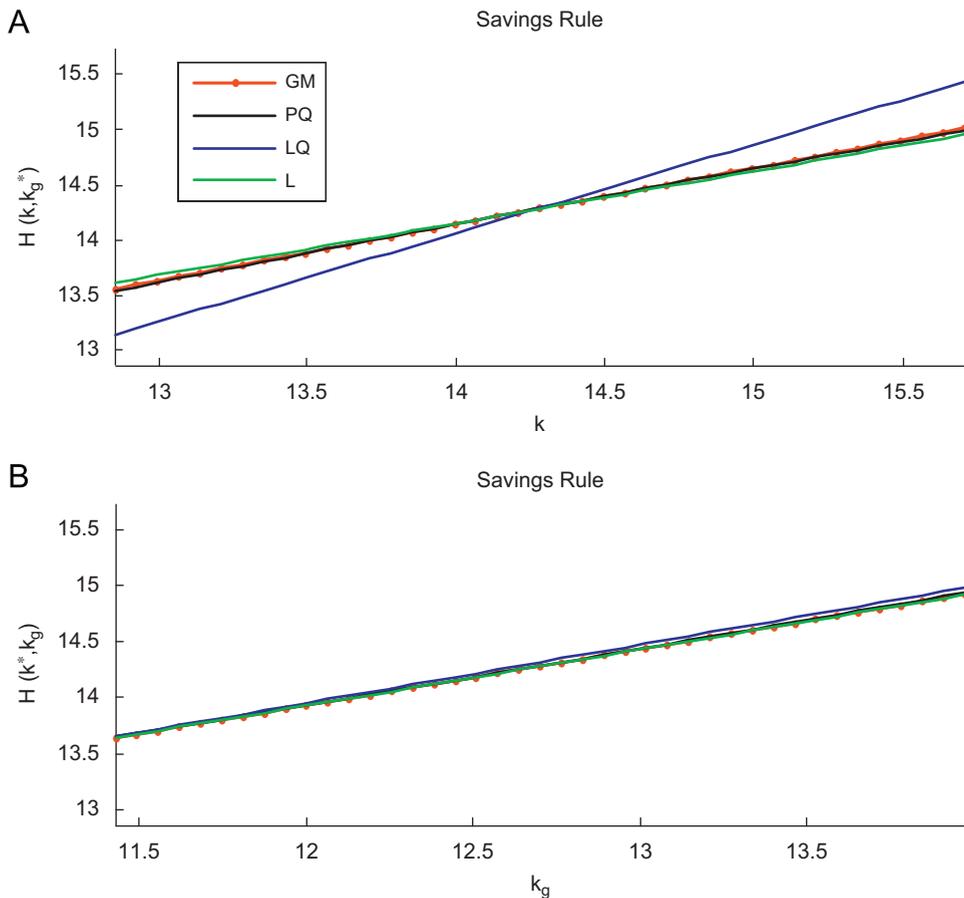


Fig. 3. Private capital accumulation— $H(k, k_g)$ —a function of the state variables. Panels A and B: savings rule.

6.1. Maximization problem

Consider a benevolent government that, at date zero, is concerned with choosing a sequence of tax rates consistent with the development of public infrastructure that maximizes household welfare. In doing so, this government takes into account the decentralized optimal behavior of households and firms, as summarized by the Euler equation and prices being equated to marginal products, and can commit to future policy. The Lagrangian corresponding to the Ramsey problem can be written as

$$\begin{aligned}
 \max_{\{c_t, \tau_t, k_{t+1}, k_{gt+1}\}_{t=0}^{\infty}} \mathcal{L} = & \sum_{t=0}^{\infty} \beta^t u(c_t) \\
 & + \sum_{t=0}^{\infty} \beta^t \mu_{1t} \{ \beta u_c(c_{t+1}) [(1 - \tau_{t+1}) F_k(k_{t+1}, k_{gt+1}) + 1 - \delta_k] - u(c_t) \} \\
 & + \sum_{t=0}^{\infty} \beta^t \mu_{2t} \{ (\tau_t - \phi) F(k_t, k_{gt}) + (1 - \delta_{k_g}) k_{gt} - k_{gt+1} \} \\
 & + \sum_{t=0}^{\infty} \beta^t \mu_{3t} \{ (1 - \tau_t) F(k_t, k_{gt}) + (1 - \delta_k) k_t - k_{t+1} - c_t \}.
 \end{aligned} \tag{P^R}$$

The constraint associated with the multiplier μ_{1t} implies that our benevolent planner takes households' consumption–savings behavior as given. He can, however, influence the intertemporal allocations they choose by altering tax policy over time. Because the capital stocks are fixed at date 0, the first-order conditions associated with (P^R) will generally differ at $t = 0$ and $t > 0$. As first noted in [Kydlan and Prescott \(1980\)](#), this suggests an incentive to take advantage of initial conditions in the first period with the promise never to do so in the future. It is exactly in this sense that the optimal policy may not be time consistent since, once date 0 has passed, a planner at date $t > 0$ who re-optimizes would want to start with a tax rate, τ_t , that differs from what was chosen for that date at time 0.

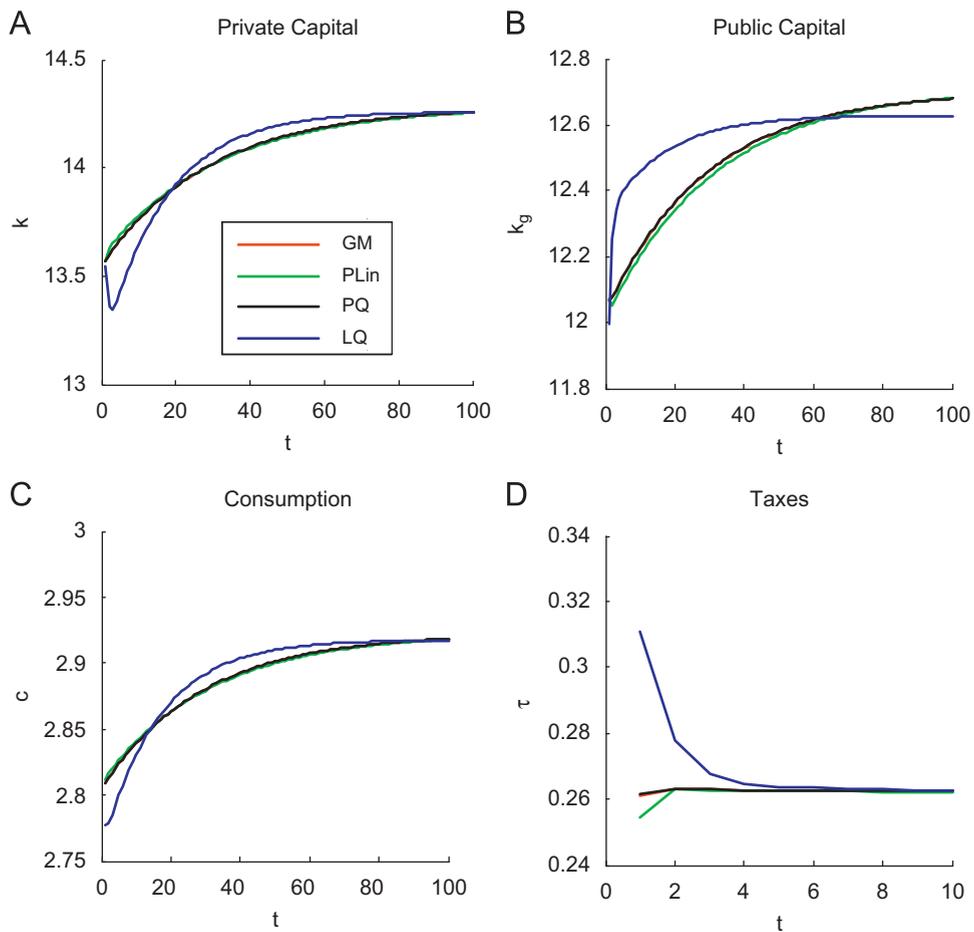


Fig. 4. Transition dynamics under alternative methods. Panel A: private capital, panel B: public capital, panel C: consumption, panel D: taxes.

While Ramsey plans are typically time inconsistent, we introduced a parameterization in Section 4 that we argued made the time consistency problem non-binding. To see why, recall that under that parameterization, household savings depended only on the current tax rate through disposable income. A Ramsey policymaker would then recognize that the choice of tax rate at any date never has an effect on either past or future savings and, in this sense, period 0 is no different than any other period. Put another way, in that example, the government never has an incentive to renege on past promises so that the resulting Ramsey policy is time consistent. We summarize this finding in the following proposition.

Proposition 2. *When preferences are logarithmic, $u(c) = \log(c)$, technology is Cobb–Douglas, $y = k^\alpha k_g^\theta$, capital depreciates fully within the period, $\delta_k = \delta_{k_g} = 1$, and there are no unproductive expenditures $\phi = 0$, the optimal sequence of tax rates with commitment is time invariant and reproduces the Markov-perfect policy, $\tau_t = \beta\theta \forall t \geq 0$.*

Proof. See Appendix C.

An implication of Proposition 2 is that in this specific example, there are no welfare gains from adopting a technology that constrains governments to fulfill their promises. In other words, whether or not governments can commit to future policy is immaterial.

The value of commitment, however, is generally related to both the elasticity of intertemporal substitution and the depreciation rate. In particular, household savings typically depend on the entire stream of tax rates, and setting $\tau_t > 0$ at some date t affects the entire sequence of capital allocations. Since Markov policymakers take the states they inherit as given, they never internalize the efficiency costs of distortionary taxes on past investment decisions. Hence, binding governments to the policy prescribed under the Ramsey plan at every date opens up the possibility of a welfare improvement.

Table 4
Long-run equilibrium allocations.

Variable	Pareto	Ramsey	Markov
Tax rate (percent)	–	26.47	26.04
Consumption	3.22	2.98	2.92
Private investment	0.92	0.73	0.72
Public investment	0.64	0.48	0.44
Output	5.63	5.04	4.92
Private capital	18.37	14.55	14.28
Public capital	15.25	13.64	12.70

6.2. Differences in long-run allocations

To gauge the importance of the friction generated by a government's inability to commit to future policy, we first present in Table 4 a comparison between long-run allocations under the Ramsey and Markov-perfect equilibria. The numbers in the table refer to the calibration described in Section 5. As a benchmark, the table also presents Pareto optimal allocations. Thus, differences between first-best and Ramsey allocations reflect the distortionary effect of proportional taxes. Differences between Ramsey and Markov-perfect allocations reflect the additional distortion introduced by the lack of commitment.

In this model economy, the inability of a policymaker to commit to future policy results in a long-run equilibrium with lower taxes and too little public capital relative to the Ramsey equilibrium. To see the intuition behind this result, consider an increase in the tax rate at some date $t > 0$ from the standpoint of a household. On the one hand, this increase in taxes reduces the return to previous investments and makes previous consumption more attractive relative to consumption at time t . Investments made in prior periods then tend to fall. On the other hand, the increase in the tax rate generates a negative wealth effect that leads households to want to reduce consumption in all periods and, therefore, increase investments made prior to date t . For log preferences and depreciation rates that are strictly less than one, the wealth effect dominates and an increase in the current tax rate lowers previous consumption. That is, the wealth effect induced by the tax rate hike raises prior investments. Moreover, by financing higher levels of public capital, higher taxes at date t raise the productivity of private capital and encourage private investment prior to t . Since a Markov government takes future policy as given, it cannot exploit this positive effect of future taxes on today's investments. A Ramsey policymaker, on the other hand, will exploit this effect and optimally choose to set higher future tax rates. The ability to commit to future policy, therefore, results in higher taxes and more public capital in the long-run.

The fact that steady-state Markov-perfect tax rates are lower than their Ramsey counterpart implies that, absent government commitment, the economy is ultimately unable to reach the second-best level of public infrastructure. This effect discourages the accumulation of private capital *in spite of the lower tax rates*. The combination of lower public and private capital results in an output loss of approximately 2.4 percent associated with the lack of a commitment technology. Under this calibration, this output loss translates into a long-run consumption difference of about 2 percent between Markov-perfect and Ramsey equilibria.

6.3. How costly is the loss of commitment?

Having described differences in long-run allocations between a world with and without commitment, we now try to assess the overall value of having a commitment technology in our environment. Specifically, we ask: what would it cost households who live in a world where the government can credibly bind itself to future policy to suddenly lose government commitment?

To answer this question, consider a household who, at date 0, lives in an economy that is in a long-run Ramsey equilibrium. Allocations in that economy solve a set of stationary equations that capture the optimality conditions and resource constraints associated with problem (P^R). If the economy were to stay in this long-run equilibrium, the Ramsey allocations would then continue unchanged from date 0 onwards and are given by the constant time paths shown in Fig. 5.¹⁵ Now, suppose instead that the economy suddenly switches to a regime characterized by discretionary policy at date zero. Fig. 5, panel D, shows that a Markov government would immediately deviate from the Ramsey policy and lower the tax rate. This implies a reduction in public capital in the following period (Fig. 5, panel B), but encourages private investment in the short-run (panel A). Because of the fall in tax rates, households are also able to enjoy higher consumption in the first few periods following the switch to a discretionary regime (panel C). However, the new discretionary policy eventually catches up with them as it implies lower levels of public infrastructure that in turn lower the marginal product of private capital. This effect discourages private capital accumulation which, along with the decline in public capital, ultimately leads to a fall in output and consumption.

¹⁵ In that sense, the transition dynamics associated with the continuation of the commitment problem are straightforward in this experiment.

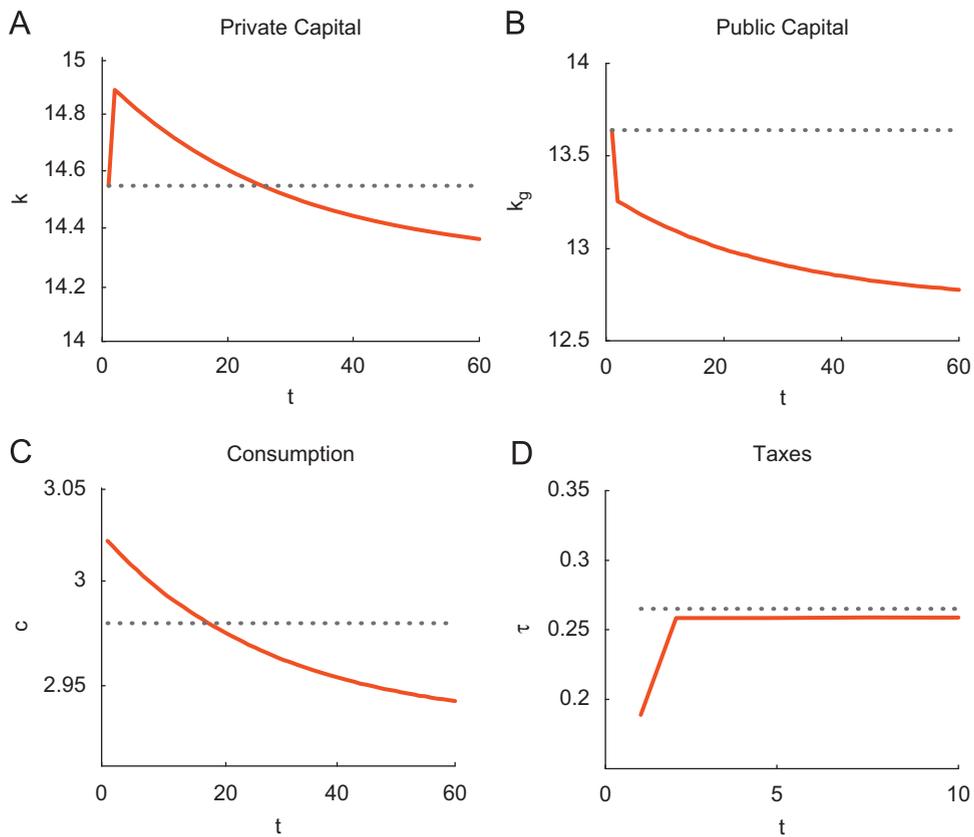


Fig. 5. Transitions from commitment (dotted line) to discretion (solid line). Panel A: private capital, panel B: public capital, panel C: consumption, panel D: taxes.

What then are the welfare implications of the transition paths that describe the move to discretionary policy? Let $\{c_t^R\}_{t=0}^{\infty}$ describe the consumption stream that emerges if the economy were to continue in the long-run Ramsey equilibrium. In this case, we have that $c_t = c^R \forall t$ where c^R denotes stationary Ramsey consumption. Analogously, let $\{c_t^M\}_{t=0}^{\infty}$ describe the consumption path induced by the move to discretionary policy. We then measure the value of government commitment as the percentage loss, ζ , of Ramsey consumption that the household would have to incur at every date in order to be indifferent between the economy with and without commitment (Lucas, 1987). Formally, let k^R and k_g^R denote the Ramsey steady-state stocks of private and public capital, respectively. We then define welfare under the alternatives of continued commitment and the move to discretion by $V^R(k^R, k_g^R)$ and $V^M(k^R, k_g^R)$, respectively. It follows that ζ solves

$$V^M(k^R, k_g^R) = \frac{u(c^R(1 - \zeta))}{1 - \beta}. \quad (21)$$

Since, under our parameterization, $u(c^R)/(1 - \beta) = \ln(c^R)/(1 - \beta) = V(k^R, k_g^R)$, we have that

$$\zeta = 1 - \exp\{(1 - \beta)[V^M(k^R, k_g^R) - V^R(k^R, k_g^R)]\}. \quad (22)$$

Table 5 presents the value of commitment computed under the global method. We find that the move to a discretionary regime implies a welfare loss of just 0.013 percent of each period's consumption. However, a jump from the Ramsey steady state to the discretionary steady state is associated with a welfare loss of 1.849 percent of consumption per period. This difference is driven in large part by the greater emphasis that Markov governments place on short-run gains relative to a Ramsey planner. In particular, although the economy with discretion achieves lower long-run consumption relative to the regime with commitment, the tax policy chosen under discretion implies higher consumption during the transition (Fig. 5, panel C) that largely offsets this long-run loss. The net welfare effect of losing government commitment, in this case, is on the same order of magnitude as the welfare costs of business cycles calculated by Lucas (1987) for similar household preferences.

Consumption of non-durables and services in the U.S. is currently \$9.086 trillion per year and, since there are around 116 million households, this amounts to \$78,328 per household. Assuming that the U.S. economy reflects a Markov-perfect steady state, Table 4 then implies that Ramsey steady state consumption would instead be \$79,937 per household.

Table 5

The value of government commitment.

	Percentage loss of Ramsey consumption, ζ	
	Long-run	Total (including transition)
Global method	1.849	0.013

Therefore, applying $\zeta = 0.013$ percent in Table 5 to this figure, we obtain a welfare loss equivalent of \$10.39 per household per year. In present value terms, this amounts to \$270.19 when using a 4 percent annual discount rate (consistent with our calibration) into the infinite future.

6.3.1. Robustness

As indicated in Section 5, there exists significant uncertainty regarding the elasticity of output with respect to public capital, θ . A question then is: How do our findings change depending on the value of θ ? We use an elasticity in our benchmark that is approximately the median among those reported in different studies, $\theta = 0.25$, which implies a ratio of public capital to output of around 2.6 in the steady state. In contrast, the ratio of public capital to output, using data published by the Bureau of Economic Analysis (BEA), is considerably lower at about 0.7. This discrepancy reflects in part the difficulties involved in measuring public capital. These arise, in particular, from the sheer variety of government activities, the different practices at different government levels, and especially the problem of valuing capital not rented out at market prices, as well as the ensuing complications for measuring depreciation rates. They also arise from the often mixed public–private nature of many activities.¹⁶ In specific cases where measurement problems are of somewhat lesser concern, such as for the infrastructure related to roads and highways, estimates of the public capital elasticity of output are around 0.35 (Fernald, 1999). In contrast, obtaining a long-run ratio of public capital to output of 0.7 in our model requires that we set $\theta = 0.07$. Because public capital matters much less in production when $\theta = 0.07$, welfare losses implied by the lack of commitment are then considerably attenuated. Specifically, long-run consumption in the Markov-perfect equilibrium falls to 0.10 percent of its Ramsey counterpart when $\theta = 0.07$ as opposed to approximately 2 percent in Table 5. Furthermore, this long-run welfare loss is almost entirely offset during the transition so that the welfare loss from moving to a regime with discretion goes from small, 0.013 percent in Table 5, to negligible, 0.0003 percent when $\theta = 0.07$.

7. Concluding remarks

In this paper, we characterize Markov-perfect equilibria in a model where the absence of government commitment affects public investment in infrastructure. We show that in choosing the tax rate, the government trades off intertemporal distortions in the provision of public and private capital over two consecutive periods only.

From a numerical standpoint, we find that both GEE-based perturbation methods and linear quadratic approximations deliver accurate stationary Markov-perfect equilibria. However, the approximation errors associated with the latter can imply misleading policy recommendations in response to changes in the state variables, as well as transition paths that are noticeably different from those produced by a global solution.

Finally, we find that moving from an economy with government commitment to one with discretion implies only a small welfare loss. This finding stems from the greater emphasis that Markov governments place on short-run gains, and ultimately imply that lack of government commitment alone does not materially amplify the distortionary effects of income taxes. This result, however, partly reflects the fact that while taxes distort the return to investment in our setting, thus discouraging private capital accumulation, they also finance goods that directly enhances the productivity of capital which has a direct opposite effect on households' incentive to accumulate capital. In a model where the policy distortion is not so directly offset, the friction associated with the lack of commitment is likely to be more critical. In addition, other political factors, including the set of fiscal instruments available, the potential non-benevolence of governments (Acemoglu, 2004), or conflicts of interest among heterogeneous households (Azzimonti et al., 2006; Battaglini and Coate, 2008), and their interactions with government discretion may also play an important role in the analysis of time-consistent policy problems.

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¹⁶ Slater and David (1998) edit a detailed report for the National Research Council on these issues. Moreover, in an effort to account for missing components in public capital, the BEA recently added a satellite account to attempt to measure the stock of public capital embodied in government sponsored Research and Development, and preliminary research on prototype additional accounts regarding innovation, health care, and education is also underway. This effort is ongoing and it is likely that such accounts will continue to be added going forward.

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Appendix A. Proof of Proposition 1

This Appendix develops and interprets the government Euler equation, the equation that represents the optimal intertemporal allocation for a Markov policymaker. Although the derivation below applies to a generic model with two state variables, it should be clear that our generalization of the method proposed in Klein et al. (2008) can be straightforwardly extended to models with a larger state space. Throughout the derivation, we make use of the fact that in equilibrium, $H(k, k_g, \Psi(k, k_g)) = \mathcal{H}(k, k_g)$ and $G(k, k_g, \Psi(k, k_g)) = \mathcal{G}(k, k_g)$.

The relevant first-order condition with respect to taxes involves the derivatives of the value function with respect to the two state variables, k and k_g ,

$$R_\tau + \beta[V'_k H_\tau + V'_{k_g} G_\tau] = 0, \quad (\text{A.1})$$

where $R_\tau = u_c[\mathcal{C}_k H_\tau + \mathcal{C}_\tau]$, $V'_k = V_k(k', k'_g)$, and $V'_{k_g} = V_{k_g}(k', k'_g)$. The envelope theorem fails to hold because in choosing taxes today, the current government is constrained by the future policy rule, Ψ .

Differentiating the value function (14) with respect to each state (where we can cancel out the terms multiplying derivatives of Ψ by Eq. (A.1)), we obtain¹⁷

$$V_k = R_k + \beta[V'_k H_k + V'_{k_g} G_k], \quad (\text{A.2})$$

$$V_{k_g} = R_{k_g} + \beta[V'_k H_{k_g} + V'_{k_g} G_{k_g}]. \quad (\text{A.3})$$

At this stage, we have a system of three functional equations, (A1)–(A3), in four unknown functions, V_k , V_{k_g} , V'_k , and V'_{k_g} . Since tomorrow's policymaker faces the same problem, however, Eqs. (A1)–(A3) updated one period must also hold. Updating these equations then yields a system of six equations in six unknowns (with the additional unknowns being V''_k and V''_{k_g}). Solving this system gives expressions for V_k and V_{k_g} in terms of the policy and decision rules only, which we can then update one period and substitute back into the first-order condition (A.1) to obtain the GEE that determines Ψ .

We can use the necessary condition (A.1) to obtain

$$V'_k = -\frac{R_\tau + \beta V'_{k_g} G_\tau}{\beta H_\tau}. \quad (\text{A.4})$$

Replacing (A.1) into Eq. (A.2), and rearranging gives

$$V_k = R_k - R_\tau \frac{H_k}{H_\tau} + \beta V'_{k_g} B_k, \quad (\text{A.5})$$

where $B_k = G_k - G_\tau(H_k/H_\tau)$. Replacing (A.1) once more into Eq. (A.3), and defining B_{k_g} analogously, we get

$$V_{k_g} = R_{k_g} - R_\tau \frac{H_{k_g}}{H_\tau} + \beta V'_{k_g} B_{k_g}. \quad (\text{A.6})$$

We can now rearrange Eq. (A.5) to get

$$V'_{k_g} = \left[V_k - R_k + R_\tau \frac{H_k}{H_\tau} \right] \frac{1}{\beta B_k}.$$

Substituting this last expression into (A.6) gives

$$V_{k_g} = R_{k_g} - R_\tau \frac{H_{k_g}}{H_\tau} + \frac{B_{k_g}}{B_k} \left[V_k - R_k + R_\tau \frac{H_k}{H_\tau} \right], \quad (\text{A.7})$$

which we can update to get an expression for V'_{k_g} as a function of V'_k ,

$$V'_{k_g} = R'_{k_g} - R'_\tau \frac{H'_{k_g}}{H'_\tau} + \frac{B'_{k_g}}{B'_k} \left[V'_k - R'_k + R'_\tau \frac{H'_k}{H'_\tau} \right]. \quad (\text{A.8})$$

¹⁷ In doing so, we make use of the fact that in equilibrium, $H(k, k_g, \Psi(k, k_g)) = \mathcal{H}(k, k_g)$ and $g(k, k_g, \Psi(k, k_g)) = \mathcal{G}(k, k_g)$.

As explained above, we can now update (A.6) a second time in order to introduce additional equations that help us solve for the unknown derivatives of the value functions,

$$V''_{k_g} = R''_{k_g} - R''_{\tau} \frac{H''_{k_g}}{H''_{\tau}} + \frac{B''_{k_g}}{B''_k} \left[V''_k - R''_k + R''_{\tau} \frac{H''_k}{H''_{\tau}} \right]. \tag{A.9}$$

By updating the first-order condition (A.1), we obtain another expression involving V''_k as a function of V''_{k_g} ,

$$V''_k = - \frac{R'_{\tau} + \beta V''_{k_g} G'_{\tau}}{\beta H'_{\tau}}. \tag{A.10}$$

Replacing (A.10) into (A.9) then yields an expression for V''_{k_g} that only depends on equilibrium decision rules (and not on the derivatives of the value function).

$$V''_{k_g} = \left(R''_{k_g} - R''_k \frac{B''_{k_g}}{B''_k} + R''_{\tau} \lambda'' - \frac{B''_{k_g} R'_{\tau}}{B''_k \beta H'_{\tau}} \right) \frac{H'_{\tau}}{H'_{\tau} + G'_{\tau} \frac{B''_{k_g}}{B''_k}}, \tag{A.11}$$

where $\lambda'' = (H''_k G''_{k_g} - H''_{k_g} G''_k) / (H''_{\tau} G''_k - H''_k G''_{\tau})$. By updating Eq. (A.6) and substituting (A.11) into it, one obtains an expression for V''_{k_g} . Following the same steps with Eq. (A.5), one arrives at an analogous expression for V''_k . These last two equations can then be substituted back into the first-order condition (A.1) to obtain the GEE. After cumbersome algebra, this GEE can be simplified to

$$R_{\tau} + \beta [R'_k H_{\tau} + R'_{k_g} G_{\tau} + R'_{\tau} A'_{\tau}] + \beta^2 [R'_k \tilde{H}'_{\tau} + R'_{k_g} \tilde{G}'_{\tau} + R'_{\tau} A''_{\tau}] = 0, \tag{GEE}$$

where

$$\begin{aligned} R_i &= u_c [\mathcal{C}'_k H_i + \mathcal{C}'_i], \quad i = \tau, k, k_g, \\ A'_{\tau} &= - \frac{B''_k (H'_k H_{\tau} + H'_{k_g} G_{\tau}) + B''_{k_g} (G'_k H_{\tau} + G'_{k_g} G_{\tau})}{B''_k H'_{\tau} + B''_{k_g} G'_{\tau}}, \\ \tilde{H}'_{\tau} &= \zeta B''_{k_g} \tilde{G}'_{\tau} = -\zeta B''_k, \text{ and} \\ A''_{\tau} &= \zeta (H''_k B''_k - H''_k B''_{k_g}) \end{aligned}$$

with $B_i = G_i - G_{\tau} (H_i / H_{\tau})$, $i = k, k_g$ and $\zeta = -H_{\tau} (B'_k H_{\tau} + B'_{k_g} G_{\tau}) / (B''_k H'_{\tau} + B''_{k_g} G'_{\tau})$. In general, $H_{\tau} < 0$, and $H_k, H_{k_g} > 0$. Since $k'_g = G(k, k_g, \tau) = (\tau - \phi) f(k, k_g)$, then $G_i > 0$ for $i = k, k_g, \tau$.

To obtain the expression in Proposition 1, we substitute the definition of R_i , $i = k, k_g, \tau$, in Eq. (GEE) to obtain

$$\begin{aligned} &u_c [-G_{\tau} - H_{\tau}] + \beta u'_c \{ [f'_k (1 - \phi) + 1 - \delta_k - G'_k - H'_k] H_{\tau} \\ &\quad + [f'_{k_g} (1 - \phi) + 1 - \delta_{k_g} - G'_{k_g} - H'_{k_g}] G_{\tau} + [-G'_{\tau} - H'_{\tau}] A'_{\tau} \} \\ &\quad + \beta^2 u''_c \{ [f''_k (1 - \phi) + 1 - \delta_k - G''_k - H''_k] \tilde{H}'_{\tau} \\ &\quad + [f''_{k_g} (1 - \phi) + 1 - \delta_{k_g} - G''_{k_g} - H''_{k_g}] A''_{\tau} + [-G''_{\tau} - H''_{\tau}] A''_{\tau} \} = 0. \end{aligned}$$

Using the definition of the wedges provided in Proposition 1, we can collect terms and write this equation as

$$H_{\tau} \Delta_k + G_{\tau} \Delta_{k_g} + \beta \{ \tilde{H}'_{\tau} \Delta'_k + \tilde{G}'_{\tau} \Delta'_{k_g} - \beta u''_c [(G''_{k_g} + H''_{k_g}) \tilde{G}'_{\tau} + (G''_k + H''_k) \tilde{H}'_{\tau} + (G''_{\tau} + H''_{\tau}) A''_{\tau}] \} = 0.$$

It can be shown, after straightforward algebraic manipulations, that the last row of this last equation is identically zero. Therefore, the GEE is simply

$$H_{\tau} \Delta_k + G_{\tau} \Delta_{k_g} + \beta \{ \tilde{H}'_{\tau} \Delta'_k + \tilde{G}'_{\tau} \Delta'_{k_g} \} = 0.$$

Appendix B. A special case: constant Markov-perfect policy

Consider here the case where households' period utility is logarithmic, $u(c) = \log c$, technology is Cobb–Douglas, $y = k^{\theta} k^{\alpha} l^{1-\alpha}$, capital depreciates fully within the period, $\delta_k = \delta_{k_g} = 1$, and there are no unproductive expenditures, $\phi = 0$. It is well known that for these specifications of preferences and technology, the household savings function in the decentralized equilibrium depends only on the current level of taxes and not the entire policy stream. Specifically, future policy changes lead to income and substitution effects on current savings that exactly offset each other. Under these assumptions, the household Euler equation reduces to

$$c' = \beta f'_k [1 - \Psi(k', k_g)] c.$$

Given policy, we guess that the recursive competitive equilibrium is such that households save a constant proportion, $s \in (0, 1)$, of after tax income, $H(k, k_g) = s(1 - \Psi(k, k_g)) k^{\theta} k^{\alpha}$. It is then straightforward to show that Eq. (EE) holds if and only

if $s = \alpha\beta$. It follows that

$$\mathcal{H}(k, k_g) = \alpha\beta(1 - \Psi(k, k_g))y, \quad (\text{B.1})$$

and that consumption is also proportional to after tax income

$$c = (1 - \alpha\beta)(1 - \Psi(k, k_g))y. \quad (\text{B.2})$$

Given Eq. (11) in the text and Eq. (B.1) above, it is straightforward to obtain expressions for H_τ , G_τ , \tilde{H}'_τ , and \tilde{G}'_τ that can be substituted into the GEE in Proposition 1. Moreover, expressions for Δ_k and Δ_{k_g} are given by Eqs. (5) and (6), respectively. Basic algebraic manipulations then immediately show that the constant tax policy $\tau = \beta\theta$ solves the GEE.¹⁸

Appendix C. Proof of Proposition 2

When preferences are logarithmic, $u(c) = \log(c)$, technology is Cobb–Douglas, $y = k^\alpha k_g^\theta$, capital depreciates fully within the period, $\delta_k = \delta_{k_g} = 1$, and there are no unproductive expenditures $\phi = 0$, the Ramsey Problem reads as

$$\begin{aligned} \max_{\{\tau_t, k_{t+1}, k_{gt+1}\}_{t=0}^\infty} & \sum_{t=0}^{\infty} \beta^t \ln((1 - \tau_t)y_t - k_{t+1}) \\ \text{s.t.} & k_{t+1} = \alpha\beta y_t(1 - \tau_t) \end{aligned} \quad (\text{C.1})$$

and

$$k_{gt+1} = \tau_t y_t, \quad (\text{C.2})$$

where we have used the closed-form solution for households' savings in the previous Appendix, and $y_t = k_t^\alpha k_{gt}^\theta$. Let λ_t and μ_t denote the Lagrange multipliers associated with constraints (C.1) and (C.2), respectively.

The Ramsey planner's optimality conditions are

$$\begin{aligned} \lambda_t &= \left(\mu_t + \frac{1}{c_t} \right) \frac{1}{\alpha\beta}, \\ \lambda_t - \frac{1}{c_t} - \frac{\alpha\beta y_{t+1}}{k_{t+1}} \left[\lambda_{t+1} \alpha\beta(1 - \tau_{t+1}) + \mu_{t+1} \tau_{t+1} - \frac{(1 - \tau_{t+1})}{c_{t+1}} \right] &= 0, \\ \mu_t + \frac{\theta\beta y_{t+1}}{k_{gt+1}} \left[\frac{(1 - \tau_{t+1})}{c_{t+1}} - \lambda_{t+1} \alpha\beta(1 - \tau_{t+1}) - \mu_{t+1} \tau_{t+1} \right] &= 0. \end{aligned}$$

Straightforward algebra allows us to dispose of the multipliers and obtain

$$\frac{1}{\alpha\beta(1 - \tau_t)} + \frac{\theta}{\alpha\beta(\alpha\tau_t - \theta(1 - \tau_t))} - \frac{\theta}{(1 - \tau_t)(\alpha\tau_{t+1} - \theta(1 - \tau_{t+1}))} = 0.$$

It is then easy to see that $\tau_t = \tau_{t+1} = \beta\theta$ satisfies the above equation. Since the government faces the same first order conditions at $t = 0$ and $t > 0$, this Ramsey solution is time consistent.

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¹⁸ See Azzimonti et al. (2003) for details of the calculations.

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