Online Appendix: Financial Globalization, Inequality, and the Rising Public Debt

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A. Data description for Figures 1, 2, and 11

1) *Debt/GDP ratio* is total (domestic plus external) gross central government debt over GDP, from Reinhart and Rogoff (2011). Sample period: 1973-2005.

2) *Financial liberalization index* is from Abiad, Detragiache, and Tressel (2008). Sample period: 1973-2005.

3) *Income share of top 1 percent* is from Alvaredo, Atkinson, Piketty, and Saez (2011).

4) *Deflator, p*, is the GDP deflator from Kaminsky, Reinhart, and Végh (2004). Sample period: 1970-2003.

5) *Inflation, π*, is computed as $\pi_t = p_t/p_{t-1} - 1$.

6) *Expected inflation, π^e*, is computed as the fitted values from the regression

$$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + \alpha_4\pi_{t-4} + \epsilon_t.$$

7) *Nominal interest rate, i*, is the long-term (10 year) interest rate on government bonds from OECD Statistics. Generally, the yield is calculated at the pre-tax level and before deductions for brokerage costs and commissions and is derived from the relationship between the present market value of the bond and that at maturity, also taking into account interest payments paid through to maturity.

8) *Real interest rate, r*, is computed as $r_t = (1 + i_t)/(1 + \pi^e_{t+1}) - 1$, where i is the nominal interest rate and π^e is expected inflation.

9) *OECD*: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Japan, Korea, Mexico, the Netherlands, New Zealand, Norway, Poland, Portugal, Spain, Sweden, Switzerland, Turkey, the United Kingdom, and the United States.

10) *EUROPE*: Austria, Belgium, Bulgaria, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Netherlands, Norway, Poland, Portugal, Russia, Spain, Sweden, Switzerland, Turkey, and UK.

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11) *Figure 2* plots the fitted values and confidence interval from the following linear regression:

$$\frac{\Delta r_{j,t}}{\Delta \ln(B_{j,t})} = \alpha_0 + \alpha_1 \cdot t,$$

where $r_{j,t}$ is the real interest rate in country j in year t, and $B_{j,t}$ is the real government debt in country j in year t. Since the dependence variable is a ratio and could take very large values when the denominator is close to zero, we have eliminated observations for which the ratio is bigger than 5. Without this selection, the graph has a similar shape but the elasticity values are bigger.

B. Proof of Lemma I.1

Terminal conditions imply $k^i_{j,T+1} = \tilde{b}^i_{j,T+1} = 0$. For $t < T$, we guess $k^i_{j,t+1} = \frac{\eta_t \phi_{j,t}}{p_{j,t}} a^i_{j,t}$ and $\tilde{b}^i_{j,t+1} = R_{j,t}\eta_t(1-\phi_{j,t})a^i_{j,t}$, where η_t is an unknown time-varying parameter. Then, $c^i_{j,t} = (1-\eta_t)a^i_{j,t}$ and $a^i_{j,t+1}$ satisfies

$$a^i_{j,t+1} = \eta_t \left[\left(\frac{A(z^i_{j,t+1}, w_{j,t+1}) + p_{j,t+1}}{p_{j,t}} \right) \phi_{j,t} + R_{j,t}(1-\phi_{j,t}) \right] a^i_{j,t}.$$

The first-order conditions with respect to land and bond holdings for $t < T$ become

$$\text{(B1)} \qquad \frac{\eta_t}{1-\eta_t} = \beta \mathbb{E} \left\{ \frac{\frac{A(z^i_{j,t+1}, w_{j,t+1}) + p_{j,t+1}}{p_{j,t}}}{(1-\eta_{t+1}) \left[\left(\frac{A(z^i_{j,t+1}, w_{j,t+1}) + p_{j,t+1}}{p_{j,t}} \right) \phi_{j,t} + R_{j,t}(1-\phi_{j,t}) \right]} \right\}$$

$$\text{(B2)} \qquad \frac{\eta_t}{1-\eta_t} = \beta \mathbb{E} \left\{ \frac{R_{j,t}}{(1-\eta_{t+1}) \left[\left(\frac{A(z^i_{j,t+1}, w_{j,t+1}) + p_{j,t+1}}{p_{j,t}} \right) \phi_{j,t} + R_{j,t}(1-\phi_{j,t}) \right]} . \right\}$$

Multiply the two conditions by $\phi_{j,t}$ and $1-\phi_{j,t}$, respectively, and add them to get

$$\frac{\eta_t}{1-\eta_t} = \beta \mathbb{E} \left(\frac{1}{1-\eta_{t+1}} \right).$$

Hence, $\eta_T = 0$ and

$$\eta_t = \beta \frac{1}{1 + \beta^{T-t} \left(\sum_{s=1}^{T-t} \beta^{s-1} \right)^{-1}} \qquad \forall t < T$$

verify the guess, and the first optimality condition becomes

$$\mathbb{E}\left[\frac{R_{j,t}}{\left(\frac{A(z^i_{j,t+1},w_{j,t+1})+p_{j,t+1}}{p_{j,t}}\right)\phi_{j,t}+R_{j,t}(1-\phi_{j,t})}\right]=1. \tag{B3}$$

C. Proof of Proposition I.1

We first show that the wage rate does not depend on the distribution and it is constant. The optimality condition for the labor input is $F_l(z^i_{j,t},k^i_{j,t},l^i_{j,t})=w_{j,t}$. Because the production function is homogeneous of degree 1, the demand of labor is linear in land, that is, $l^i_{j,t}=l(z^i_{j,t},w_{j,t})k^i_{j,t}$. If we integrate over all i and average over z, we obtain the aggregate demand of labor

$$\int_i\sum_\ell l(z_\ell,w_{j,t})k^i_{j,t}\mu_\ell=\sum_\ell l(z_\ell,w_{j,t})\mu_\ell\int_i k^i_{j,t},$$

where the expression on the right-hand side uses the law of large numbers. Since in equilibrium the demand for labor must be equal to the supply, which is 1, and total land is also 1, the above condition can be rewritten as $1=\sum_\ell l(z_\ell,w_{j,t})\mu_\ell$. This implicitly defines the wage, which does not depend on endogenous variables. Therefore, the wage is constant. Since the distribution of z is the same across countries, the wage rate must also be equal across countries, that is, $w_{j,t}=\bar{w}$.

Eq. (11) follows from replacing the government's budget constraint (5) into the workers' budget constraint (eq. (4)). Eq. (7) is obtained from eq. (B1) after replacing $R_{j,t}(1-\phi_{j,t})=\phi_{j,t}\tilde{b}_{j,t+1}/p_{j,t}$. This expression is derived from Lemma I.1. To obtain eq. (8), combine aggregate asset holdings $\bar{a}_{j,t}=\sum_\ell A(z_\ell,\bar{w})\mu_\ell+p_{j,t}+\tilde{b}_{j,t}$ with the aggregated choice of land, $p_{j,t}\bar{k}=\eta_t\phi_{j,t}\bar{a}_{j,t}$. Taking into account that the wage is $\bar{w}$, $\bar{k}=1$, and defining $\sum_\ell A(z_\ell,\bar{w})\mu_\ell=\bar{A}$, we obtain eq. (8).

To derive eq. (9), consider the aggregate entrepreneurs' budget constraint $c^e_{j,t}+\frac{\tilde{b}_{j,t+1}}{R_{j,t}}=\bar{A}+\tilde{b}_{j,t}$. We can now use the aggregate policy $c^e_{j,t}=(1-\eta_t)\bar{a}_{j,t}$ and aggregate asset holdings $\bar{a}_{j,t}=\bar{A}+p_{j,t}+\tilde{b}_{j,t}$ to eliminate consumption and use eq. (8) to eliminate $p_{j,t}$ and solve for $R_{j,t}$.

To derive eq. (10), aggregate individual entrepreneurs' budgets to obtain $c^e_{j,t}+\frac{\tilde{b}_{j,t+1}}{R_{j,t}}=\bar{A}+\tilde{b}_{j,t}$, and rearrange the terms to get eq. (10).

D. Proof of Corollary I.1

Proof of Proposition I.1 established that $\eta_T=0$, which, together with eq. (8), implies that $p_{j,T}=0\ \forall j$. Replacing this in eq. (7), we obtain

$$\phi_{j,T-1}=\mathbb{E}_{T-1}\left[\frac{A(z^i_{j,T})}{A(z^i_{j,T})+\tilde{b}_{j,T}}\right]. \tag{D1}$$

Rewriting eq. (9) at date $t = T-1$, and using $R_{j,t} = R_t$, $\forall j$ in an integrated equilibrium, delivers

$$\tilde{b}_{j,T} = R_{T-1}(\bar{A} + \tilde{b}_{j,T-1})\frac{\eta_{T-1}(1-\phi_{j,T-1})}{(1-\eta_{T-1}\phi_{j,T-1})}. \tag{D2}$$

Replacing eq. (D2) in eq. (D1) results in $\phi_{j,T-1}$ being a function of $\tilde{b}_{j,T-1}$. Notice that this holds because we have assumed a common shock structure across countries. Using the function $\phi_{j,T-1}(\tilde{b}_{j,T-1})$ in eq. (D2) delivers $\tilde{b}_{j,T}(\tilde{b}_{j,T-1})$. Substituting these two functions in eq. (8), evaluated at $t = T-1$, yields $p_{T-1}(\tilde{b}_{j,T-1})$.

Evaluating eqs. (7) and (9) at $t = T-2$, and using the functions derived above, we can show that $\tilde{b}_{j,T-1}(\tilde{b}_{j,T-2})$. By repeated substitution over time (following the same steps) we can obtain an expression for $\tilde{b}_{j,2}$ that depends only on $\tilde{b}_{j,1}$. Since $\tilde{b}_{j,1} = \tilde{b}_1$, $\forall j$, then $\tilde{b}_{j,2} = \tilde{b}_2$, $\forall j$. Substituting forward, we can easily show that $\tilde{b}_{j,t+1} = \tilde{b}_{t+1}\,\forall j$. Adding up across countries, and using the bond-market equilibrium condition yields $\sum_{j=1}^{N}\tilde{b}_{j,t+1} = N\tilde{b}_{t+1} = \nu\sum_{j=1}^{N}B_{j,t+1}$.

E. of Lemma II.1

Follow the steps in the proof of Proposition I.1 (see section C) to derive

$$\frac{B}{R(B)} = \frac{\beta\bar{A}(1-\phi(B))}{\nu[1+\beta(1-\phi(B))]} > 0, \quad \text{where} \tag{E1}$$

$$\phi(B) = \mathbb{E}\left(\frac{A(z)}{A(z)+\nu B}\right) \leq 1. \tag{E2}$$

i. Let B^A satisfy the FOC $\frac{\partial W(B)}{\partial B} = 0$, with $\frac{\partial W(B)}{\partial B} = \frac{\nu}{c_1^w}\frac{\partial(B/R(B))}{\partial B} - \beta\frac{\nu}{c_2^w}$, where $c_1^w = \bar{w} + \nu B/R(B)$ and $c_2^w = \bar{w} - \nu B$ are aggregate workers' consumption. Recall that $\frac{\partial B/R(B)}{\partial B} = \frac{1}{R(B)}[1-\epsilon(B)]$, where $\epsilon(B) = \frac{B}{R(B)}\frac{\partial R(B)}{\partial B}$. Since

$$\frac{\partial R(B)}{\partial B} = \frac{\nu}{\beta\bar{A}(1-\phi(B))}\left[1+\beta(1-\phi(B)) + \frac{B}{1-\phi(B)}\frac{\partial\phi(B)}{\partial B}\right], \text{ then}$$

$$\epsilon(B) = 1 + \frac{B\frac{\partial\phi(B)}{\partial B}}{(1-\phi(B))[1+\beta(1-\phi(B))]} \Rightarrow \frac{\partial B/R(B)}{\partial B} = -\frac{\beta\bar{A}\frac{\partial\phi(B)}{\partial B}}{\nu[1+\beta(1-\phi(B))]^2} > 0.$$

The inequality holds because $\frac{\partial\phi(B)}{\partial B} = -\mathbb{E}\frac{A(z)}{\nu(A(z)+B)^2} < 0$. Since $\frac{\partial W(B)}{\partial B} > 0|_{B=0}$ and $\frac{\partial W(B)}{\partial B} \to -\infty$ as $B \to \frac{\bar{w}}{\nu}$, then $B^A \in \left[0, \frac{\bar{w}}{\nu}\right]$.

Existence and uniqueness follow from the concavity of $W(B)$. Differentiating eq. (13) twice yields

$$\frac{\partial^2 W(B)}{\partial B^2} = -\frac{\nu^2}{(c_1^w)^2}\left[\frac{\partial(B/R(B))}{\partial B}\right]^2 + \frac{\nu}{c_1^w}\frac{\partial^2(B/R(B))}{\partial B^2} - \frac{\beta\nu^2}{(c_2^w)^2}. \tag{E3}$$

Since

$$\frac{\partial^2 \phi(B)}{\partial B^2} = 2\mathbb{E}\left[\frac{A(z)\nu^2}{(A(z)+\nu B)^3}\right] > 0, \tag{E4}$$

we obtain

$$\frac{\partial^2 (B/R(B))}{\partial B^2} = -\frac{\beta \bar{A}}{\nu(1+\beta[1-\phi(B)])^3}\left[\frac{\partial^2 \phi(B)}{\partial B^2}(1+\beta[1-\phi(B)]) + 2\beta\left(\frac{\partial \phi(B)}{\partial B}\right)^2\right] < 0.$$

ii. Replace eq. (E1) in the representative entrepreneur's consumption and obtain $c_1^e = \frac{\bar{A}}{1+\beta[1-\phi(B)]}$. Then, differentiate the resulting indirect utility

$$\frac{\partial V(B)}{\partial B} = \frac{\beta}{1+\beta(1-\phi(B))}\frac{\partial \phi(B)}{\partial B} + \beta\mathbb{E}\left(\frac{\nu}{A(z)+\nu B}\right).$$

Substitute

$$\frac{\partial \phi(B)}{\partial B} = -\mathbb{E}\left[\frac{\nu A(z)}{(A(z)+\nu B)^2}\right] \tag{E5}$$

in the expression above and collect terms to show

$$\frac{\partial V(B)}{\partial B} = \beta\nu\mathbb{E}\left[\frac{\nu B + \beta[1-\phi(B)]\,(A(z)+\nu B)}{(A(z)+\nu B)^2\,(1+\beta[1-\phi(B)])}\right] > 0.$$

F. Proof of Proposition II.1

Suppose that $\Phi > \frac{(1+\beta)\bar{w}}{\bar{A}} + \beta$, and let the government's objective be defined by

$$G(B) \equiv \Phi W(B) + V(B),$$

where $W(B)$ and $V(B)$ are given by eqs. (13) and (12). To prove concavity, differentiate $G(B)$ twice, where $\frac{\partial^2 W(B)}{\partial B^2}$ is defined in eq. (E3) and

$$\frac{\partial^2 V(B)}{\partial B^2} = -\frac{\nu^2}{(c_1^e)^2}\left[\frac{\partial (B/R(B))}{\partial B}\right]^2 - \frac{\nu}{c_1^e}\frac{\partial^2 (B/R(B))}{\partial B^2} - \beta\mathbb{E}\frac{\nu^2}{(c_2^e)^2}.$$

After some manipulations, we can show that

$$\begin{aligned}\frac{\partial^2 G(B)}{\partial B^2} &= -\left[\frac{\partial (B/R(B))}{\partial B}\right]^2 \nu^2 \left[\frac{\Phi}{(c_1^w)^2} + \frac{1}{(c_1^e)^2}\right] - \beta\nu^2\left[\frac{\Phi}{(c_2^w)^2} + \mathbb{E}\frac{1}{(c_2^e)^2}\right] \\ &+ \frac{\partial^2 (B/R(B))}{\partial B^2}\nu\left[\frac{\Phi}{c_1^w} - \frac{1}{c_1^e}\right].\end{aligned}$$

The first row is negative for all B. Hence, a sufficient condition for $\frac{\partial^2 G(B)}{\partial B^2} < 0$ is that the second row is non-positive. We established that $\frac{\partial^2 (B/R(B))}{\partial B^2} < 0$ in Section E (Part *i.*). In addition, we need

$$\frac{\Phi}{c_1^w} - \frac{1}{c_1^e} \quad = \quad \frac{\Phi c_1^e - c_1^w}{c_1^w c_1^e} > 0,$$

since $c_1^e = \frac{\bar{A}}{1+\beta[1-\phi(B)]}$ and $c_1^w = \bar{w} + \nu B/R(B)$. Substituting for $R(B)$ we obtain

$$\begin{aligned} c_1^e - c_1^w/\Phi \quad &= \quad \frac{1}{1+\beta(1-\phi(B))}\left[\bar{A} - \frac{1}{\Phi}\left([1+\beta(1-\phi(B))]\bar{w} + \beta\bar{A}(1-\phi(B))\right)\right] \\ &\geq \quad \frac{1}{\Phi}\frac{1}{1+\beta(1-\phi(B))}[\bar{A}(\Phi-\beta) - \bar{w}(1+\beta)]. \end{aligned}$$

Since $0 \leq \phi(B) \leq 1$, the denominator of the above equation is positive. Moreover, the assumption that $\Phi > \frac{(1+\beta)\bar{w}}{\bar{A}} + \beta$ is sufficient for the numerator to be positive. This establishes concavity.

Let B^A satisfy $\frac{\partial G(B)}{\partial B} = 0$. From Lemma II.1, $V(B)$ is increasing in B $\forall B \in \left[0, \frac{\bar{w}}{\nu}\right]$ and $\frac{\partial W(B)}{\partial B}|_{B=0} > 0 \Rightarrow \frac{\partial G(B)}{\partial B}|_{B=0} > 0$. Additionally, $\frac{\partial V(B)}{\partial B}$ is finite at $\frac{\bar{w}}{\nu}$ and $\frac{\partial W(B)}{\partial B} \to -\infty$ as $B \to \frac{\bar{w}}{\nu}$, so $\frac{\partial G(B)}{\partial B} \to -\infty$. Hence $B^A \in \left[0, \frac{\bar{w}}{\nu}\right]$. Because $G(B)$ is strictly concave, B^A must be unique.

G. Proof of Proposition II.2

Let the relative size of workers $\nu = 1$. To show that debt is increasing in N, replace $\Phi/(1+\Phi) = 1$ in eq. (20) to obtain

$$G(\mathbf{B}, N) \equiv \Phi\left[\frac{\partial(B_j/R(\mathbf{B}))}{\partial B_j}\left(\frac{1}{c_1^w}\right) - \beta\left(\frac{1}{c_2^w}\right)\right] = 0.$$

Since $\tilde{b} = b = \frac{\sum_{j=1}^N B_j}{N}$ in this case, the interest rate can be rewritten as

$$R(b) = \nu b\left[\frac{1+\beta(1-\phi(b))}{\beta(1-\phi(b))\bar{A}}\right], \quad \text{implying}$$

(G1)
$$\frac{\partial(B_j/R(\mathbf{B}))}{\partial B_j} = \frac{1}{R(b)}\left(1 - \frac{B}{R(b)}\frac{\partial R(b)}{\partial b}\frac{1}{N}\right) \equiv \gamma(b) \quad \text{and} \quad \frac{\partial R(b)}{\partial b} = R(b)\left[\frac{1}{b} + \frac{\partial \phi(b)}{\partial b}\frac{1}{[1+\beta(1-\phi(b))](1-\right.$$

Claim G.1: The interest rate is increasing in b, $\frac{\partial R(b)}{\partial b} > 0$.

Proof: Rewrite eq. (G1) as

$$\frac{\partial R(b)}{\partial b} = \frac{R(b)}{b[1+\beta(1-\phi(b))](1-\phi(b))}\left[[1+\beta(1-\phi(b))](1-\phi(b)) + b\frac{\partial \phi(b)}{\partial b}\right]$$

$$> \frac{R(b)}{b[1+\beta(1-\phi(b))](1-\phi(b))}\left[1-\phi(b)+b\frac{\partial\phi(b)}{\partial b}\right]=0.$$

The inequality follows from $\beta(1-\phi(b))<1$. Replace eqs. (E2) and (E5) in the bracketed term to show equality.

Claim G.2: *(i.)* $\partial G(\mathbf{B},N)/\partial B_j<0$ and *(ii.)* $\partial G(\mathbf{B},N)/\partial N>0$

Proof:

(i.) We can show that

$$\partial G(\mathbf{B},N)/\partial B_j=\Phi\left[\frac{\partial\gamma(b)}{\partial b}\frac{1}{c_1^w}-\gamma(b)^2\frac{\Phi}{(c_1^w)^2}-\beta\frac{\Phi}{(c_2^w)^2}\right], \tag{G2}$$

where

$$\frac{\partial\gamma(b)}{\partial b}=-\left(1-\frac{B}{Nb}\right)\frac{2}{NR(b)^2}\frac{\partial R(b)}{\partial b}-\frac{B}{Nb}\frac{\beta\bar{A}}{(1+\beta(1-\phi(b)))^2}\left[\frac{\partial^2\phi(b)}{\partial b^2}+\frac{2\beta\left(\frac{\partial\phi(b)}{\partial b}\right)^2}{1+\beta(1-\phi(b))}\right].$$

Since $\frac{\partial^2\phi(b)}{\partial b^2}>0$ from eq. (E4) and $\frac{\partial R(b)}{\partial b}>0$ from Claim G.1, then $\frac{\partial\gamma(b)}{\partial b}<0$. Because all terms in eq. (G2) are negative, the result follows.

(ii.) We can show that

$$\frac{\partial G(\mathbf{B},N)}{\partial N}=\Phi\left[\frac{\partial\gamma(b)}{\partial N}\frac{1}{c_1^w}-\gamma\frac{\Phi}{(c_1^w)^2}\frac{\partial(B/R(b))}{\partial N}\right].$$

The first term is positive. Noting that since $b=B$ in a symmetric equilibrium, then $\frac{\partial b}{\partial N}=\frac{b-B}{N^2}=0$, and performing some algebraic manipulations, we obtain $\frac{\partial\gamma(b)}{\partial N}=\frac{B}{R(b)^2N^2}\frac{\partial R(b)}{\partial b}>0$ from Claim G.1. The second term is zero, since

$$\frac{\partial(B/R(b))}{\partial N}=-\left[\frac{1-\phi(b)}{b}+\frac{1}{[1+\beta(1-\phi(b))]}\frac{\partial\phi(b)}{\partial b}\right]\frac{B\beta\bar{A}}{[1-\beta(1-\phi(b))]b}\frac{\partial b}{\partial N}$$

and $\frac{\partial b}{\partial N}=0$.

Using Claim G.2 and the implicit function theorem, we show that domestic debt B_j is increasing in N:

$$\frac{\partial B_j}{\partial N}=-\frac{\partial G(\mathbf{B},N)/\partial N}{\partial G(\mathbf{B},N)/\partial B_j}>0.$$

For the limiting case, let $N\to\infty$ in eq. (21). Substituting c_1^w and c_2^w and rearranging, we obtain

$$\beta R(b)=1-\frac{1+\beta}{\bar{w}}B. \tag{G3}$$

This equation determines country 1's supply of debt given $R(b)$.

In equilibrium, $B_1 = B_2 = ..B_N = B = b$, where the per capita demand for debt b satisfies eq. (18). The financially integrated equilibrium levels of b and R are thus determined by eqs. (18) and (G3).

Existence and uniqueness follow from (i) the RHS of eq. (G3) is decreasing in b and equals 1 at the origin, and (ii) the LHS of eq. (G3) is increasing in b (since $R_b > 0$) and has an intercept at $\left[\mathbb{E}\left(\frac{\bar{z}}{z}\right)\right]^{-1} < 1$. Denote the intersection point by B^{FI}. From (i) and (ii) it also follows that B^{FI} is bounded and $\beta R(b) < 1$ when $b = B^{FI}$.

Under autarky, eq. (G3) is instead

$$\beta R(b) = 1 - \frac{1+\beta}{\bar{w}} b - \epsilon(b)\left(1 - \frac{b}{\bar{w}}\right). \tag{G4}$$

The LHS is the same as before. The RHS is also equal to 1 at the origin because $\epsilon(0) = 0$. Since $\epsilon(b) > 0$ and $\bar{w} - b = c_2^w > 0$ when $b > 0$, the new term in the RHS is positive. Hence, the intersection of the two curves in eq. (G4) occurs at $B^A < B^{FI}$, since the RHS is steeper.

Since debt is larger and V is increasing in b, $V(B^A) < V(B^{FI})$. Since W is concave in b and $W(b)$ is decreasing when $b > B^A$, then $W(B^A) > W(B^{FI})$.

What is left to prove is that the equilibrium must be symmetric. This can be shown starting from the first-order condition of the government, which must be satisfied for all countries:

$$\Phi \cdot \left[\frac{1 - \frac{\epsilon(\overline{B})B}{N\overline{B}}}{c_1^w} - \frac{\beta R(\overline{B})}{c_2^w}\right] = \left(\frac{1}{N}\right) \cdot \left[\frac{1-\epsilon(\overline{B})}{c_1^e} - \mathbb{E}_t\left(\frac{\beta R(\overline{B})}{c_2^e}\right)\right]. \tag{G5}$$

An equilibrium is characterized by a worldwide debt $\overline{B}$. Given $\overline{B}$, the elasticity ϵ and the interest rate R are determined. Also notice that the right-hand side of (G5) is the same for all countries, since entrepreneurs choose to hold the same stock of bonds in all countries. The left-hand side could differ, since governments could choose different B. However, since the left-hand side is strictly decreasing in B (keeping $\overline{B}$ constant), the fact that the right-hand side is the same for all countries implies that B must be the same for all countries. Otherwise, the first-order condition (G5) will not hold for all countries. Notice that this result applies for any value of Φ, not only for the limiting case $\Phi/(1+\Phi) = 1$.

H. Proof of Proposition II.3

Setting $\Phi/(1+\Phi) = 1$, the first-order conditions for the domestic and foreign country become

$$1 - \frac{\alpha B_1}{\overline{B}}\epsilon(\overline{B}) = \beta R(\overline{B})\left(\frac{c_1^w(B_1)}{c_2^w(B_1)}\right) \tag{H1}$$

$$1 - \frac{(1-\alpha)B_2}{\overline{B}}\epsilon(\overline{B}) = \beta R(\overline{B})\left(\frac{c_1^w(B_2)}{c_2^w(B_2)}\right), \tag{H2}$$

where we have made it explicit that the interest rate elasticity, $\epsilon(\overline{B})$, and the interest rate, $R(\overline{B})$, are functions of the average worldwide debt $\overline{B} = \alpha B_1 + (1-\alpha)B_2$.

An equilibrium will be characterized by B_1 and B_2 (and $\overline{B}$) that satisfy conditions (H1) and (H2). We want to show that in an integrated economy $B_1 > B_2$ if $\alpha < 1/2$, that is, the per capita debt of the large country is lower than the per capita debt of the small country.

Subtracting (H2) from (H1) and substituting $(1-\alpha)B_2 = \overline{B} - \alpha B_1$ we obtain

$$\left(1 - \frac{2\alpha B_1}{\overline{B}}\right)\epsilon(\overline{B}) = \beta R(\overline{B})\left(\frac{c_1^w(B_1)}{c_2^w(B_1)} - \frac{c_1^w(B_2)}{c_2^w(B_2)}\right) \tag{H3}$$

For a given $\overline{B}$ that characterizes the equilibrium, the left-hand-side term is decreasing in B_1. Since $\overline{B}$ is the equilibrium worldwide debt taken as given in this exercise, an increase in B_1 must be associated with a decline in B_2. Therefore, it is the ratio B_1/B_2 that matters. The right-hand-side term, instead, is increasing in B_1. To see this, we can define aggregate workers' consumption using the budget constraints as

$$c_1^w(B_1) = \bar{w} + \frac{B_1}{R(\overline{B})}, \qquad c_2^w(B_1) = \bar{w} - B_1 \tag{H4}$$

$$c_1^w(B_2) = \bar{w} + \frac{B_2}{R(\overline{B})}, \qquad c_2^w(B_2) = \bar{w} - B_2. \tag{H5}$$

From these equations it is clear that $c_1^w(B_1)/c_2^w(B_1)$ is increasing in B_1 and $c_1^w(B_2)/c_2^w(B_2)$ is increasing in B_2. Since an increase in B_1 must be associated with a decline in B_2, then $c_1^w(B_2)/c_2^w(B_2)$ is decreasing in B_1. Thus, the right-hand side of eq. (H3) must be increasing in B_1.

So far, we have established that the LHS of eq. (H3) is decreasing and the RHS is increasing in B_1. Next, we observe that, if $\alpha < 1/2$, then the LHS is positive when $B_1 = B_2$. The RHS, instead, is zero. Therefore, to equalize the LHS (which is decreasing in B_1) to the RHS (which is increasing in B_1) we have to increase B_1 (which must be associated with a decrease in B_2). Therefore, if $\alpha < 1/2$, $B_1 > B_2$.

Finally, since in the autarky equilibrium both countries had the same debt, the growth in debt following financial liberalization is bigger for the small country.

I. Proof of Proposition II.4

Let $\Phi/(1+\Phi) = 1$. Then the autarky equilibrium satisfies the government's first-order condition

$$\frac{1-\epsilon(B)}{R(B)c_1^w} = \frac{\beta}{c_2^w},$$

where we made it explicit that the interest elasticity ϵ and the interest rate R are functions of debt B. Since $c_1^w = \bar{w} + B/R(B)$ and $c_2^w = \bar{w} - B$, the first-order condition can be rewritten as

$$\frac{1-\epsilon(B)}{\bar{w}R(B) + B} = \frac{\beta}{\bar{w} - B}. \tag{I1}$$

The right-hand side of (I1) is clearly increasing in B. We now show that the left-hand side is decreasing in B. First let's rewrite the left-hand side as

$$\frac{1-\epsilon(B)}{\bar{w}R(B)+B} = \left(\frac{1-\epsilon(B)}{R(B)}\right)\cdot\left(\frac{1}{\bar{w}+B/R(B)}\right), \tag{I2}$$

which is the product of two terms. We want to show that both terms are decreasing in B. Let's start with the first term, which is equal to

$$\frac{1-\epsilon(B)}{R(B)} = -\frac{\beta\phi'(B)\bar{A}}{[1+\beta(1-\phi(B))]^2}.$$

Since $\phi(B) = \mathbb{E}[A(z)/(A(z)+B)]$ and $-\phi'(B) = \mathbb{E}[A(z)/(A(z)+B)^2]$ are both decreasing in B, then the first term in (I2) is also decreasing in B. The second term in (I2) depends negatively on $B/R(B) = \beta(1-\phi(B)\bar{A}/[1+\beta(1-\phi(B))]$. As we have already observed, $\phi(B) = \mathbb{E}A(z)/(A(z)+B)$ depends negatively on B and, therefore, $B/R(B)$ increases in B. Thus, the second term in (I2) decreases with B. This proves that (I2) is decreasing in B.

To summarize, we have shown that the left-hand side of first-order condition (I1) decreases with B, while the right-hand side increases with B. Therefore, if an increase in the mean-preserving spread of z raises the term $(1-\epsilon(B))/[\bar{w}R(B)+B]$ on the left-hand side, to reestablish equality B has to rise.

J. Proof of Proposition III.1

Let B^* denote the optimal level of debt in the benchmark case, where $\overline{D} = 0$. Denote by R^*, c_t^{w*}, and c_t^{e*} the associated interest rate and allocations.

Consider a case where $\overline{D} > 0$ and let $\tilde{d} = d + \frac{B}{1+\Phi}$ denote workers' excess supply of debt. Workers choose $\tilde{d}$ in order to maximize (1) subject to $c_1^w = \bar{w}\frac{1}{\Phi} + \frac{\tilde{d}}{R}$, $c_2^w = \bar{w}\frac{1}{\Phi} - \tilde{d}$, and the borrowing constraint $\tilde{d} \leq \overline{d} + \frac{B}{1+\Phi}$. The interest rate is determined by the entrepreneurs' optimality condition,

$$R = \frac{1}{A}\left(\frac{1}{\beta E\frac{1}{A(z)+\tilde{b}}} + \tilde{b}\right). \tag{J1}$$

In equilibrium, $b = B + D$, where $D = \Phi d$, implying $\tilde{b} = \Phi\tilde{d} = \nu B + D$. When workers are fully unconstrained ($\overline{D} \to \infty$), their FOC holds with equality: $c_2^w = \beta R c_1^w$. This equation, together with eq. (J1), determines $\tilde{d}$ and R independently of B, so public debt is irrelevant (e.g., Ricardian equivalence holds). Since only $d - B/(1+\Phi)$ is determined, we can set $B = 0$ without loss of generality and denote the optimal aggregate amount of debt when workers are unconstrained by D^*. Clearly, $\nu B^* < D^*$.

Our conjecture is that, for $\overline{D} \in (0,\infty)$, the government will optimally set $B = B^* - \overline{D}/\nu$. To verify this, note that under the conjecture $\tilde{b} = \nu B^* + D - \overline{D} \leq \nu B^* < D^*$. But then, the FOC of workers is slack, so $D = \overline{D}$. Because $\tilde{b} = \nu B^*$, the interest rate and allocations will be R^*, c_t^{w*}, and c_t^{e*}, which satisfy the government optimality condition (15).

K. Proof of Proposition III.2

Let τ_t denote a proportional tax on income and T_t a lump-sum transfer. Entrepreneurs' consumption is $c_1 = \pi(\bar{z}, k_1)(1-\tau_1) + k_1 p_1 - p_1 k_2 - \frac{b}{R} + T_1$ and $c_2^i = \pi(z^i, k_2)(1-\tau_2) + b + T_2$, while workers' consumption becomes $c_t^w = (1-\tau_t) w_t n_t + T_t$. When $\Phi = \frac{1-\theta}{\theta}$, first-period labor income $w_1 n_1 = \bar{z}^\theta (1-\theta)\frac{1}{\Phi}$ equals entrepreneurs' income $\pi(\bar{z}, k_1) = \bar{z}^\theta \theta$. In equilibrium, $c_1 = e - \nu\frac{B}{R}$ and $c_1^w = e + \nu\frac{B}{R}$. In the second period, $c_2^w = e - \nu B$ while $c_2^i = e\frac{z_i}{z}(1-\tau) + \tau e + \nu B$. Setting $\tau_2 = 1$ and $B = 0$ maximizes welfare, since the government can efficiently redistribute resources eliminating entrepreneurs' inequality in $t = 2$, without distorting the economy.

L. Proof of Proposition III.3

Entrepreneurs maximize the expected value of lifetime utility, $\ln(c_1 - h_1) + \beta E \ln(c_2^i - h_2^i)$, where $c_1 = \pi(\bar{z}, k_1)(1-\tau_1) + k_1 p_1 - p_1 k_2 - \frac{b}{R} + T_1$ and $c_2^i = \pi(z^i, k_2)(1-\tau_2) + b + T_2$. Profits satisfy $\pi(z_t, k_t) = (z_t k_t)^{\theta\eta} l_t^{(1-\theta)\eta} h_t^{1-\eta} - w_t l_t$. As before, we assume $k_1 = 1$ and $z_1 = \bar{z}$.

The problem of a worker is identical to the benchmark model, but government constraints become

$$T_1 = \tilde{\tau}_1 + \frac{B}{R}\frac{1}{(1+\Phi)} \qquad \text{and} \qquad T_2 = \tilde{\tau}_2 - B\frac{1}{(1+\Phi)},$$

where $\tilde{\tau}_t = \tau_t \frac{\left[w_t l_t \Phi + \int \pi(z_t, k_t)\right]}{(1+\Psi)}$ denotes tax revenues per capita.

LEMMA L.1: *Given policy $\{B, \tau_1, \tau_2\}$, the equilibrium wage is constant $\bar{w}$ and the remaining prices and allocations in an autarky equilibrium satisfy*

$$\text{(L1)} \qquad h_t^i = k_t \frac{z_t^i}{\bar{z}^{1-\theta}}[(1-\eta)(1-\tau_t)]^{1/\eta}$$

$$\text{(L2)} \qquad c_1 - h_1 = A(\bar{z}, \tau_1) + \tilde{\tau}_1 - \frac{\nu B}{R}$$

$$\text{(L3)} \qquad c_2^i - h_2^i = A(z^i, \tau_2) + \tilde{\tau}_2 + \nu B,$$

$$\text{(L4)} \qquad c_1^w = A(\bar{z}, \tau_1) + \tilde{\tau}_1 + \frac{\nu B}{\Phi R},$$

$$\text{(L5)} \qquad c_2^w = A(\bar{z}, \tau_2) + \tilde{\tau}_2 - \frac{\nu B}{\Phi},$$

$$\text{(L6)} \qquad R = \left(\frac{1}{\beta E\left(\frac{1}{c_2^i - h_2^i}\right)} + \nu B\right)\frac{1}{\tilde{\tau}_1 + A(\bar{z}, \tau_1)}$$

where $A(z,\tau) = \frac{z\theta\eta(1-\eta)^{(1-\eta)/\eta}(1-\tau)^{1/\eta}}{\bar{z}^{1-\theta}}$ and $\tilde{\tau}_t = \tau_t \frac{\bar{z}^\theta[(1-\eta)(1-\tau_t)]^{(1-\eta)/\eta}}{1+\Psi}$.

PROOF L.1: *The demands for labor l_t^i and h_t^i of entrepreneur i are*

$$h_t^i = \left[(z_t^i k_t)^{\theta\eta} l_t^{i(1-\theta)\eta}(1-\eta)(1-\tau_t)\right]^{1/\eta},$$

$$l_t^i = z_t^i k_t \left(\frac{(1-\theta)\eta[(1-\eta)(1-\tau_t)]^{(1-\eta)/\eta}}{w_t} \right)^{1/\theta}.$$

In equilibrium, $\int_i l_t^i = 1$*, implying* $\bar{w} = \bar{z}^\theta (1-\theta)\eta[(1-\eta)(1-\tau_t)]^{(1-\eta)/\eta}$*. Replacing this and* l_t^i *in the expression for* h_t^i *above delivers eq. (L1). Eqs. (L2) and (L3) are obtained by replacing: (i) government transfers, (ii) the bond market and land market equilibrium conditions,* $\tilde{b} = \nu B$ *and* $k_2 = 1$*, and (iii) after-tax profits net of the disutility of effort in the entrepreneurs' budget constraints. Expressions (L4) and (L5) are obtained by replacing government transfers in the workers' budget constraints and noting that* $w_1 l_1 (1-\tau_1) = \pi(\bar{z}, k_1)(1-\tau_1) - h_1$ *when* $\Phi = \frac{1-\theta}{\theta}$*. That is, after-tax income of workers and entrepreneurs (net of the disutility of exerting effort) is identical in period 1. Finally, the interest rate, eq. (L6), arises from rearranging the entrepreneur's FOC w.r.t. the excess demand for bonds* $\tilde{b}$ *and replacing in other equilibrium conditions.*

The lemma characterizes the autarky equilibrium given government policies. Taxes, transfers, and public debt are chosen in the same fashion as in the benchmark model. Because elections are held every period, we can solve the Markov perfect equilibrium of this finite-horizon economy by backward induction.

LEMMA L.2: *The optimal tax in the second period is positive and bounded away from 1,* $\tau_2 \in (0,1)$*.*

PROOF L.2: *The government's objective in the second period, given B, is*

$$\max_{\tau_2} \Phi \ln c_2^w + \int_i \ln(c_2^i - h_2^i)$$

s.t. eq. (L3), (L5) and

$$\tilde{\tau}_2 = \tau_2 \frac{\bar{z}^\theta [(1-\eta)(1-\tau_2)]^{(1-\eta)/\eta}}{1+\Phi}.$$

The necessary condition w.r.t. τ_2 *simplifies to*

$$\Phi \frac{1}{c_2^w} \left(-\frac{\tau_2(1-\eta)}{(1-\tau_2)\eta} \right) + \int_i \frac{1}{c_2^i - h_2^i} \left(\frac{-z^i}{\bar{z}} + 1 - \frac{\tau_2(1-\eta)}{(1-\tau_2)\eta} \right) = 0.$$

Clearly, the solution for τ_2 *is interior,* $\tau_2 \in (0,1)$*.*

Notice that this results from the fact that the government can redistribute resources across agents in the economy when entrepreneurs' effort is endogenous. This is not the case in period 1.

LEMMA L.3: *The government does not tax in the first period,* $\tau_1 = 0$*.*

PROOF L.3: *The government's objective in the first period is*

$$\max_{\tau_1, B} \Phi \left(\ln c_1^w + \beta \ln c_2^w \right) + \ln(c_1 - h_1) + \beta \int_i \ln(c_2^i - h_2^i),$$

s.t. $\tau_2 = \Psi(B)$ *and eqs. (L6), (L2), (L3), (L4), and (L5) which are now all functions of* τ_1 *and B.*

The optimality condition w.r.t τ_1 is

$$\Phi\frac{1}{c_1^w}\left(-\frac{\tau_1(1-\eta)}{(1-\tau_1)\eta}-\frac{\nu B}{R^2}\frac{\partial R}{\partial\tau_1}\frac{1}{\Phi}\right)+\frac{1}{c_1-h_1}\left(-\frac{\tau_1(1-\eta)}{(1-\tau_1)\eta}+\frac{\nu B}{R^2}\frac{\partial R}{\partial\tau_1}\right)=0,$$

Simplifying, this reduces to

$$\frac{-\tau_1(1-\eta)}{(1-\tau_1)\eta}\left[\frac{\Phi}{c_1^w}\left(1-\xi(\nu B)\right)+\frac{1}{c_1}\left(1+\xi(\nu B)\right)\right]=0$$

with $\xi(\nu B)=\frac{\nu B}{\nu B+\frac{1}{\beta\mathbb{E}\frac{1}{c_2^i-h_2^i}}}$. *Since the term in brackets is strictly positive, then* $\tau_1=0$.

Because both agents have the same income in the first period, there is no redistribution from the affine tax system (that is, labor income and lump-sum transfers). The change in the interest rate caused by changes in τ_1 would result in some redistribution between workers and entrepreneurs. The government, however, can achieve this by simply changing B, a less distortionary instrument than τ_1.

LEMMA L.4: *Debt is relevant,* $B\neq 0$.

PROOF L.4: *By contradiction. The optimality condition w.r.t* νB *can be written as*

$$\left[\frac{1}{c_1^w}\left(1-\epsilon(\nu B)\right)-\beta R\frac{1}{c_2^w}\right]+\left[-\frac{1}{c_1-h_1}\left(1-\epsilon(\nu B)\right)+\beta R\mathbb{E}\frac{1}{c_2^i-h_2^i}\right]=0.$$

Suppose that $\tilde{B}=0$, *then* $c_1-h_1=c_1^w$, *so the equation collapses to*

$$-\frac{1}{c_2^w}+\mathbb{E}\frac{1}{c_2^i-h_2^i}=0.$$

Rearranging and substituting in consumption, this results in

$$\mathbb{E}\frac{A(\bar{z},\tau_2)+\tilde{\tau}_2}{A(z^i,\tau_2)+\tilde{\tau}_2}=1\;\Rightarrow\;\mathbb{E}\frac{\eta+\frac{\tau_2(1-\eta)}{(1-\tau_2)\eta}}{\eta\frac{z^i}{\bar{z}}+\frac{\tau_2(1-\eta)}{(1-\tau_2)\eta}}=1,$$

a contradiction.

M. Proof of Proposition IV.1

Let's first derive the value for workers. Individual workers' consumption is equal to

$$c_{j,t}=\left(\frac{1}{\Phi}\right)\bar{w}+\tau_{j,t}.$$

Since total government transfers are equal to $B_{j,t+1}/R_{j,t} - B_{j,t}$ and the population is $1+\Phi$, each worker gets $\tau_{j,t} = (B_{j,t+1}/R_{j,t} - B_{j,t})/(1+\Phi)$. Substituting and collecting $1/\Phi$ we obtain

$$c_{j,t} = \left(\frac{1}{\Phi}\right)\left[\bar{w} + \nu\left(\frac{B_{j,t+1}}{R_{j,t}} - B_{j,t}\right)\right]. \tag{M1}$$

To derive the workers' value, we start from the terminal period $t = T$, where

$$\widetilde{W}_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}) = \ln(c_{j,T}).$$

Substituting (M1) we can rewrite it as

$$W_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}) = \ln\left(\bar{w} + \frac{\nu B_{j,T+1}}{R_{j,T}} - \nu B_{j,T}\right),$$

where $W_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}) = \widetilde{W}_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}) + \ln(\Phi)$.

We now consider the earlier period $t = T-1$, where the workers' value is

$$\widetilde{W}_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) = \ln(c_{j,T-1}) + \beta\widetilde{W}_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}).$$

Substituting (M1), the workers' value can be rearranged as

$$W_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) = \ln\left(\bar{w} + \frac{\nu B_{j,T}}{R_{j,T-1}} - \nu B_{j,T-1}\right) + \beta W_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}),$$

where $W_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) = \widetilde{W}_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) + (1+\beta)\ln(\Phi)$.

Continuing with $t = T-2, .., 1$ we can derive the general expression

$$W_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) = \ln\left(\bar{w} + \frac{\nu B_{j,t+1}}{R_{j,t}} - \nu B_{j,t}\right) + \beta W_{j,t+1}(\mathbf{B}_{t+1}, \mathbf{B}_{t+2}), \tag{M2}$$

where

$$W_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) = \widetilde{W}_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) + \left(\frac{1}{1-\eta_t}\right)\ln(\Phi). \tag{M3}$$

Replacing $\mathbf{B}_{t+2} = \mathcal{B}_{t+1}(\mathbf{B}_{t+1})$, we obtain the expression for $W_{j,t}$ reported in Proposition IV.1.

We now derive the value for entrepreneurs. By Lemma I.1, entrepreneurs' consumption is equal to $c^i_{j,t} = (1-\eta_t)a^i_{j,t}$, where $a^i_{j,t} = [A(z^i_{j,t}) + p_{j,t} + \tilde{b}^i_{j,t}/k^i_{j,t}]k^i_{j,t}$. Since $\tilde{b}^i_{j,t}/k^i_{j,t}$ is the same across entrepreneurs and aggregate land is 1, we can write $a^i_{j,t} = [A(z^i_{j,t}) + p_{j,t} + \tilde{b}_{j,t}]k^i_{j,t} = \hat{a}^i_{j,t}k^i_{j,t}$, with consumption equal to

$$c^i_{j,t} = (1-\eta_t)\hat{a}^i_{j,t}k^i_{j,t}. \tag{M4}$$

Using Lemma I.1 we can also write the individual gross growth rate of land as

$$\frac{k^i_{j,t+1}}{k^i_{j,t}} = \frac{\eta_t \phi_{j,t} \hat{a}^i_{j,t}}{p_{j,t}}. \tag{M5}$$

Since we have a finite number of periods, we start with the terminal period $t = T$. The indirect utility of an entrepreneur i at $t = T$ is equal to

$$\widetilde{V}^i_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}) = \ln(c^i_{j,T}).$$

Substituting (M4) for $t = T$ we have

$$\widetilde{V}^i_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}) = \ln(1 - \eta_T) + \ln(\hat{a}^i_{j,T}) + \ln(k^i_{j,T}).$$

Subtracting $\ln(k^i_{j,T})$ on both sides and integrating over $z^i_{j,t}$, we define the expected normalized value

$$V_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}) = \mathbb{E}\Big[\widetilde{V}^i_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}) - \ln(k^i_{j,T})\Big] = \ln(1 - \eta_T) + \mathbb{E}\ln(\hat{a}^i_{j,T}).$$

Here the expectation operator $\mathbb{E}$ represents the integration over all individual entrepreneurs indexed by i. Once we integrate, the resulting value does not depend on individual characteristics nor the distribution of $k^i_{j,t}$. Thus, we have dropped the superscript i.

We move next to the earlier period $t = T - 1$. The indirect utility of an entrepreneur i can be written as was derived above

$$\widetilde{V}^i_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) = \ln(c^i_{j,T-1}) + \beta\mathbb{E}\widetilde{V}^i_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}).$$

Substituting (M4) for $t = T - 1$ we have

$$\widetilde{V}^i_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) = \ln(1 - \eta_{T-1}) + \ln(\hat{a}^i_{j,T-1}) + \ln(k^i_{j,T-1}) + \beta\mathbb{E}\widetilde{V}^i_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}).$$

Subtracting $(1+\beta)\ln(k^i_{j,T-1})$ from both sides and adding and subtracting $\beta\ln(k^i_{j,T})$ on the right-hand side we obtain

$$\begin{aligned}\widetilde{V}^i_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) - (1+\beta)\ln(k^i_{j,T-1}) \quad &= \quad \ln(1 - \eta_{T-1}) + \ln(\hat{a}^i_{j,T-1}) + \beta\ln\left(\frac{k^i_{j,T}}{k^i_{j,T-1}}\right) \\ &\quad + \beta\mathbb{E}\Big[\widetilde{V}^i_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}) - \ln(k^i_{j,T})\Big].\end{aligned}$$

Using eq. (M5) to eliminate $k^i_{j,T}/k^i_{j,T-1}$ and integrating over i we get,

$$V_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) = \ln(1 - \eta_{T-1}) + (1+\beta)\mathbb{E}\ln(\hat{a}^i_{j,T-1}) + \beta\ln\left(\frac{\eta_{T-1}\phi_{j,T-1}}{p_{j,T-1}}\right) + \beta V_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}),$$

where we have defined

$$V_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) = \mathbb{E}\Big[\widetilde{V}^i_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) - (1+\beta)\ln(k^i_{j,T-1})\Big].$$

The next step is to consider $t = T - 2$ and continue backward until $t = 1$. For a generic t we have

$$\text{(M6)} \quad V_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) = \ln(1 - \eta_t) + \left(\frac{1}{1 - \eta_t}\right) \left[\eta_t \ln\left(\frac{\eta_t \phi_{j,t}}{p_{j,t}}\right) + \mathbb{E}_t \ln \hat{a}^i_{j,t}\right] + \beta V_{j,t+1}(\mathbf{B}_{t+1}, \mathbf{B}_{t+2}),$$

where

$$\text{(M7)} \qquad V_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) = \mathbb{E}\left[\widetilde{V}^i_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) - \left(\frac{1}{1 - \eta_t}\right) \ln(k^i_{j,t})\right].$$

Replacing $\mathbf{B}_{t+2} = \mathcal{B}_{t+1}(\mathbf{B}_{t+1})$, we obtain the expression for $V_{j,t}$ reported in Proposition IV.1.

The government objective is

$$\Phi \tilde{W}_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) + \mathbb{E}\widetilde{V}^i_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}).$$

Remember that the objective of the government is the integral of the "non-normalized" values for workers and entrepreneurs. Using (M3) and (M7), the objective can be rewritten as

$$\Phi W_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) + V_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) + \left(\frac{1}{1 - \eta_t}\right) \Big(\mathbb{E} \ln(k^i_{j,t}) - \Phi \ln(\Phi)\Big).$$

The last term does not depend on $\mathbf{B}_{t+1}$. Thus, the optimal debt does not depend on this term and we can focus on the first two terms as reported in Proposition IV.1.

N. Numerical algorithm

To simplify the exposition we describe the numerical procedure when there are only two countries ($N = 2$). We first form a two-dimensional, equally spaced grid over the states B_1 and B_2, which we denote by $\mathbf{S}$. We then solve the model at any grid point for the states $(B_1, B_2) \in \mathbf{S}$ backward, starting from the terminal period $t = T$. Before describing the specific solution at any $t = T, T - 1, ..., 1$, we state the following property based on Lemma 2.1, Proposition 2.1, Corollary 2.1, and Proposition 4.1.

PROPERTY 1: *Given $B_{j,t}$, $B_{j,t+1}$, $p_{j,t+1}$, $V_{j,t+1}$, $W_{j,t+1}$, $j \in \{1, 2\}$, we can solve for all variables* *at time t using the equations*

$$\eta_t = \frac{\beta}{1+\frac{\beta^{T-t}}{\sum_{s=1}^{T-t}\beta^{s-1}}} \tag{N1}$$

$$\tilde{b}_{j,t} = \begin{cases} \nu B_{j,t}, & \text{In autarky} \\ \nu\left(\frac{B_{1,t}+B_{2,t}}{2}\right), & \text{With mobility} \end{cases} \tag{N2}$$

$$\tilde{b}_{j,t+1} = \begin{cases} \nu B_{j,t+1}, & \text{In autarky} \\ \nu\left(\frac{B_{1,t+1}+B_{2,t+1}}{2}\right), & \text{With mobility} \end{cases} \tag{N3}$$

$$\phi_{j,t} = \mathbb{E}_t\left[\frac{A(z^i_{j,t+1})+p_{j,t+1}}{A(z^i_{j,t+1})+p_{j,t+1}+\tilde{b}_{j,t+1}}\right], \tag{N4}$$

$$p_{j,t} = \frac{\eta_t\phi_{j,t}(\bar{A}+\tilde{b}_{j,t})}{(1-\eta_t\phi_{j,t})}, \tag{N5}$$

$$R_{j,t} = \frac{(1-\eta_t\phi_{j,t})\tilde{b}_{j,t+1}}{\eta_t(1-\phi_{j,t})(\bar{A}+\tilde{b}_{j,t})}, \tag{N6}$$

$$\hat{a}^i_{j,t} = A(z^i_{j,t})+p_{j,t}+\tilde{b}_{j,t}, \tag{N7}$$

$$V_{j,t} = \ln(1-\eta_t)+\left(\frac{1}{1-\eta_t}\right)\left[\eta_t\ln\left(\frac{\eta_t\phi_{j,t}}{p_{j,t}}\right)+\mathbb{E}_t\ln\hat{a}^i_{j,t}\right]+\beta V_{j,t+1}, \tag{N8}$$

$$W_{j,t} = \ln\left(\bar{w}+\nu\left(\frac{B_{j,t+1}}{R_{j,t}}-B_{j,t}\right)\right)+\beta W_{j,t+1}, \tag{N9}$$

where $\nu = \frac{\Phi}{1+\Phi}$, $\bar{A} = \sum_\ell A(z_\ell)\mu_\ell$, $\bar{w} = (1-\theta)\bar{z}^\theta$.

We can solve exactly for the above variables sequentially, once we know $B_{j,t}$, $B_{j,t+1}$, $p_{j,t+1}$, $V_{j,t+1}$ and $W_{j,t+1}$. For the following description of the computational algorithm, it will be convenient to define the vectors $\mathbf{X}_t = \{B_{j,t}, B_{j,t+1}, p_{j,t+1}, V_{j,t+1}, W_{j,t+1}\}_{j=1}^2$ and $\mathbf{Y}_t = \{p_{j,t}, V_{j,t}, W_{j,t}\}_{j=1}^2$. Using Property 1, we can express $\mathbf{Y}_t$ as a function of $\mathbf{X}_t$,

$$\mathbf{Y}_t = \Upsilon(\mathbf{X}_t).$$

We describe next the solution at each time t, starting from the terminal period T.

Solution at $t = T$

In the terminal period governments fully repay their debts. Thus, $B_{j,T+1} = 0$. Furthermore we know that $p_{j,T+1} = V_{1,T+1} = W_{j,T+1} = 0$. Therefore, $\mathbf{X}_T = \{B_{j,T}, 0, 0, 0, 0\}_{j=1}^2$. We can then use the function $\Upsilon(\mathbf{X}_T)$ from property 1 to solve for $\mathbf{Y}_T = \{p_{j,T}, V_{j,T}, W_{j,T}\}_{j=1}^2$ at each grid point $(B_1, B_2) \in \mathbf{S}$.

The solution obtained for $\mathbf{Y}_T$ at each grid point $(B_1, B_2) \in \mathbf{S}$ is used to form the approximate function

$$\mathbf{Y}_T = \Gamma_T(B_{1,T}, B_{2,T}).$$

The reason we need to create this approximate function is that, when we move to the next step $t = T - 1$, we need to determine $\mathbf{Y}_T$ also for values of $B_{1,T}$ and $B_{2,T}$ that are not on the grid $\mathbf{S}$. The approximate function is created with bilinear interpolation of the solutions $\mathbf{Y}_T$ obtained at the grid points $(B_1, B_2) \in \mathbf{S}$. Armed with the approximate function $\Gamma_T(B_{1,T}, B_{2,T})$, we can move to period $t = T - 1$.

Solution at $t < T$

The main difference from the terminal period T is that now we need to solve for the optimal debts $B_{1,t+1}$ and $B_{2,t+1}$ chosen by governments. To find the optimal debt chosen by each government at each grid point $(B_1, B_2) \in \mathbf{S}$, we implement the following steps.

1) We first solve for the optimal response functions to the debt chosen by the other country. To find the optimal response functions we need to find the government objective $O_{j,t}(B_{1,t+1}, B_{2,t+1}) = \Phi W_{j,t} + V_{j,t}$. The response function of country 1 and country 2 are defined, respectively, as

$$\begin{aligned}\varphi_{1,t}(B_{2,t+1}) &= \max_{B_{1,t+1}} O_{1,t}(B_{1,t+1}, B_{2,t+1}) \\ \varphi_{2,t}(B_{1,t+1}) &= \max_{B_{2,t+1}} O_{2,t}(B_{1,t+1}, B_{2,t+1}).\end{aligned}$$

The optimization is performed by searching over an equally spaced grid $\mathbf{B}$. This grid is finer than the two-dimensional grid for the states $\mathbf{S}$ so we can obtain a more accurate approximation to the maximization problem. The detailed steps are as follows:

a) Given $(B_{1,t+1}, B_{2,t+1}) \in \mathbf{B}\times\mathbf{B}$, we find $\mathbf{Y}_{t+1}$ using the approximate function $\Gamma_{t+1}(B_{1,t+1}, B_{2,t+1})$ found in the previous step $t + 1$.

b) Given $\mathbf{Y}_{t+1}$ we have all the terms we need to construct the vector $\mathbf{X}_t$. Thus we can find $\mathbf{Y}_t$ using the function $\Upsilon_t(\mathbf{X}_t)$ from Property 1.

c) The vector $\mathbf{Y}_t$ contains the necessary elements to compute the government objectives $O_{j,t}(B_{1,t+1}, B_{2,t+1})$ for $(B_{1,t+1}, B_{2,t+1}) \in \mathbf{B} \times \mathbf{B}$.

d) Now that we know the government objectives at the grid points, we compute the optimal response functions as

$$\begin{aligned}\varphi_{1,t}(B_{2,t+1}) &= \max_{B_{1,t+1}\in\mathbf{B}} O_{1,t}(B_{1,t+1}, B_{2,t+1}) \\ \varphi_{2,t}(B_{1,t+1}) &= \max_{B_{2,t+1}\in\mathbf{B}} O_{2,t}(B_{1,t+1}, B_{2,t+1}),\end{aligned}$$

which are defined only over the grid $\mathbf{B}$. To make the response functions continuous, we join the grid values with piece-wide linear segments.

2) The optimal response functions allow us to compute the equilibrium policies chosen by the two governments. They are the fix point $(B^*_{1,t+1}, B^*_{2,t+1})$ to

$$\begin{aligned} B^*_{1,t+1} &= \varphi_{1,t}(B^*_{2,t+1}) \\ B^*_{2,t+1} &= \varphi_{2,t}(B^*_{1,t+1}). \end{aligned}$$

After making the response function continuous, we find the solution $(B^*_{1,t+1}, B^*_{2,t+1})$ using a nonlinear solver. Of course, the solution is not necessarily on the grid $\mathbf{B} \times \mathbf{B}$.

3) Given the solution for $(B^*_{1,t+1}, B^*_{2,t+1})$, we can compute $\mathbf{Y}_{t+1} = \Gamma_{t+1}(B^*_{1,t+1}, B^*_{2,t+1})$ and construct the vector $\mathbf{X}_t$ associated with the equilibrium policies. This allows us to compute $\mathbf{Y}_t = \Upsilon_t(\mathbf{X}_t)$.

Once we have completed the above steps and found the vector $\mathbf{Y}_t$ for each grid point $(B_1, B_2) \in \mathbf{S}$, we can then construct the approximate (bi-linearly interpolated) function

$$\mathbf{Y}_t = \Gamma_t(B_{1,t}, B_{2,t}).$$

We can then move to the earlier step until we reach the initial period $t = 1$.

N1. Some properties of the equilibrium

Figure N1 plots the best response functions of the two countries in the autarky regime and in the regime with capital mobility. The parameterization of the model is described in Section 5.2 of the paper. The response functions are computed at steady-state values—under autarky and mobility, respectively—of debt in each country. The best response function under autarky does not depend on the other country's choice of debt, as shown in the left panel of Figure N1. In the regime with capital mobility, however, there is strategic interaction between the two countries, and therefore the response functions depend on the debt chosen by the other country. The right panel of Figure N1 shows that the response functions intersect only once, suggesting that the Nash equilibrium is unique. The shape of the response functions when current debt differs from steady state is similar (graph available upon request).

Figure N2 plots the equilibrium policy function $B_{j,t+1}$ as a function of $B_{j,t}$, under the two financial arrangements (autarky and mobility) and assuming that the two countries are symmetric (so they choose the same level of debt). As can be seen, the two functions cross the 45 degree line only once, implying that the steady state is unique. The fact that with mobility the crossing point is at a higher level of debt means that governments borrow more when financial markets are integrated, as shown in the two-period version of the model. The policy function in the regime with capital mobility also illustrates the dynamics of debt after liberalization. Starting from the autarkic steady state (where the "autarky" policy function crosses the 45 degree line), capital liberalization induces immediate higher borrowing. This follows from the fact that the policy function with mobility is above the 45 degree line when current debt is at the autarkic steady state. This jump brings the two economies closer to the new steady state with mobility (where the "mobility" policy function intersects the 45 degree line), but does not imply immediate convergence. Concavity of the policy function implies that debt increases, at a decreasing rate, until the new steady state is reached.

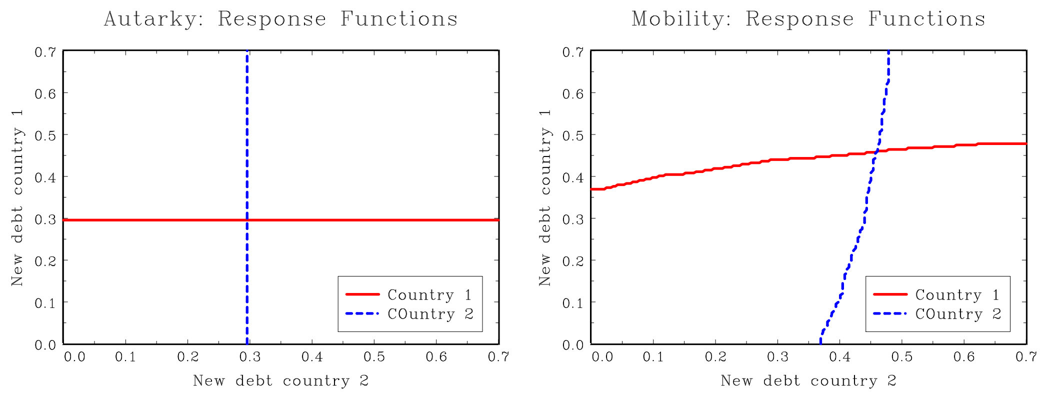

Figure N1. Optimal response functions at the steady state with autarky and with mobility.

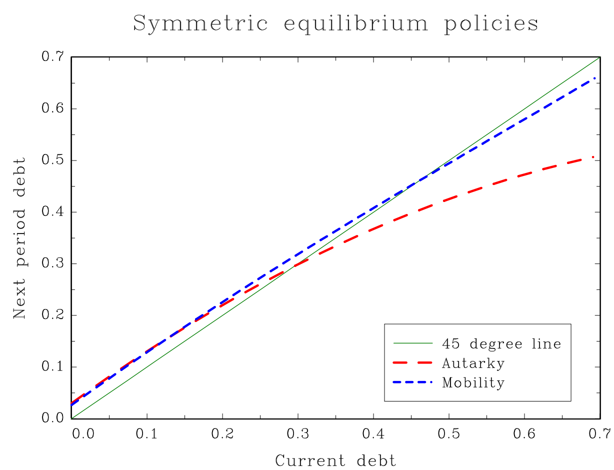

Figure N2. Equilibrium policy function at the steady state with autarky and with mobility.

O. Empirical analysis

The theoretical analysis has shown that greater mobility of capital and higher income inequality raise government borrowing. In this section we conduct an empirical investigation of these theoretical predictions using cross-country data for the OECD countries. In subsection O.O1 we test whether the issuance of government debt is positively associated with capital market liberalization and income inequality. In subsection O.O2 we test whether the elasticity of the interest rate to the issuance of public debt declines with capital market liberalization.

O1. Liberalization, inequality, and the supply of debt

To check whether there are statistically significant links between indices of capital market liberalization, income inequality, and government borrowing, we regress the growth rate of real government debt on two main variables: (i) an index that captures the change in capital mobility, dMOB, and (ii) changes in some measure of inequality, $dINEQ$. We estimate the following fixed effect regression equation:

$$\begin{aligned}\text{dDEBT}_{j,t} &= \alpha_D \cdot \text{DEBT}_{j,t-1} + \alpha_G \cdot \text{dGDP}_{j,t-1} + \alpha_M \cdot \text{dMOB}_t \\ &\quad + \alpha_I \cdot dINEQ_t + \alpha_X \cdot X_{j,t} + u_{j,t}.\end{aligned}$$

- $\text{dDEBT}_{j,t}$: Log-change in real public debt of country j in year t.
- $\text{DEBT}_{j,t-1}$: Ratio of public debt to the GDP of country j in year $t-1$.
- $\text{dGDP}_{j,t}$: Log-change in the GDP of country j in year t.
- dMOB_t: Change in the index of capital mobility in year t or $t-1$.
- dINEQ_t: Change in a measure of inequality in year t or $t-1$.
- $X_{j,t}$: Set of control variables for country j.
- $u_{j,t}$: Residuals containing country and year fixed effects.

A few remarks are in order. First, we relate the *change* in public debt to the *change* in the liberalization index, instead of the *level* of the index. This better captures the dynamics predicted by the model. In fact, in the long run, there is no relation between the degree of capital mobility and the change in debt, since the stock of debt converges to the steady state.

The second remark pertains to the construction of the index of financial liberalization. This index is not country-specific as can be noticed from the absence of the country subscript j. Instead, we construct the index as the average of country-specific indices for all countries included in the sample, weighted by their size (measured by total GDP). The motivation for adopting this measure of capital liberalization can be explained as follows.

Indicators of financial liberalization refer to the private sector, not the public sector. Thus, the fact that one country has very strict international capital controls does not mean that the government is restrained from borrowing abroad. What is relevant for the government's ability to borrow abroad is the openness of other countries. Therefore, to determine the ease with which the government can sell its debt to foreign (private) investors, we have to look at the capital controls imposed by other countries. This is done by computing an average index for all countries included in the sample.[1]

A related issue is whether in computing the weighted average of the liberalization index we should exclude the country of reference. For example, to evaluate the importance of capital mobility for the U.S. public debt, we should perhaps average the indices of the OECD countries excluding the U.S. We have chosen not to do so for the following reason. Although the liberalization of other countries is what defines the foreign market for government bonds, the domestic liberalization can still affect domestic issuance through an indirect channel. However, we also tried the alternative index and the results (not reported) are robust.

Regarding the data for the liberalization variable, we use two indices, both based on de jure measures. The first is the liberalization index constructed by Abiad, Detragiache, and Tressel

[1] Another way of showing the irrelevance of the country's own indicator is with the following example. Suppose that country A liberalizes its capital markets, allowing free international mobility of capital. However, all other countries maintain strict controls. Obviously, the government of country A does not have access to the foreign market even if it had liberalized its own market.

(2008). The results based on this index are reported in Table O1. The second index uses the capital account openness indicator constructed by Chinn and Ito (2008), with results reported in Table O2. Income inequality in Table O1 is proxied by the share of income earned by the top 1 percent of the population (specification (5)), compiled by Atkinson, Piketty, and Saez (2011), and by the averages of the gross Gini coefficients (specification (6)) obtained from the "Standardized World Income Inequality Database, version 3.0, July 2010" compiled by Frederick Solt. The data sources are described in the tables.

We estimate the regression equation on a sample that includes 22 OECD countries. The selection of countries in the first set of regressions is based on data availability for government debt and the financial index, which restricts the sample to 26 countries. From this selected group, we exclude four countries: Hungary, Poland, Mexico, and Turkey. The first two countries are excluded, since the available data start in the 1990s, when they became market-oriented economies. Mexico and Turkey are excluded because they were at a lower stage of economic development compared to the other countries in the sample and they experienced various degrees of market turbulence during the sample period. For robustness, however, we also repeated the estimations for the whole sample with 26 countries, and the results are consistent with those obtained with the restricted sample, including 22 countries.

We start by analyzing the effects of financial integration on debt accumulation, but initially excluding inequality dINEQ_t. By doing so we can use a larger sample, since the inequality variable is unavailable for Austria, Belgium, Switzerland, Germany, Greece, and Korea. The sample size consists of 677 observations. In the simplest specification, we also abstract from any controls $X_{j,t}$. In the second specification we include a dummy for the countries that joined the European Monetary System. Since the membership was conditional on fulfilling certain requirements in terms of public debt (Maastricht Treaty), it is possible that the government debt of certain European countries has been affected by joining the EMU.

As can be seen in the first two columns of Tables O1 and O2, the coefficient on the financial index is positive and highly significant, meaning that the change in capital market integration is positively correlated with the change in public debt. Although we do not claim that this proves causation, there is a strong conditional correlation between these two variables. As far as the EMU dummy is concerned, the coefficient is negative, consistent with the view that EMU countries were forced to adjust their public finances before becoming full members.

Next, we add the interaction term between the financial index and the size of the country, measured by real GDP. The motivation to include this term is dictated by the theory. We have seen in Section 3.2 that the effect of capital liberalization is stronger for smaller countries. Since small countries have a lower ability to affect the world interest rate, their governments have more incentive to borrow once they have access to the world financial market. The third column of Tables O1 and O2 shows that the coefficient of the interaction term is negative, as expected from the theory, and statistically significant in some cases.

The fourth specification adds a demographic variable. This is the *old dependency ratio* between the population in the age group 65 and over and the population in the age group 15-64. Although our model abstracts from demographic considerations, there is a widespread belief that aging in industrialized countries is an important force for the rising public debt. This is because the political weight shifts toward older generations that may prefer higher debt. As can be seen from the fourth column of Tables O1 and O2, the coefficient associated with the change in this variable is positive.

However, the inclusion of the old dependency ratio does not affect the sign and significance of the financial index, confirming the importance of capital market liberalization for government borrowing.

The final specification introduces income inequality. With the inclusion of the inequality index we lose some observations, since the index is not available for all countries. As a result, the sample shrinks to 435 observations. The coefficient is positive and statistically significant, indicating that rising income inequality is associated with higher borrowing.

As far as the other variables are concerned, we find that the lagged stock of debt is negatively correlated with its change. This is what we expect if the debt tends to converge to a long-term level. The change in GDP is meant to capture business cycle effects, and it has the expected negative sign: When the economy does well, government revenues increase and automatic expenditures decline so that government debt increases less.

O2. Liberalization and interest rate elasticity

We have shown in the introduction of the paper that the interest rate elasticity has declined over time (see Figure 2 in the paper). We have also observed that we should be cautious in interpreting the empirical elasticities. As already remarked in the first footnote of the paper, the empirical estimation provides only "suggestive" evidence about the demand elasticity of the interest rate since movements in the interest rates can be driven by movements in both demand and supply. Only if the position (but not necessarily the slope) of the demand function has remained stable during the sample period—relative to the supply—can the estimated elasticities reflect the slope of the demand function. With this in mind, we now investigate whether the declining elasticity is associated with increased financial globalization as predicted by the theory.

To check the importance of financial globalization, we regress the percentage change in a country's interest rate on the percentage change in the country's real public debt and its interaction with measures of financial market integration. More specifically, we estimate the fixed-effect regression

$$\mathrm{dINT}_{j,t} = \alpha_1 \mathrm{dDEBT}_{j,t} + \alpha_2 \mathrm{dDEBT}_{j,t}\mathrm{MOB}_{j,t} + \alpha_3 \mathrm{dDEBT}_{j,t}\mathrm{DEBT}_{-j,t} + \alpha_4 X_{j,t} + u_{j,t},$$

where

- $\mathrm{dINT}_{j,t}$: percentage change in the interest rate of country j in year t.
- $\mathrm{dDEBT}_{j,t}$: percentage change in public real debt of country j in year t.
- $\mathrm{MOB}_{j,t}$: financial integration index in year t from Abiad, Detragiache, and Tressel (2008).
- $\mathrm{DEBT}_{-j,t}$: average global public real debt in year t excluding country j.
- $X_{j,t}$: set of control variables for country j in year t.
- $u_{j,t}$: residuals containing country and year fixed effects.

The most important variable is the interaction term between the growth of public debt and the financial index. According to the theory, we expect the coefficient of this interaction term to be negative. The regression is estimated using the same sample used in the previous subsection.

By looking at the time series for each country, it is clear that in 1999 and 2000 the interest rate for the "satellite" European countries that joined the Euro zone (Spain, Ireland, Italy, and Belgium) converged to the prevalent interest rates of core European countries such as France and Germany. This justifies the inclusion of the EMU dummy. We have also included a dummy variable for Germany for the years 1989, 1990, and 1991, to account for the consolidation of the public debt of East and West Germany.

The results support our main hypothesis: Interest rate elasticity decreases with financial liberalization or global market size. This is true regardless of whether we use a country-specific or the average financial liberalization index from the previous section, or whether we use an actual measure of the size of the global market. In all cases, the coefficients on the interaction terms are negative and statistically significant. The goodness of fit, however, is quite small.

These findings complement the observation of higher cross-country convergence in interest rates during the last three decades. See Obstfeld and Taylor (2005).

Table O1—Country fixed-effect regression. The dependent variable is real public debt growth. The financial index is based on Abiad, Detragiache, and Tressel (2008).

	(1)	(2)	(3)	(4)	(5)	(6)
Lag debt to GDP ratio	-0.149^{***}	-0.146^{***}	-0.149^{***}	-0.170^{***}	-0.162^{***}	$-.159^{***}$
	(0.0374)	(0.0375)	(0.0378)	(0.0383)	(0.0253)	(0.0373)
Lag real GDP growth	-1.235^{***}	-1.210^{**}	-1.216^{***}	-1.159^{**}	-1.381^{**}	-0.958^{*}
	(0.433)	(0.430)	(0.429)	(0.413)	(0.571)	(0.473)
Lag change in financial index	0.688^{**}	0.697^{**}	0.966^{***}	1.180^{***}	1.555^{***}	1.309^{***}
	(0.269)	(0.270)	(0.281)	(0.278)	(0.331)	(0.272)
Lag EMU dummy		-0.0478^{**}	-0.0474^{**}	-0.0521^{**}	-0.084^{***}	-0.055^{***}
		(0.0189)	(0.0190)	(0.0185)	(0.0259)	(0.0183)
Size × Lag change in FI			-6.136	-6.602^{*}	-7.883^{*}	-6.506^{*}
			(3.818)	(3.554)	(3.932)	(3.556)
Change in dependency ratio				0.0695^{**}	0.0636^{**}	0.0707^{**}
				(0.0256)	(0.0223)	(0.0267)
Change in inequality					0.128^{**}	0.013^{*}
					(0.0536)	(0.0071)
Observations	677	677	677	677	435	648
R-squared	0.130	0.132	0.137	0.150	0.199	0.165
Number of countries	22	22	22	22	16	21

Notes: The variable *Financial Index* (FI) is constructed using the liberalization index of Abiad, Detragiache, and Tressel (2008). We compute the financial index for a year as a weighted average of all the country indexes where weights are given by their relative GDP shares. The ratio of debt to GDP is from Reinhart and Rogoff (2011), and real GDP and population data are from the World Development Indicators (World Bank). Real debt is constructed by multiplying the ratio of debt to GDP by real GDP. Size is the lagged logarithm of real GDP weights. The EMU dummy is equal to 1 in the year the country joined the European Monetary Union and 0 otherwise. The old dependency ratio is the population 65 and above divided by the population in the age group 15-64. In specification (5), the inequality index is measured by the logarithm of the top 1 percent income share calculated by Atkinson, Piketty, and Saez (2011), and we include its change at t. In specification (6), inequality is measured by the gross Gini coefficient obtained from the "Standardized World Income Inequality Database, version 3.0, July 2010," and we include its change at $t-1$. The sample period is 1973-2005 and includes the following countries for specifications (1) to (4): Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Korea, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States. Due to data availability, Austria, Belgium, Switzerland, Germany, Greece, and Korea are excluded in specification (5), and only Korea is excluded in specification (6). Robust standard errors are in parentheses.

* Significant at 10 percent. ** Significant at 5 percent. *** Significant at 1 percent.
Source: Authors' calculations.

TABLE O2—COUNTRY FIXED-EFFECT REGRESSION. THE DEPENDENT VARIABLE IS REAL PUBLIC DEBT GROWTH. THE FINANCIAL INDEX IS BASED ON CHINN AND ITO (2008).

	(1)	(2)	(3)	(4)	(5)
Lag debt to GDP ratio	−0.150***	−0.147***	−0.148***	−0.166***	−0.157***
	(0.0366)	(0.0366)	(0.0368)	(0.0380)	(0.0267)
Lag real GDP growth	−1.262***	−1.235***	−1.230***	−1.189***	−1.400**
	(0.428)	(0.425)	(0.423)	(0.410)	(0.585)
Change in financial index	0.113**	0.116**	0.177***	0.205***	0.253***
	(0.0539)	(0.0539)	(0.0575)	(0.0630)	(0.0606)
Lag EMU dummy		−0.0485**	−0.0487**	−0.0528**	−0.0854***
		(0.0189)	(0.0192)	(0.0187)	(0.0264)
Size × Change in fin index			−1.375**	−1.428**	−1.437**
			(0.728)	(0.680)	(0.617)
Change in dependency ratio				0.0594**	0.0535**
				(0.0259)	(0.0250)
Change in top 1 percent share					0.106*
					(0.0599)
Observations	677	677	677	677	435
R-squared	0.130	0.132	0.137	0.150	0.199
Number of countries	22	22	22	22	16

Notes: The variable *Financial Index* is constructed using the capital account openness index of Chinn and Ito (2008). For the other variables, see notes in Table O1.
Source: Authors' calculations.

TABLE O3—COUNTRY FIXED-EFFECT REGRESSION. THE DEPENDENT VARIABLE IS PERCENTAGE CHANGE OF REAL INTEREST RATES. THE FINANCIAL INDEX MEASURES ARE BASED ON ABIAD, DETRAGIACHE, AND TRESSEL (2008).

	(1)	(2)	(3)
Percent change in B_t	0.112*	0.0504*	0.0541**
	(0.0588)	(0.0245)	(0.0253)
Percent change in $B_t \times MOB_{j,t}$	−0.129*		−0.1486**
	(0.0683)		(0.0631)
Percent change in $B_t \times B_{-j,t}$		$-7.8e^{-16}$**	$-7.9e^{-16}$**
		($3.7e^{-16}$)	($3.8e^{-16}$)
Lag EMU dummy	0.0008	0.0008	0.0009
	(0.0020)	(0.0020)	(0.0020)
German reunification dummy	0.0084***	0.0086***	0.0083***
	(0.0005)	(0.0004)	(0.0004)
Constant	−0.0013***	−0.0014***	−0.0013**
	(0.0004)	(0.0004)	(0.0004)
Observations	459	459	459
R-squared	0.013	0.012	0.012
Number of countries	22	22	22

Notes: The variable $MOB_{j,t}$ in specifications (1) and (3) uses the liberalization index of Abiad, Detragiache, and Tressel (2008). In specification (1), $MOB_{j,t}$ is the weighted average index of financial liberalization used in Tables O1 and O2 where weights are given by their relative GDP shares. In specification (2), $B_{-j,t}$ is the average of the real debt at time t of the countries in the sample excluding country j. And finally specification (3) includes two interaction terms, $MOB_{j,t}$ and $B_{-j,t}$, where $MOB_{j,t}$ is the country-specific index of financial liberalization weighted by size as before, and $B_{-j,t}$ is defined exactly as in specification (2). The sample period is 1974-2003. For the other variables, see notes in Table O1. Robust standard errors are in parentheses.

* Significant at 10 percent. ** Significant at 5 percent. *** Significant at 1 percent.
Source: Authors' calculations.