Financial Globalization, Inequality, and the Rising Public Debt*

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Abstract

During the last three decades the stock of government debt has increased in most developed countries. During the same period we also observe a significant liberalization of international financial markets. In this paper we propose a multi-country model with incomplete markets and show that governments may choose higher levels of debt when financial markets become internationally integrated. We also show that public debt increases with the volatility of uninsurable idiosyncratic income (risk). To the extent that the increase in income inequality observed in some industrialized countries during the last three decades has been associated with higher idiosyncratic risk, the paper suggests another potential mechanism for the rise in public debt.

Keywords: Government debt, financial integration, income inequality.

JEL classification: E60, F59.

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1 Introduction

During the last three decades, we have observed an increase in the stock of public debt in most developed countries. As shown in the top panel of Figure 1, the stock of public debt in OECD countries has increased from around 30 percent of GDP in the early 1980s to about 50 percent in 2005. Similar increases are observed in the United States and Europe.

Historically, the dynamics of public debt have been closely connected to war financing and business cycle fluctuations, where budget deficits and surpluses are instrumental in minimizing the distortionary effects of taxation. The tax-smoothing theory developed by Barro (1979) provides a rationale for such dynamics. However, when we look at the upward trend in public debt that started in the early 1980s, it is difficult to rationalize this trend with the tax-smoothing argument, since this period has been characterized by relatively peaceful times and low macroeconomic volatility.

The last three decades have also been characterized by two additional trends. The first trend is the international liberalization of financial markets as shown in the second panel of Figure 1. This panel plots the index of financial liberalization constructed by Abiad, Detragiache and Tressel (2008) for the group of OECD countries, the United States and Europe, and shows that world financial markets have become much less regulated starting in the early 1980s. This pattern can also be seen with other indicators of international capital mobility as shown in Obstfeld and Taylor (2005). The second trend that took place during the last three decades is the increase in inequality as shown in the last panel of Figure 1, which plots the share of income earned by the top 1 percent of the population as reported by Atkinson, Piketty, and Saez (2011).

In this paper we propose a theory in which government borrowing responds positively to financial liberalization. Furthermore, to the extent that income inequality is associated with higher uninsurable income risks, public debt also responds positively to income inequality.

We study a multi-country model where agents face uninsurable idiosyncratic risks and public debt is held to smooth consumption. To keep tractability, we assume that there are two types of agents: those who face idiosyncratic risks (entrepreneurs) and those who are less exposed to these risks (workers). Both agents have some preferences for public debt. Agents who face higher idiosyncratic risk (entrepreneurs) benefit from public debt because it provides an instrument to smooth consumption. The demand for safe assets is reflected in the equilibrium interest rate being lower than the intertemporal discount rate. Because of the lower interest rate, agents who face lower risk, the workers, would also benefit from public debt if they cannot borrow directly in private markets. Effectively, government debt acts as a substitute for private debt. The benefits for workers, however, fade away as the stock of debt increases because this requires higher interest rates. Thus, once the debt has reached a certain level, workers no longer support further increases in public debt; the internalization of the increasing cost of debt serves as a limit to its growth.

How does financial integration affect the government's incentive to issue debt? The central mechanism is the elasticity of the interest rate to the supply of debt. In a globalized world, both the demand and supply of government debt come not only from domestic agents (investors and governments) but also from their foreign counterparts. Therefore, when governments do not coordinate their policies, each country faces a lower elasticity of the interest rate to the supply of 'their own' government debt. Since the interest rate is less responsive to a country's debt, governments

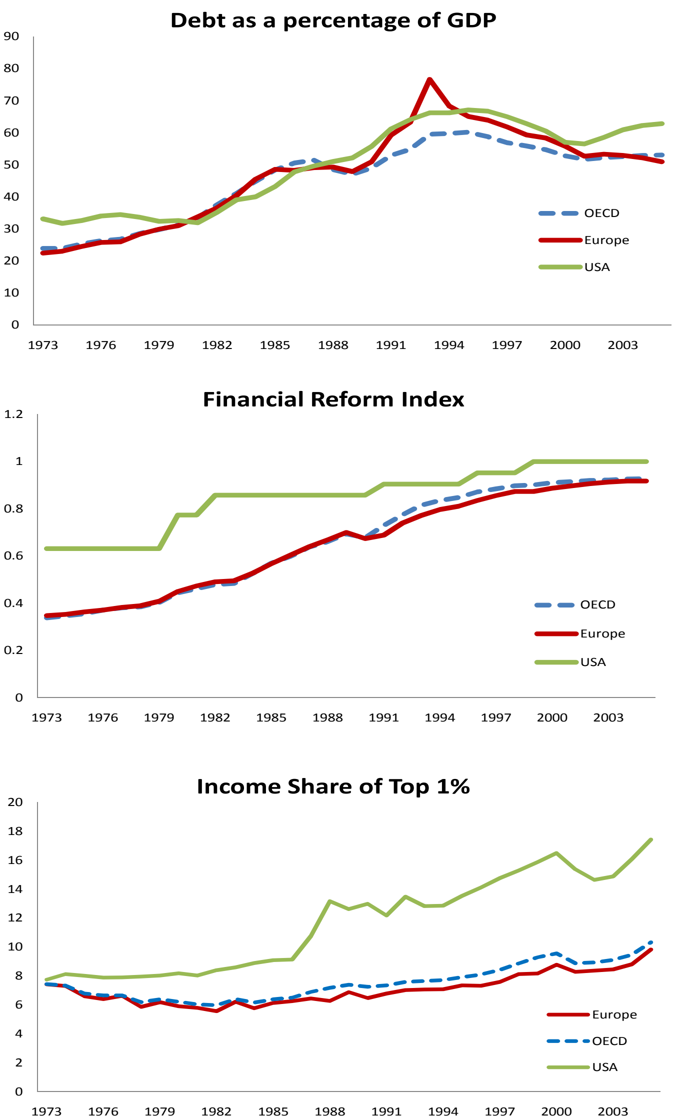

Figure 1: Public debt, financial liberalization, and inequality in advanced economies. Appendix A provides the definition of variables and the data sources.

have more incentives to increase borrowing provided that workers have sufficient political influence.

Figure 2 suggests that the responsiveness of the interest rate to public debt has indeed declined over time. This figure plots the fitted values of a simple cross-country regression where the dependent variable is the ratio of the change in the real interest rate over the change in real government debt; the dependent variable is time (calendar year). A more extensive analysis of the impact of international liberalization on the elasticity of the interest rate is provided in the online Appendix. The findings of the empirical analysis complement the observation of higher cross-country convergence in interest rates. See Obstfeld and Taylor (2005).

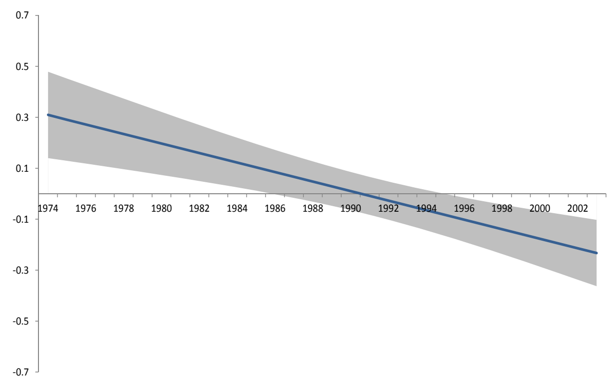

Figure 2: Interest rate elasticity over time. The solid line is the fitted value of a linear regression in which the dependent variable is the ratio of the change in the real interest rate over the change in real government debt; the dependent variable is time (calendar year). The shaded area is the 95% confidence band. Appendix A provides the definition of variables and the data sources.

How does income risk affect preferences for public debt? When entrepreneurs face higher risk, they increase the demand for safe assets (government bonds). Then the issuance of more debt is beneficial for both entrepreneurs and workers. For entrepreneurs, it is beneficial because additional public bonds provide safe assets available for consumption smoothing and counterbalance the decline in the interest rate induced by the increased demand for bonds. For workers, it is beneficial because, through government debt, they can borrow at a lower interest rate.

An increase in uninsurable income risk leads to higher government borrowing independently of the international capital markets regime. However, if financial markets are integrated, our model could generate an increase in the government debt of all countries even if the increase in income risk arises in a subset of countries. This is an important property of our model because, although the increase in public debt has been observed in most of the developed countries, the increase in income inequality took place only in a few countries. Provided that the observed income inequality is associated with increased uninsurable risk and financial markets are integrated, a worldwide rise in government borrowing could emerge in response to an increase in income inequality in only a subset of countries. This is another dimension in which financial globalization plays an important role in the determination of government borrowing.

The organization of the paper is as follows. We first describe how the paper relates to the literature. After the literature review, Section 2 describes the model and defines the equilibrium. Section 3 explores a simplified version of the model with few periods, providing simple analytical intuition for the key results of the paper. Section 4.2 performs a quantitative analysis with the infinite horizon model. Section 5 concludes. All technical proofs are relegated to the Appendix.

1.1 Literature review

An influential theoretical literature studies the optimal choice of public debt over the business cycle with contributions by Barro (1979), Lucas and Stokey (1983), Aiyagari, Marcet, Sargent, and Seppala (2002), Angeletos (2002), Chari, Christiano, and Kehoe (1994), and Marcet and Scott (2009). We depart from the tax-smoothing mechanism because we abstract from aggregate fluctuations and distortionary taxation. Instead, we focus on the role of heterogeneity within a country that is assumed away in these papers.

The structure of our model is closer to models studied in Aiyagari and McGrattan (1998) and Shin (2006). In these papers the role of government debt is to partially complete the assets market when agents are subject to uninsurable idiosyncratic risk. The government accumulates debt in order to crowd out private capital, which is inefficiently high due to precautionary savings. In our model, however, we abstract from capital accumulation and the government choice of debt is based on redistribution considerations. In this sense, our paper is related to the literature on optimal redistributive policy in economies with heterogeneous agents such as Krusell and Rios-Rull (1999), Corbae, D'Erasmo, and Kuruscu (2009), and Bachmann and Bai (2012). In our model, however, we do not impose that the government budget has to balance in every period and inter-temporal redistribution (across times) is more important than intra-temporal redistribution (between groups at a particular point in time).

The paper is also related to the literature on the political economy of debt initiated by Alesina and Tabellini (1990), Persson and Svensson (1989), and further developed by Battaglini and Coate (2008), Caballero and Yared (2008), Ilzetzki (2011), Aguiar and Amador (2011) and Song, Storesletten, and Zilibotti (2012). A common feature of these studies is the strategic use of public debt in economies where the interest rate is exogenous and governments with different preferences alternate in power. We abstract from political turnover and consider instead how the supply of government bonds endogenously affects interest rates and redistribution. The interest rate manipulation channel is also present in Krusell, Martin, and Rios-Rull (2006) and in Azzimonti, de Francisco, and Krusell (2008), but it relies on the use of distortionary taxation, which we assume away here.

Another difference with many of the papers that study the optimal public debt is that we consider an open economy environment with large countries. An exception is Chang (1990), who also studies how the international liberalization of capital markets affects government borrowing in an economy with overlapping generations. Although the structure of the model is different, the mechanism through which capital liberalization leads to higher government borrowing is similar. The analysis of Chang (1990), however, abstracts from risk. Kehoe (1989), Mendoza and Tesar (2005) and Quadrini (2005) also study equilibrium government policies with capital mobility, but in models without public debt or with public debt that is not chosen optimally.

The paper is also related to the literature that explores the importance of market incompleteness for international financial flows. Caballero, Farhi, and Gourinchas (2008), Mendoza, Quadrini, and Rios-Rull (2009), and Angeletos and Panousi (2011) have all emphasized the importance of cross-country heterogeneity in financial markets for global imbalances. Our study differs from these contributions in two dimensions. First, our focus is on public debt, while the above contributions have focused on private debt. There is an important difference between public and private debt that is crucial for our results: while in private borrowing atomistic agents do not internalize the impact that the issuance of debt has on the interest rate, governments do. Part of our results are driven by the fact that governments do not take the interest rate as given, as individual agents do. The second difference is that the goal of our study is to explain the gross stocks of (public) debt, while the contributions mentioned above focus on net volumes. In these models financial liberalization leads to higher liabilities in one country but lower liabilities in others, with the difference defining the imbalance. The global volume of credit, however, does not change significantly. In contrast, in our model capital liberalization (and income risk) generates an increase in the global stock of debt even if countries are symmetric and liberalization does not generate international imbalances.

2 Theoretical environment

Consider an economy composed of N symmetric countries indexed by $j \in \{1, .., N\}$ that lasts for T periods. The infinite horizon case is obtained as a special case with $T \to \infty$. Agents face uninsurable idiosyncratic risk but some agents are more exposed to risk than others.

To model heterogeneous exposure to risk in a tractable manner, we assume that there are two types of agents: a measure Φ of workers and a measure 1 of entrepreneurs. Workers do not face idiosyncratic uncertainty, while entrepreneurs encounter investment risk. In modeling the entrepreneurs, we adopt the approach proposed by Angeletos (2007), which allows for linear aggregation. We can then analyze the general equilibrium by focusing on a representative worker and a representative entrepreneur, without the need to track the wealth distribution of entrepreneurs.

Although we focus on heterogeneity between workers and entrepreneurs and make the extreme assumption that workers do not face any risk, the model should be interpreted more generally as an environment in which some agents face more risk than others. Because of the different exposure to risk, preferences over government debt differ between workers and entrepreneurs. Thus, the debt chosen by the government will depend on the relative size (or political power) of these two groups.

Both types of agents maximize the expected lifetime utility

$$\mathbb{E}_0 \sum_{t=1}^{T} \beta^t \ln(c_t), \tag{1}$$

where c_t is consumption and $\beta \in (0, 1)$ is the intertemporal discount factor.

In each country j there is a unit supply of land, an international immobile asset traded at price $p_{j,t}$. Entrepreneurs are individual owners of firms, each operating the production function $F(z, k, l)$, where k is the input of land chosen in the previous period, l the input of labor chosen in the current period and supplied by workers, and z is an idiosyncratic productivity shock that is observed after the input of land but before the choice of labor. The productivity shock is independently and identically distributed among agents and over time and takes values in the set $\{z_1, .., z_m\}$, with probabilities $\{\mu_1, .., \mu_m\}$. This is the only source of risk in the model. The function $F(z, k, l)$ is strictly increasing in z, k, l and homogeneous of degree 1 in k and l (constant returns).

Entrepreneurs hire workers paying the wage rate w. The hiring decision is static because it affects only current profits. Given productivity z and land k, the marginal product of labor is equalized to the wage rate, that is, $F_l(z, k, l) = w$. Because the production function is homogeneous of degree 1, the demand for labor is linear in the input of land and can be expressed as $l = l(z, w)\,k$. The entrepreneurial profits are also linear in the input of land and can be expressed as

$$F(z, k, l) - wl = A(z, w)k. \tag{2}$$

Entrepreneur i in country j enters period t with risk-free bonds $b^i_{j,t}$, land $k^i_{j,t}$ and productivity $z^i_{j,t}$, and receives lump-sum transfers $\tau_{j,t}$ from the government. The budget constraint is

$$c^i_{j,t} + p_{j,t}k^i_{j,t+1} + \frac{b^i_{j,t+1}}{R_{j,t}} = A(z^i_{j,t}, w_{j,t})k^i_{j,t} + p_{j,t}k^i_{j,t} + b^i_{j,t} + \tau_{j,t}. \tag{3}$$

Entrepreneurs also face the terminal condition $b^i_{j,T+1} \geq 0$, imposing that any outstanding debt needs to be fully repaid in the terminal period T. In the limiting case with $T \to \infty$, this is replaced by a transversality condition.

Workers are endowed with $1/\Phi$ units of labor[1] supplied inelastically for the wage $w_{j,t}$. Labor is internationally immobile. Workers also receive the same lump-sum transfers $\tau_{j,t}$ received by entrepreneurs but they do not hold assets or borrow. Thus, workers' consumption is equal to

$$c^w_{j,t} = w_{j,t}\left(\frac{1}{\Phi}\right) + \tau_{j,t}. \tag{4}$$

The assumption that workers do not hold assets is without loss of generality. As we will see, the equilibrium interest rate is typically smaller than the intertemporal discount rate, that is, $R_{j,t} < 1/\beta$. Since workers do not face any risk, they will not hold bonds in the long run. The inability to borrow can be rationalized by limited enforcement, leading to a borrowing limit. Here we set the upper bound to zero but later we will consider less tight borrowing constraints.

Governments raise revenues by issuing one-period bonds. The proceeds are used to pay lump-sum transfers and to repay the outstanding debt. Thus, the government budget constraint is

$$(1 + \Phi)\tau_{j,t} + B_{j,t} = \frac{B_{j,t+1}}{R_{j,t}}, \tag{5}$$

where $B_{j,t}$ are the bonds issued at time $t - 1$ and due in period t, and $B_{j,t+1}$ are the new bonds. Governments face the terminal condition $B_{j,T+1} = 0$.

2.1 Competitive equilibrium for given policies

In this section we characterize the competitive equilibrium *given* government policy. This is a necessary first step to characterize *optimal* government debt. The endogenous derivation of government policies will be described in Sections 3 and 4.

The decision problem of workers is trivial because transfers are taken as given and the supply of labor is inelastic. They simply consume their income. The decision problem of entrepreneurs

[1] The assumption that the individual labor supply is $1/\Phi$ is a normalization that keeps the ratio of total land over the aggregate supply of labor equal to 1.

is more complex. Given the initial holdings of land and bonds, the choices of labor, consumption and portfolio holdings (land and bonds) are functions of individual histories $\mathbf{z}^i_{j,t} = \{z^i_{j,1}, ..., z^i_{j,t}\}$. Following is the formal definition of a competitive equilibrium starting from a regime in which bonds cannot be traded in international markets (financial autarky).

Definition 2.1 (Autarky Equilibrium) *Given a sequence of government debt $\{B_{j,t}\}_{t=1}^T$ and $B_{j,T+1} = 0$, a competitive equilibrium without mobility of capital is defined as a sequence of prices $\{w_{j,t}, p_{j,t}, R_{j,t}\}_{t=1}^T$, entrepreneurs' decisions $\{c^i_{j,t}(\mathbf{z}^i_{j,t}), l^i_{j,t}(\mathbf{z}^i_{j,t}), k^i_{j,t+1}(\mathbf{z}^i_{j,t}), b^i_{j,t+1}(\mathbf{z}^i_{j,t})\}_{t=1}^T$, consumption of workers $\{c^w_{j,t}\}_{t=1}^T$, and transfers $\{\tau_{j,t}\}_{t=1}^T$ for $j \in \{1, .., N\}$ such that:*

i. Entrepreneurs' decisions maximize (1) subject to the budget constraint (3) and the terminal condition $b^i_{j,T+1} \geq 0$. Workers' consumption satisfies the budget constraint (4).

ii. Prices clear domestic markets for labor, $\int_i l^i_{j,t}(\mathbf{z}^i_{j,t})di = 1$, land, $\int_i k^i_{j,t+1}(\mathbf{z}^i_{j,t})di = 1$, and bonds, $\int_i b^i_{j,t+1}(\mathbf{z}^i_{j,t})di = B_{j,t+1}$.

iii. Domestic bonds and transfers satisfy the government's budget (5).

If financial markets are financially integrated, governments can sell their bonds to (borrow from) domestic and foreign entrepreneurs. The definition of a competitive equilibrium is similar. The only modification is that the bond market clears internationally instead of country by country, that is, $\sum_{j=1}^N \int_i b^i_{j,t+1}(\mathbf{z}^i_{j,t})\mathrm{d}i = \sum_{j=1}^N B_{j,t+1}$, and interest rates are equalized worldwide, $R_{1,t} = ... = R_{N,t}$.

For the analysis that follows, it will be convenient to define

$$\tilde{b}^i_{j,t} = b^i_{j,t} - \frac{B_{j,t}}{1+\Phi}.$$

The variable $b^i_{j,t}$ is the individual demand from entrepreneur i and $B_{j,t}/(1+\Phi)$ is the per capita stock of domestic public debt (remember that $1+\Phi$ is the country population). Thus, $\tilde{b}^i_{j,t}$ is the demand of bonds from entrepreneur i in *excess* of the per capita stock of domestic public debt. We will refer to $\tilde{b}^i_{j,t}$ as the 'excess demand for bonds'. The aggregate excess demand is $\tilde{b}_{j,t} = \int_i \tilde{b}^i_{j,t}$.

Using $\tilde{b}^i_{j,t}$ and the government budget (5), we rewrite the entrepreneurs' budget constraint as

$$c^i_{j,t} + p_{j,t}k^i_{j,t+1} + \frac{\tilde{b}^i_{j,t+1}}{R_{j,t}} = A(z^i_{j,t}, w_{j,t})k^i_{j,t} + p_{j,t}k^i_{j,t} + \tilde{b}^i_{j,t}. \tag{6}$$

We can now show that entrepreneurs' decision rules are linear in wealth $a^i_{j,t} = A(z^i_{j,t}, w_{j,t})k^i_{j,t} + p_{j,t}k^i_{j,t} + \tilde{b}^i_{j,t}$, which generalizes the result of Angeletos (2007) to an economy with fiscal transfers.

Lemma 2.1 *Given the sequence of prices $\{w_{j,t}, p_{j,t}, R_{j,t}\}_{t=1}^T$, entrepreneurs' policies are*

$$\begin{aligned} c^i_{j,t} &= (1-\eta_t)a^i_{j,t}, \\ p_{j,t}k^i_{j,t+1} &= \phi_{j,t}\eta_t a^i_{j,t}, \\ \frac{\tilde{b}^i_{j,t+1}}{R_{j,t}} &= (1-\phi_{j,t})\eta_t a^i_{j,t}, \end{aligned}$$

where $\eta_t = \frac{\beta}{1+\frac{\beta^{T-t}}{\sum_{s=1}^{T-t}\beta^{s-1}}}$ *and* $\phi_{j,t}$ *satisfies* $\mathbb{E}_t\left[\frac{R_{j,t}}{\left(\frac{A(z^i_{j,t+1},w_{j,t+1})+p_{j,t+1}}{p_{j,t}}\right)\phi_{j,t}+R_{j,t}(1-\phi_{j,t})}\right] = 1.$

Proof 2.1 *Appendix B.*

Aggregating agents' decisions using Lemma 2.1 and imposing market clearing, we establish the following result, which is also a generalization of Angeletos (2007).

Proposition 2.1 *Given the sequence of public debt* $\{B_{1,t},..,B_{N,t}\}_{t=1}^T$, $B_{1,T+1} =,..,= B_{N,T+1} = 0$, *the equilibrium wage is constant,* $w_{j,t} = \bar{w}$, *and the remaining prices and aggregate allocations are*

$$\phi_{j,t} = \mathbb{E}_t\left[\frac{A(z^i_{j,t+1}) + p_{j,t+1}}{A(z^i_{j,t+1}) + p_{j,t+1} + \tilde{b}_{j,t+1}}\right], \tag{7}$$

$$p_{j,t} = \frac{\eta_t\phi_{j,t}(\bar{A} + \tilde{b}_{j,t})}{(1-\eta_t\phi_{j,t})}, \tag{8}$$

$$R_{j,t} = \frac{(1-\eta_t\phi_{j,t})\tilde{b}_{j,t+1}}{\eta_t(1-\phi_{j,t})(\bar{A}+\tilde{b}_{j,t})}, \tag{9}$$

$$c^e_{j,t} = \bar{A} + \tilde{b}_{j,t} - \frac{\tilde{b}_{j,t+1}}{R_{j,t}}, \tag{10}$$

$$c^w_{j,t} = \bar{w} + \left(\frac{\Phi}{1+\Phi}\right)\left(\frac{B_{j,t+1}}{R_{j,t}} - B_{j,t}\right), \tag{11}$$

where $\bar{A} = \sum_\ell A(z_\ell)\mu_\ell$ *and the variables* $c^e_{j,t} = \int_i c^i_{j,t}$ *and* $c^w_{j,t}$ *are, respectively, the aggregate consumption of entrepreneurs and workers.*

Proof 2.1 *Appendix C.*

The proposition holds with and without capital mobility. Without mobility (autarky), the bonds held by residents must be equal to the bonds issued by the domestic government, that is, $\int_i b^i_{j,t} = B_{j,t}$. In terms of aggregate excess demand for bonds, $\tilde{b}_{j,t} = \int_i \tilde{b}^i_{j,t} = \nu B_{j,t}$. To simplify notation we have defined $\nu = \Phi/(1+\Phi)$ the population share of workers. When financial markets are integrated, the bonds held by residents of a country could differ from the bonds issued by their own government. A corollary to Proposition 2.1 characterizes bond holdings with capital mobility.

Corollary 2.1 *With capital mobility, if* $\tilde{b}_{j,1} = \tilde{b}_1$, *then* $\tilde{b}_{j,t} = \nu\left(\frac{\sum_{j=1}^N B_{j,t}}{N}\right)$ *for all* $t > 1$.

Proof 2.1 *Appendix D.*

If the initial aggregate excess demands of bonds are equal across countries, then future excess demands are also equal. This follows from the assumption that countries are homogeneous in endowments and technology and, with integrated financial markets, interest rates are equalized across countries. Since the excess demand $\tilde{b}_{j,t}$ is the difference between the bonds purchased by entrepreneurs and the outstanding government liabilities, this property implies that in countries where governments issue more liabilities, entrepreneurs save more because they anticipate higher payments of future taxes in the form of negative transfers. Notice that this result does not apply if the risk faced by entrepreneurs differs across countries.

3 Optimal policy in the two-period model

We start by analyzing optimal government policy in a special version of the model with only two periods, $T = 2$. This allows us to characterize several properties of the model analytically.

To further simplify the analysis, we assume that in period 1 governments have zero debt, that is, $B_{j,1} = 0$. Furthermore, all entrepreneurs start period 1 with one unit of land, $k^i_{j,1} = 1$, zero bonds, $b^i_{j,1} = 0$, and the same productivity $z^i_{j,1} = \bar{z}$. Under these assumptions, initial entrepreneurs' wealth, including current profits, is $a^i_{j,1} = \bar{A} + p_{j,1}$. Wealth in period 1 is allocated between consumption and savings in the form of bonds, $\tilde{b}^i_{j,2}$, and land, $k^i_{j,2}$. Thus, wealth in period 2 is $a^i_{j,2} = A(z^i_{j,2}) + b^i_{j,2} - B_{j,2}/(1+\Phi) = A(z^i_{j,2}) + \tilde{b}^i_{j,2}$, which is stochastic because profits depend on the realization of the idiosyncratic shock $z^i_{j,2}$. Land has no value in period 2 after production.

3.1 Financial autarky

We first characterize the equilibrium with financial autarky. Since in period 1 entrepreneurs are homogeneous, we drop the individual superscript i. We also ignore country and time subscripts and let k and b denote the individual land and bonds purchased at time 1. Furthermore, we use p, R, and B, without subscripts, to denote the price of land, the gross interest rate and the bonds issued in period 1. The idiosyncratic shock realized in period 2 is denoted by z. Total government transfers paid in period 1 equal government borrowing B/R, while total government transfers paid in period 2 equal the repayment of debt, $-B$. Since the population is $1+\Phi$, per capita transfers are $\tau_1 = (B/R)/(1+\Phi)$ in period 1 and $\tau_2 = -B/(1+\Phi)$ in period 2.

Workers earn the wage $\bar{w}$ in both periods on labor $1/\Phi$ and receive transfers τ_1 and τ_2. Consumption is $(\bar{w} + \nu B/R)/\Phi$ in period 1 and $(\bar{w} - \nu B)/\Phi$ in period 2, and the lifetime utility is

$$W(B) = \chi + \ln\left(\bar{w} + \nu\frac{B}{R}\right) + \beta\ln\Big(\bar{w} - \nu B\Big), \tag{12}$$

where $\chi = -(1+\beta)\ln\Phi$ is a constant and we have used $\nu = \Phi/(1+\Phi)$.

Entrepreneurs start period 1 with wealth $a = \bar{A} + p$ and consume $c^e_1 = a - \tilde{b}/R - pk$. Since entrepreneurs start with the same wealth, they all choose the same next period land and bonds. Thus, $k = 1$ and $b = B$, which implies $\tilde{b} = \nu B$. This also implies that $c^e_1 = \bar{A} - \nu B/R$. Next period consumption depends on the realization of the idiosyncratic shock and it is equal to $c^e_2 = A(z) + \nu B$.

Therefore, entrepreneurs' lifetime utility is

$$V(B) = \ln\left(\bar{A} - \nu\frac{B}{R}\right) + \beta\mathbb{E}\ln\Big(A(z) + \nu B\Big). \tag{13}$$

Entrepreneurs are identical in period 1 but heterogeneous in period 2, after the realization the idiosyncratic shock z. This creates a precautionary saving motive and, since workers cannot borrow, the government is the only provider of safe assets available for consumption smoothing. Apart from the effects that the issuance of debt has on the interest rate R, eqs. (12) and (13) make clear that public debt redistributes consumption intertemporally between workers and entrepreneurs.

Lemma 3.1 *The indirect utility of workers (12) is strictly concave in B with a unique maximum in the interval $\left(0, \frac{\bar{w}}{\nu}\right)$. The indirect utility of entrepreneurs (13) is strictly increasing in B.*

Proof 3.1 *Appendix E.*

The indirect utilities of workers and entrepreneurs are plotted in Figure 3 for particular parameter values. Starting from $B = 0$, workers would like to increase public debt because the interest rate is lower than the intertemporal discount rate. In fact, we can verify that $R < 1/\beta$ at $B = 0$. However, as the government borrows more, it reaches a point in which workers' welfare starts to decrease. This happens for two reasons. First, keeping the interest rate fixed, the marginal utility of consumption in the next period becomes bigger than the marginal utility of consumption in the current period. Second, as the government borrows more, the interest rate increases, raising the cost of borrowing. Entrepreneurs, on the other hand, always prefer higher debt because it increases the interest rate and, therefore, the return on their financial wealth.

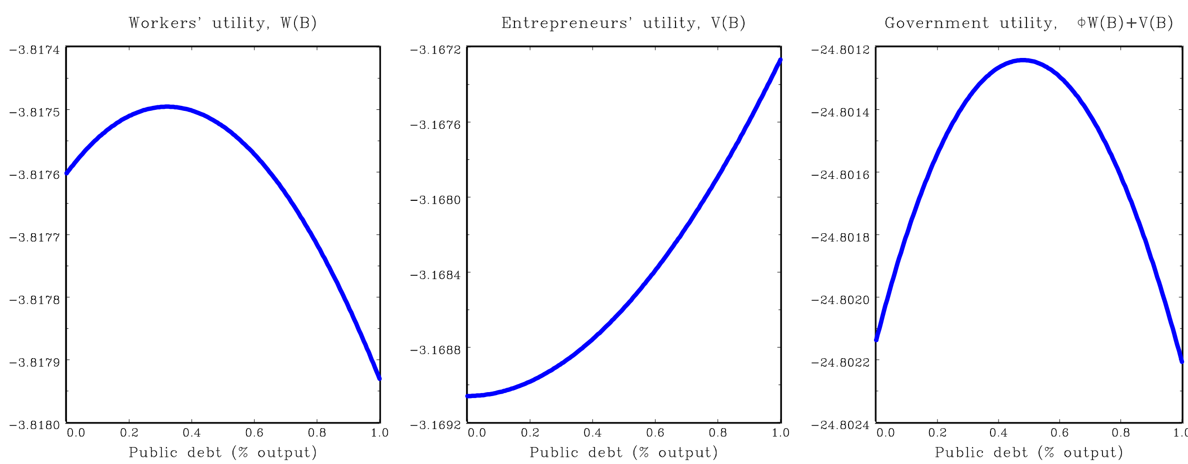

Figure 3: Indirect utilities in autarky. Baseline parameters: $\beta = 0.95$, $\theta = 0.2$, $z \in \{0.75, 1.25\}$ with equal probabilities, $\nu = \Phi/(1 + \Phi) = 0.85$.

The government maximizes the weighted sum of workers' and entrepreneurs' utilities,[2]

$$\max_{B} \Big\{ \Phi W(B) + V(B) \Big\}. \tag{14}$$

Since the indirect utility of entrepreneurs $V(B)$ is not concave, the government's objective is not necessarily concave. We can establish concavity only for large values of Φ, that is, when the government assigns a large weight to workers and a small weight to entrepreneurs.

Proposition 3.1 *If $\Phi > \frac{(1+\beta)\bar{w}}{A} + \beta$, the government's objective is strictly concave, and there is a unique maximum in the interval $\left(0, \frac{\bar{w}}{\nu}\right)$.*

Proof 3.1 *Appendix F.*

Two remarks are in order here. First, the condition on Φ is sufficient, not necessary. Second, even if the government's objective is not strictly concave, the maximum is still interior in the interval $\left[0, \frac{\bar{w}}{\nu}\right]$. This is the case because the objective function is continuous and converges to minus infinity as B converges to $\frac{\bar{w}}{\nu}$. Since the objective function is also differentiable, its derivative must be zero at the optimum. Differentiating (14) we obtain

$$\Phi \cdot \left[\frac{\partial \left(\frac{B}{R}\right)}{\partial B} \left(\frac{1}{c_1^w} \right) - \beta \left(\frac{1}{c_2^w} \right) \right] = \left[\frac{\partial \left(\frac{B}{R}\right)}{\partial B} \left(\frac{1}{c_1^e} \right) - \beta \mathbb{E} \left(\frac{1}{c_2^e(z)} \right) \right], \tag{15}$$

where c_1^w and c_2^w are the aggregate consumptions of workers (per capita consumption multiplied by the mass of workers Φ); c_1^e and $c_2^e(z)$ are the individual consumption of entrepreneurs. In period 2, entrepreneurs' consumption is stochastic because it depends on the idiosyncratic shock z.

A additional unit of debt issued in period 1 transfers consumption from entrepreneurs (who buy the bonds net of transfers) to workers (who receive government transfers). In period 2, the government pays back the debt with negative transfers. This reduces workers' consumption, c_2^w, and increases the consumption of entrepreneurs, $c_2^e(z)$. As the size of workers Φ increases, the left-hand-side of (15) receives more weight. Thus, the effect of public borrowing on workers' welfare becomes more important for the government.

Because the government is a monopolist in the supply of bonds, it takes into account that its debt affects the interest rate. Remember that the total transfers made by the government in period 1 are B/R. Thus, when the government increases B marginally by one unit, the increase in current transfers is not $1/R$ because the interest rate R also changes. More specifically, the marginal change in the transfers made in period 1 is

$$\frac{\partial \left(\frac{B}{R}\right)}{\partial B} = \frac{1}{R}\Big(1 - \epsilon^A(B)\Big),$$

[2] The interpretation is that policies are chosen by representatives who are selected through democratic elections. Under standard assumptions in probabilistic voting, if there are two candidates who only care about gaining power and have commitment to some platforms, political competition leads to convergence in policy proposals and government policies maximize the weighted sum of agents' welfare. See, for example, Persson and Tabellini (2000). The government behaves, de facto, as a benevolent planner but without commitment to future policies.

where $\epsilon^A(B) = \frac{\partial R}{\partial B}\frac{B}{R}$ is the elasticity of the interest rate R to the supply of bonds in autarky. Clearly, higher values of the elasticity imply smaller transfers allowed by higher borrowing.

The internalization of the interest rate elasticity in the decision of governments is the key difference between public and private borrowing. With private borrowing, atomistic agents take the interest rate as given and $\epsilon^A(B)$ is zero in their individual optimality condition. In this case the perceived increase in consumption in period 1 from private borrowing would be $1/R$.

3.2 The effects of financial integration

When N countries are financially integrated, entrepreneurs can purchase both domestic and foreign bonds, while transfers are only a function of domestic debt. Thus, b is not necessarily equal to B. However, according to corollary 2.1, the excess holdings of bonds will be equalized across countries. Therefore, $\tilde{b} = \nu \sum_{j=1}^{N} B_j/N$. Effectively, in countries where governments make larger transfers in period 1, entrepreneurs save more because they anticipate higher taxes in period 2. Using this result, the indirect utility of entrepreneurs in country j is

$$V_j(\mathbf{B}) = \ln\left(\bar{A} - \frac{\tilde{b}}{R}\right) + \beta\mathbb{E}\ln\left(A(z) + \tilde{b}\right), \tag{16}$$

where $\mathbf{B} = (B_1, .., B_N)$ is the vector of public debts in all countries. Since $\tilde{b} = \nu \sum_{j=1}^{N} B_j/N$ (see Corollary 2.1), entrepreneurs' welfare depends on the average worldwide public debt.

The properties of $V_j(\mathbf{B})$ are similar to the autarky case. Keeping the debts in all other countries constant, entrepreneurs still prefer higher B_j since this increases the equilibrium interest rate and, therefore, the return on the risk-free bonds held to hedge the idiosyncratic risk.

The indirect utility of workers is similar to (12) for the autarky case and can be written as

$$W_j(\mathbf{B}) = \chi + \ln\left(\bar{w} + \nu\frac{B_j}{R}\right) + \beta\ln\left(\bar{w} - \nu B_j\right). \tag{17}$$

The interest rate is now determined in the world market. Using (9), this is equal to

$$R = \nu\Big(\frac{\sum_{j=1}^{N} B_j}{N}\Big)\left[\frac{1+\beta(1-\phi)}{\beta(1-\phi)\bar{A}}\right], \tag{18}$$

where $\phi = \mathbb{E}\left(\frac{A(z)}{A(z)+\nu(\sum_{j=1}^{N} B_j)/N}\right)$. This expression makes clear that it is the worldwide debt that determines the interest rate. Thus, the debt of an individual country affects the interest rate in proportion to its worldwide share.

In a Nash equilibrium, each government chooses its own debt taking as given the debts issued by all other countries,[3] that is,

$$\max_{B_j}\Big\{\Phi W_j(\mathbf{B}) + V_j(\mathbf{B})\Big\}. \tag{19}$$

[3] Each government takes as given the level of debt issued by all other countries but internalizes the fact that by changing its own debt it affects the interest rate and, therefore, the transfers made by other governments. Therefore, they take into account that the budget constraints of all governments need to be satisfied in and out of the equilibrium.

The optimal debt is denoted by $B_j = \varphi_j(\mathbf{B}_{-j})$, where $\mathbf{B}_{-j}$ is the vector of government debt of all other countries, except country j. The function $\varphi_j(.)$ is the best response function of country j to other countries' policies. A Nash policy equilibrium is a vector $\mathbf{B}^* = (B_1^*, .., B_N^*)$ that satisfies

$$B_j^* = \varphi_j(\mathbf{B}_{-j}^*), \qquad \text{for all } j = 1, .., N.$$

For each country j, the optimal debt B_j^* satisfies the first-order condition

$$\Phi \cdot \left[\frac{\partial(\frac{B_j}{R})}{\partial B_j} \left(\frac{1}{c_1^w} \right) - \beta \left(\frac{1}{c_2^w} \right) \right] = \left[\frac{\partial \left(\frac{\sum_{j=1}^N B_j}{NR} \right)}{\partial B} \left(\frac{1}{c_1^e} \right) - \beta \frac{1}{N} \mathbb{E} \left(\frac{1}{c_2^e(z)} \right) \right], \tag{20}$$

which is derived by differentiating the government objective (19). As in the autarky regime, this condition is necessary but not sufficient unless Φ is sufficiently large.

While the government still faces the trade-off between the benefits and costs of transferring consumption from entrepreneurs to workers in the first period, this expression differs from eq. (15) in several respects. First, transfers depend only on the domestic supply of government bonds B_j, while entrepreneurs' utility depends on both domestic and foreign bonds. Hence, an extra unit of B_j increases c_1^w by $\frac{\partial(B_j/R)}{\partial B_j}$ but decreases c_1^e by only $\frac{\partial(\sum_{j=1}^N B_j/(NR))}{\partial B_j} < \frac{\partial(\nu B_j/R)}{\partial B_j}$. This is because part of the extra bonds issued by the domestic government are absorbed by entrepreneurs in the rest of the world. In the second period, the government repays B_j (with negative transfers), which reduces c_2^w by the same amount as in the autarky case. The increase in $c_2^e(z)$, however, is smaller because domestic entrepreneurs hold only part of the increase in domestic bonds.

Another difference between eqs. (15) and (20) is that the effect of a unilateral change in B on the interest rate is smaller when financial markets are integrated (see eq. (18)). In a symmetric equilibrium, $B_j = \sum_{j=1}^N B_j/N = B$ and condition (20) becomes

$$\Phi \cdot \left[\frac{1}{R} \left(1 - \frac{\epsilon^A(B)}{N} \right) \left(\frac{1}{c_1^w} \right) - \frac{\beta}{c_2^w} \right] = \frac{1}{N} \left[\frac{1}{R} \Big(1 - \epsilon^A(B) \Big) \left(\frac{1}{c_1^e} \right) - \mathbb{E} \left(\frac{\beta}{c_2^e(z)} \right) \right], \tag{21}$$

where $\epsilon^A(B)$ is the elasticity of the interest rate in autarky.

Compared to the autarky case, the cost of increasing debt unilaterally is smaller because the perceived elasticity is $\epsilon^A(B)/N$. The costs and benefits for entrepreneurs are also different since the new bonds are shared by domestic and foreign residents. More specifically, the marginal effects on $V(\mathbf{B})$ are reduced when the economy is financially integrated. Thus, whether financial integration leads to more or less public debt depends on the relative size of workers and entrepreneurs.

Proposition 3.2 *If $\Phi/(1+\Phi) \simeq 1$, then (i) per capita debt is strictly increasing in the number of countries N, (ii) as $N \to \infty$ there exists a unique symmetric equilibrium where debt is bounded and $\beta R < 1$, (iii) financial integration generates welfare losses for workers and gains for entrepreneurs.*

Proof 3.2 *Appendix G.*

When the size of workers is large, the objective of the government is approximately equal to the utility of domestic workers. Since the interest rate is less elastic in an integrated world, workers would like the government to borrow more. However, if the government assigns a large weight to entrepreneurs, public debt may decline with liberalization. This is because, when financial markets are integrated, the benefits of issuing public debt for entrepreneurs are shared with foreign entrepreneurs. Thus, the government may have a lower incentive to borrow. This is shown in the first panel of Figure 4 for given parameter values.

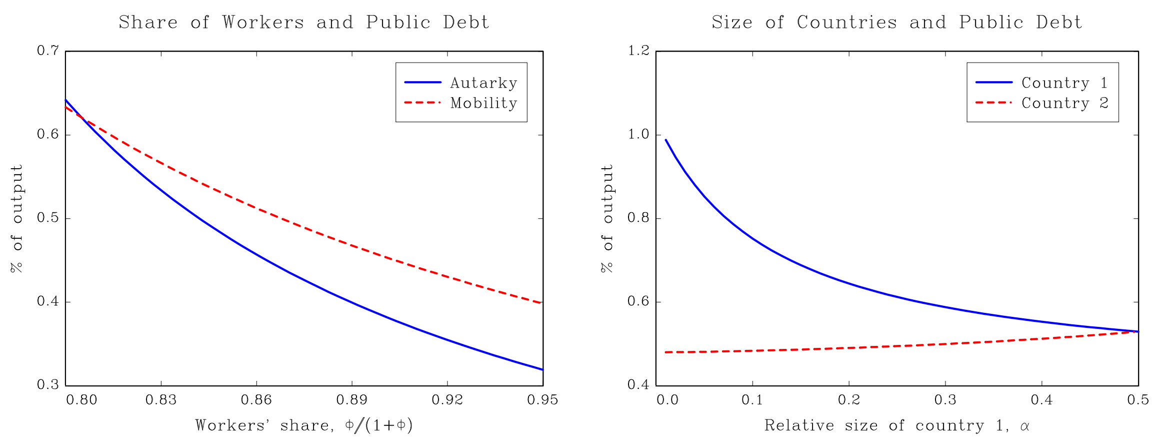

Figure 4: Dependence of government debt on workers' share (first panel) and countries' size with capital mobility (second panel). Baseline parameters: $\beta = 0.95$, $\theta = 0.2$, $z \in \{0.75, 1.25\}$ with equal probabilities, $\nu = \Phi/(1+\Phi) = 0.85$ and $\alpha = 0.5$.

The channel through which capital mobility affects public borrowing derives from the non-atomistic nature of governments—which is also emphasized in Chang (1990)—and it is essential to differentiate the equilibrium with public borrowing from an equilibrium with private borrowing. This is because private issuers do not internalize the impact of their own borrowing on the equilibrium interest rate, since each agent is too small to affect aggregate prices. Therefore, with only private borrowers, the autarky equilibrium would not be different from the equilibrium with capital mobility. With public borrowing, on the contrary, the equilibrium debt differs in the economy with and without mobility of capital. As a result, our model predicts that financial integration affects the equilibrium outcome even if countries are homogeneous. This property differentiates our study from the recent literature on global imbalances where liberalization affects the equilibrium because countries are heterogeneous in some important dimension.[4]

The effects of financial integration on the public debt depend on the relative size of the integrating countries. To show this, suppose that there are only two countries, $N = 2$. The population and land endowment of country 1 is a proportion α of the worldwide endowment. If $\alpha = 0.5$, we revert to the symmetric case.

[4]Examples are Fogli and Perri (2006), Caballero, Farhi, and Gourinchas (2008), Mendoza, Quadrini, and Rios-Rull (2009), and Angeletos and Panousi (2011).

Proposition 3.3 *Suppose that $\Phi/(1+\Phi) \simeq 1$. If $\alpha < 0.5$, in the regime with capital mobility country 1 issues higher per capita debt than country 2, that is, $B_1 > B_2$.*

Proof 3.3 *Appendix H.*

Since small countries face a larger world market relative to their own economy, they perceive the interest rate as less sensitive to their own debt and borrow more. This is shown in the second panel of Figure 4. However, if the relative size of workers is not large, the government's objective is dominated by the benefit of providing safe assets to entrepreneurs, and since these benefits are shared with foreign entrepreneurs, the government may borrow less.

3.3 The effects of rising income risk

The fact that entrepreneurs face idiosyncratic risk implies that their incomes become unequal in period 2. Here we want to study how changes in income risk affect the choice of public debt.

Proposition 3.4 *Consider the autarky regime and $\Phi/(1+\Phi) \simeq 1$. If an increase in the mean preserving spread of the distribution of z raises the term $\frac{1-\epsilon(B)}{\bar{w}R(B)+B}$, then B increases.*

Proof 3.4 *Appendix I.*

In general, an increase in the volatility of the idiosyncratic shock implies that entrepreneurs face higher risk. This strengthens the demand for safe assets (government bonds) and reduces the interest rate. Because of the lower interest rate, workers would like more public debt. The government, however, takes into account not only the level of the interest rate but also how the interest rate changes when it changes the debt (elasticity). At the same time, the government also finds it optimal to increase public debt to improve entrepreneurs' welfare. In general, we cannot establish unambiguously whether government debt increases in response to an increase in risk. However, as long as the term $(1-\epsilon(B))/(\bar{w}R(B)+B)$ increases, public debt does rise as we show in the proof of the proposition. Therefore, to the extent that the increase in income inequality was at least in part associated to an increase in income risk, the paper provides another potential mechanism for the rising public debt.

The dependence of public debt on risk is shown in the first panel of Figure 5 which plots the equilibrium debt as a function of the volatility of the idiosyncratic shock, $z_2 - z_1$. The second panel plots the debt when the volatility of the idiosyncratic shock increases only in country 1. Even if income inequality changes only in one country, debt increases in both countries. This happens because the higher risk faced by entrepreneurs in country 1 increases their demand for bonds and reduces the world interest rate. If the government's weight assigned to workers is sizable—as assumed in the numerical example—the lower interest rate makes public debt more attractive for the governments of both countries.

The finding that the increase in risk in a few countries may trigger an increase in public debt in other countries is important to reconcile the theory with the data. In fact, the increase in income

Figure 5: Dependence of government debt on rising risk in all countries (first panel) and rising risk in country 1 only with capital mobility (second panel). Baseline parameters: $\beta = 0.95$, $\theta = 0.2$, $z \in \{0.75, 1.25\}$ with equal probabilities, $\nu = \Phi/(1+\Phi) = 0.85$ and $\alpha = 0.5$.

inequality since the early 1980s has been observed only in a few countries (see Atkinson, Piketty and Saez (2011)), while the cross-country increase in public debt was more general. The fact that in the 1980s capital markets were liberalized may explain why the increase in inequality in a few countries may have affected other countries, provided that the rising inequality was associated with higher risk in large economies such as the United States.

This point raises an important question: was the rising income inequality observed in the United States the result of an increase in individual income risk? To answer this question we should decompose individual income and separate the predictable components (such as education premium and age dependence) from the stochastic components (both permanent and transitory). Although this is a very difficult task, there are several empirical studies showing that the increase in the volatility of the stochastic components of income (both permanent and transitory) has played an important role in the increasing income inequality in the United States. For example, using PSID data, Blundell, Pistaferri and Preston (2008) find that the variance of the permanent income shock increased significantly in the first half of 1980s. This is especially important because in our model the productivity shock z acts as a permanent income shock: thanks to the linearity of the production and saving functions, individual entrepreneurial income follows a random walk even if the productivity shock is iid. Also relevant are the empirical studies of DeBacker, Heim, Panousi, and Vidangos (2011) and DeBacker, Heim, Panousi, Ramnath and Vidangos (2012), which are based on US income tax returns over the period 1987-2009. They find that part of the increase in income inequality can be attributed to the volatility of the stochastic components of income, especially for business and investment incomes. Considering that business and investment incomes are more volatile and tend to be concentrated at the top of the distribution, this finding justifies our modeling choice of focusing on the rising income risk of entrepreneurs. Entrepreneurs are interpreted broadly as 'investment decision makers', including many managerial occupations.

3.4 On the relevance of public debt

The well-known Ricardian equivalence result states that, absent any frictions, public debt is irrelevant. However, public debt could become relevant if there are frictions in the market structure or restrictions in the set of policy instruments. For example, the tax smoothing theory of Barro (1979) is based on the assumption that agents are constrained in their borrowing and taxation is distortionary (no lump-sum taxes). Chang (1990) and Song, Storesletten and Zilibotti (2012) consider missing markets for trades between generations. Aiyagari and McGrattan (1998) and Shin (2006) assume that markets are incomplete because agents cannot trade state-contingent claims. In our model, the relevance of public debt is based on three assumptions: (i) some agents (entrepreneurs) face more risk than others (workers); (ii) there is no market for state-contingent claims and workers cannot borrow (market incompleteness); and (iii) fiscal policies are limited to public debt and lump-sum transfers (limited fiscal instruments). We now discuss each of these three assumptions.

Risk heterogeneity. The assumption that workers do not face any risk is not important per se. It is only made to capture, parsimoniously, the idea that some agents face more risk than others. We could relax this assumption and assume that workers also face earning risks. As long as the risk faced by workers is lower than the risk faced by entrepreneurs, we obtain similar results.

To show this point, we now assume that the efficiency units of labor supplied by each worker are stochastic and take the form $\eta = (1/\Phi)\epsilon$, where $\epsilon \in \{\epsilon_1, \epsilon_2\}$ with probabilities $\{0.5, 0.5\}$, and $(\epsilon_1 + \epsilon_2)/2 = 1$. Figure 6 shows the dependence of public debt on $\epsilon_2 - \epsilon_1$, which captures the idiosyncratic risk faced by workers. The risk faced by entrepreneurs is kept at the baseline value $z_2 - z_1 = 0.5$. As the volatility of earnings increases, the optimal level of public debt goes down. When $\epsilon_2 - \epsilon_1 = 0.5$, workers and entrepreneurs face the same risk and optimal debt is zero. Therefore, as long as the risk faced by workers is not too large, government debt remains positive.

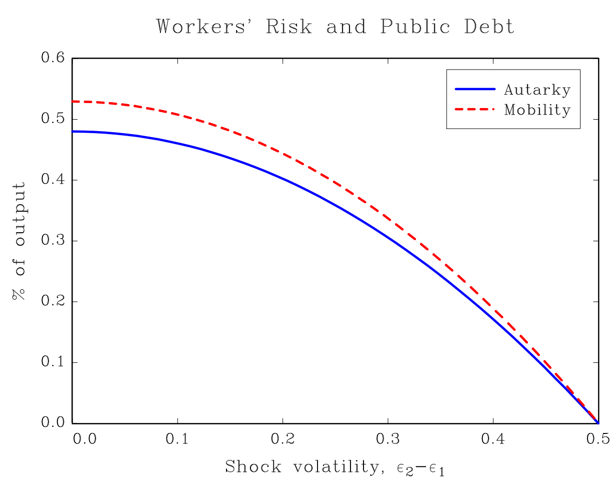

Figure 6: Dependence of government debt on workers' risk. Baseline parameters: $\beta = 0.95$, $\theta = 0.2$, $z \in \{0.75, 1.25\}$ with equal probabilities, $\nu = \Phi/(1+\Phi) = 0.85$ and $\alpha = 0.5$.

What would happen if the risk faced by workers was bigger than the risk faced by entrepreneurs? In this case we would have that entrepreneurs borrow from workers and, if entrepreneurs face a borrowing limit, government debt would still emerge. This case would only reverse the role of workers and entrepreneurs in financial markets but would not change the basic theory of the paper. For the theory to work, we only need that some agents face more risk than others.

Market for contingent claims and borrowing constraint. The assumption of incomplete markets is key for the results of the paper. If entrepreneurs could trade securities that are contingent on individual realizations of income, public debt would not play any role in the model.

The borrowing constraint for workers is also important but the debt limit does not have to be zero. To see this, consider an environment where workers are allowed to borrow $D \leq \overline{D}$, where $\overline{D}$ is the borrowing limit. Aggregate workers' consumption in the first period is then $c_1^w = \bar{w} + (D + \nu B)/R$, while entrepreneurs' consumption is $c_1^e = \bar{A} - (D + \nu B)/R$. In the benchmark model we imposed $\overline{D} = 0$, in which case optimal debt is denoted by B^*. Let D^* denote aggregate debt when workers are fully unconstrained ($\overline{D} \to \infty$) and there is no public debt. We can then characterized the optimal public debt as follows.

Proposition 3.5 *The optimal debt issued by the government is* $B = B^* - \frac{\overline{D}}{\nu} \leq \frac{D^*}{\nu}$.

Proof 3.5 *Appendix J.*

From the point of view of the government, what is relevant is the total debt adjusted by transfers, that is, $D + \nu B$, since this is what determines individuals' consumption. When $\overline{D} = 0$, aggregate consumptions equal $c_1^w = \bar{w} + \nu B^*/R$ and $c_1^e = \bar{A} - \nu B^*/R$. Clearly, the government would target the same allocation, that is, the same consumptions for workers and entrepreneurs. This is achieved when $D + \nu B = \nu B^*$. Eventually the workers will be pushed to the borrowing limit, that is, $D = \overline{D}$, and the optimal debt will be determined by the equation reported in the proposition.

The first panel of Figure 7 plots government debt as a function of the borrowing limit for workers. As the limit is relaxed, workers borrow more, which is compensated by lower borrowing from the government. If the borrowing limit is not too loose, as we have assumed in the baseline model, B remains positive. However, B may take negative values when $\overline{D}$ is large. When workers are fully unconstrained, on the other hand, no matter what the value of B is, they will always choose $D^* > \nu B^* - \nu B$ since they do not internalize the impact of their borrowing on the interest rate. In such a case, the government will not be able to reach its target. As long as the borrowing limit is finite, however, government intervention in financial markets remains relevant.[5]

The second panel of Figure 7 plots private debt in the absence of government intervention, $D/\nu = \min\{D^*/\nu, \overline{D}/\nu\}$ with $B = 0$ (dotted line) and total debt, $D/\nu + B$, when the government chooses public debt optimally (solid line). Private debt in the absence of government intervention increases with the borrowing limit $\overline{D}$ until the limit is sufficiently large and workers become unconstrained. Total debt with government intervention, instead, is constant independently of the

[5]The international liberalization of capital markets may have been associated with a relaxation of the financial constraint on private borrowers. In our model this is captured by an increase in $\overline{D}$, which would have the opposite effect on government debt.

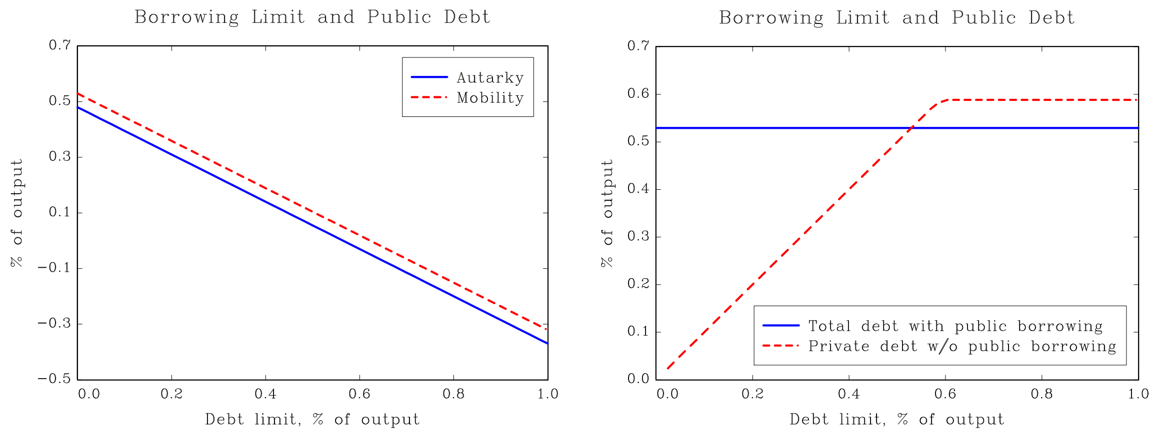

Figure 7: Dependence of government debt on private borrowing limit (first panel) and total borrowing (second panel). Baseline parameters: $\beta = 0.95$, $\theta = 0.2$, $z \in \{0.75, 1.25\}$ with equal probabilities, $\nu = \Phi/(1 + \Phi) = 0.85$ and $\alpha = 0.5$.

borrowing limit on workers. Consistent with Proposition 3.5, the debt chosen by unconstrained workers is larger than B^*.[6]

Finally, we would like to stress that, from the point of view of a positive analysis, the consideration of public debt is not a substitute for private debt. Of course, we could allow governments to intervene with other policies, such as taxes or restrictions on borrowing, ensuring that private agents choose the same amount of debt as the one desired by the government. Provided that the borrowing constraint on private borrowing is not too tight, these policies could replicate the equilibrium with public debt (see, for example, Kocherlakota (2007) and Yared (2013)). In the example above, this could be achieved by setting $\overline{D} = \nu B^*$ (the point where the dotted and solid lines intersect in the right panel of Figure 7). In the absence of these policies, however, the equilibrium with private borrowing will be different from the equilibrium with public debt.

Limited policy instruments. If governments had access to type-specific taxes and transfers, debt would also be irrelevant. By taxing agents with high realizations of income and giving transfers to those with low realizations, the government would provide insurance and mimic the complete market equilibrium. This would make public debt irrelevant as stated in the next proposition.

Proposition 3.6 *Suppose that* $\Phi = \frac{1-\theta}{\theta}$ *and the government can also use proportional income taxes, in addition to public debt. Then the optimal tax rate is 100 percent and public debt is zero.*

Proof 3.6 *Appendix K.*

[6] An immediate implication of this is that in the infinite horizon model, private borrowing would increase every period (recall that $\beta R < 1$), up to the point where the natural debt limit is reached. As a result, workers' consumption would converge to zero in the long run. This outcome differs from the economy with government debt (and no private borrowing) where workers' consumption is strictly positive in the long run even if the government is unconstrained.

The condition $\Phi = \frac{1-\theta}{\theta}$ ensures that the wage income of workers equals the profits earned by entrepreneurs. Therefore, there are no gains from redistributing income between the two groups and we can separate the purely redistributionary scope of taxes from their insurance role.

The optimality of 100 percent taxation follows from the fact that income taxes are not distortionary in our model: by taxing income at 100 percent rate and redistributing the revenues as lump-sum transfers, the government could provide full insurance, eliminating the effects of idiosyncratic risk. In such a case, debt would be irrelevant. Of course, this result is valid only because taxes are not distortionary. In reality, however, it is not plausible to think that 100 percent taxes are not distortionary. Once we extend the model to allow for distortions, perfect redistribution is no longer optimal and public debt continues to play an important role.

Suppose that production requires effort h from entrepreneurs according to the production function $F(k, l, h) = (zk)^{\theta\eta} l^{(1-\theta)\eta} h^{1-\eta}$. Furthermore, the utility of entrepreneurs takes the form $\ln(c - h)$. Workers continue to have the same characteristics as in the baseline two-period model.[7] In addition to issuing public debt, the government can tax individual incomes at rate τ_1 in period 1 and at rate τ_2 in period 2. We then have the following proposition.

Proposition 3.7 *Suppose that* $\Phi = \frac{1-\theta}{\theta}$. *The optimal government policy satisfies* $\tau_1 = 0$, $\tau_2 < 1$, *and* $B \neq 0$.

Proof 3.7 *Appendix L.*

Because all agents have the same income (net of the disutility of effort) when $\Phi = \frac{1-\theta}{\theta}$, positive taxes in the first period would distort the economy but generate no redistribution. It is then optimal to set them to zero. In the second period, entrepreneurs earn different income and redistribution would be desirable for the government. However, taxes also discourage effort, which in turn reduces income for both workers and entrepreneurs. The government trades off the gains from redistribution against the costs of distortions and chooses a positive but non-confiscatory tax rate. Since the insurance provided by taxes is partial and entrepreneurs still face uncertain consumption, issuing public debt in period 1 is still optimal even though the government has access to a richer set of instruments.[8] This result differs from that of Golosov and Sargent (2012), who find that public debt is irrelevant. This is because they assume that agents have full access to financial markets, while, in our model, market incompleteness plays a central role.

3.5 Time consistency

In the two-period model optimal policy is time-consistent, since debt is chosen only once. However, when there are more than two periods, public debt is chosen at different dates. Hence, it would

[7] We follow the traditional public finance literature in assuming that effort is publicly observable but the government cannot force agents to exert the socially optimal effort. An alternative would be to follow the public finance literature where effort is private information as, for example, Golosov, Kocherlakota, and Tsyvinski (2003), Albanesi and Sleet (2006), and Farhi and Werning (2008). We can then characterize the constrained socially optimal allocation and public debt could be one of the instruments used to decentralize the allocation.

[8] This is robust to the consideration of progressive taxes as long as the fiscal system is not discriminatory. In particular, lump-sum transfers cannot differ between workers and entrepreneurs. Otherwise, public borrowing could be replicated with positive (negative) transfers to workers (entrepreneurs) in period 1, and the reverse in period 2.

make a difference whether the debt is chosen under commitment (the optimization problem is solved only in period 1) or under discretion (the problem is solved sequentially). This can be easily illustrated in a three-period economy.

Suppose that the economy lasts for three periods, that is, $T = 3$. To simplify the discussion we abstract from uncertainty and assume that $z_t^i = \bar{z}$ for all $t = 1, 2, 3$. Entrepreneurs are endowed with the same initial assets, $k_1^i = \bar{k} = 1$ and $b_1^i = b_1 \geq 0$. Since there is no uncertainty, profits are constant and equal to $\pi_t = \bar{A}$. Consumption of entrepreneurs and workers is defined, respectively, in eqs. (10) and (11). Because $B_4 = 0$, interest rates satisfy

$$R_1 = \frac{\bar{A} + \nu\left[(1+\beta)B_2 - \frac{B_3}{R_2}\right]}{\beta(\bar{A} + \nu B_1)} \qquad \text{and} \qquad R_2 = \frac{\bar{A} + \nu(1+\beta)B_3}{\beta(\bar{A} + \nu B_2)}. \tag{22}$$

Since we abstract from idiosyncratic shocks, there is no insurance motive for holding debt. As a result, if $b_1 = B_1 = 0$, it is optimal to set $B_2 = B_3 = 0$. However, when the initial debt is different from zero, that is, $B_1 > 0$, the government's problem amounts to the optimal timing of repaying B_1. We now show that the optimal B_2 and B_3 differ under commitment and discretion. To simplify the analysis, we focus on the autarky regime with only one country.

Optimal policy with commitment. Under commitment, the government chooses B_2 and B_3 in period 1 to solve the problem

$$\max_{B_2, B_3} \left\{ \Phi\Big[\ln c_1^w + \beta \ln c_2^w + \beta^2 \ln c_3^w\Big] + \ln c_1^e + \beta \ln c_2^e + \beta^2 \ln c_3^e \right\},$$

subject to (10), (11), and (22). The first order conditions are

$$B_2: \qquad \Phi\left(\frac{1-\epsilon(B_2)}{c_1^w R_1} - \frac{\beta}{c_2^w}\right) - \left(\frac{1-\epsilon(B_2)}{c_1^e R_1} - \frac{\beta}{c_2^e}\right) + \frac{\partial\left(\frac{B_3}{R_2}\right)}{\partial B_2}\beta\left(\frac{\Phi}{c_2^w} - \frac{1}{c_2^e}\right) = 0, \tag{23}$$

$$B_3: \qquad \Phi\left(\frac{1-\epsilon(B_3)}{c_2^w R_2} - \frac{\beta}{c_3^w}\right) - \left(\frac{1-\epsilon(B_3)}{c_2^e R_2} - \frac{\beta}{c_3^e}\right) + \frac{\partial\left(\frac{B_2}{R_1}\right)}{\partial B_3}\beta^{-1}\left(\frac{\Phi}{c_1^w} - \frac{1}{c_1^e}\right) = 0. \tag{24}$$

Optimal policy with discretion. Without commitment, the problem can be solved backward starting at $t = 2$ when the government chooses B_3. This solves

$$\max_{B_3} \left\{ \Phi\Big[\ln c_2^w + \beta \ln c_3^w\Big] + \ln c_2^e + \beta \ln c_3^e \right\}$$

subject to (10), (11), and (22). The first order condition is

$$\Phi\left(\frac{1-\epsilon(B_3)}{c_2^w R_2} - \frac{\beta}{c_3^w}\right) - \left(\frac{1-\epsilon(B_3)}{c_2^e R_2} - \frac{\beta}{c_3^e}\right) = 0. \tag{25}$$

This characterizes the government policy in period 2, which we denote by $B_3 = \mathfrak{B}_3(B_2)$. Because the government takes past decisions as given, it does not internalize the redistributive effects of B_3 on first period consumption. With commitment, instead, when the government chooses B_3, it takes into account that this policy also affects consumption in period 1 as captured by the last term in eq. (24). This term, which is positive, is absent in condition (25).[9] By ignoring this effect, the government would *under-borrow* in period 2 compared to a government that chooses the whole sequence of public debts in period 1.

Going back to period 1, the government solves the problem

$$\max_{B_2} \left\{ \Phi \Big[\ln c_1^w + \beta \ln c_2^w + \beta^2 \ln c_3^w \Big] + \ln c_1^e + \beta \ln c_2^e + \beta^2 \ln c_3^e \right\},$$

taking as given the optimal policy rule in period 2, that is, $B_3 = \mathfrak{B}_3(B_2)$.

The main difference between this problem and the one under commitment is that next period's policy, $B_3 = \mathfrak{B}_3(B_2)$, is taken as given. The first order condition is

$$\Phi\left(\frac{1-\epsilon(B_2)}{c_1^w R_1} - \frac{\beta}{c_2^w}\right) - \left(\frac{1-\epsilon(B_2)}{c_1^e R_1} - \frac{\beta}{c_2^e}\right) + \frac{\partial\left(\frac{B_3}{R_2}\right)}{\partial B_2}\beta\left(\frac{\Phi}{c_2^w} - \frac{1}{c_2^e}\right) + \frac{\partial\left(\frac{B_2}{R_1}\right)}{\partial B_3}\frac{\partial \mathfrak{B}_3(B_2)}{\partial B_2}\left(\frac{\Phi}{c_1^w} - \frac{1}{c_1^e}\right) = 0, \tag{26}$$

which defines the optimal policy rule in period 1, $B_2 = \mathfrak{B}_2(B_1)$,

The last term in eq. (26), which is absent in eq. (24), captures the 'strategic manipulation' of the policy chosen in period 2 when the government chooses the optimal policy in period 1. The government understands that the future choice of B_3 has an effect on first period consumption, which it will ignore in period 2 when B_3 will actually be chosen. Anticipating this, the government chooses B_2 to affect (manipulate) the future choice of B_3.

Figure 8 shows the difference in public debt with and without commitment, for particular parameter values. The continuous lines denote the time-consistent solution, while the dashed lines denote the solution with commitment. We can see that, for any initial level of debt B_1, the time-consistent solution lies below the solution with commitment.

4 Infinite horizon model

In this section we show that the main properties of the two-period model extends to a model with a large number of periods T. In particular, financial liberalization can lead to an increase in public debt and this is exacerbated by a rise in income risk. In addition, by considering more periods, we can also study the transitions dynamics induced by capital liberation and a rise in income risk.

Since entrepreneurs face idiosyncratic shocks, the model generates a complex distribution of income and wealth. By virtue of the linearity of the production function, the model admits aggregation. An implication of this property is that income and wealth follow random walk processes and their economy-wide distributions are not stationary. This property becomes problematic if we

[9] As in the two-period economy, $\Phi/c_1^w > 1/c_1^e$ is a sufficient condition for the interest rate to be increasing, as will be assumed here. In addition, $\partial(B_3/R_1)/\partial B_3 > 0$, implying that the last term in eq. (24) is positive.

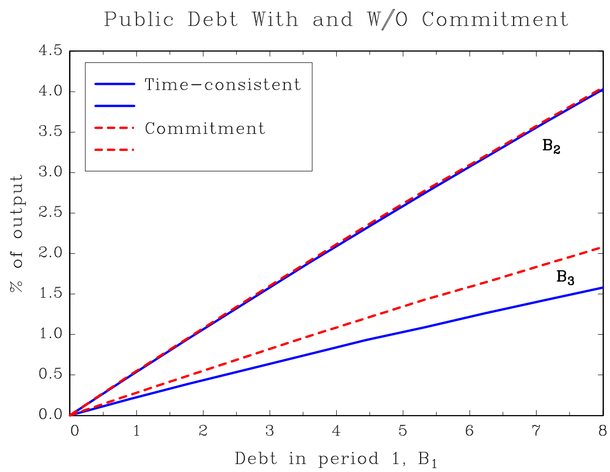

Figure 8: Time-consistent vs. commitment policy functions for $T = 3$ in the autarky equilibrium. Parameters are $\beta = 0.95$, $\theta = 0.2$, $z = 1$, and $\nu = \Phi/(1 + \Phi) = 0.85$.

want to compare the inequality generated by the model with the inequality observed in the data. Thus, to have a stationary distribution of income and wealth, we make the additional assumption that agents survive with some probability $\omega < 1$ and they are replaced by the same number of newborn agents. The discount factor β then results from the product of two terms: the intertemporal discount factor in preferences, $\hat{\beta} \in (0, 1)$, and the survival probability, $\omega \in (0, 1)$. The assets left by exiting entrepreneurs are redistributed equally (lump-sum) to the newborn entrepreneurs.[10]

4.1 Politico-economic equilibrium

We think of the infinite horizon model as the limit of $T \to \infty$. Policies are chosen in every period and they are functions of the relevant aggregate states. Suppose that at $t = 1$, countries start with the same aggregate excess holdings of bonds, that is, $\tilde{b}_{j,1} = \nu \sum_{j=1}^{N} B_{j,1}$. Using corollary 2.1, the sufficient set of states are the stocks of debt issued by the N countries, $\mathbf{B}_1 = (B_{1,1}, .., B_{N,1})$.

To characterize the strategic interaction between governments, we restrict attention to Nash equilibria where each country chooses its own public debt simultaneously and without coordination. The politico-economic equilibrium is characterized by a sequence of policy functions $\mathbf{B}_{t+1} = \mathcal{B}_t(\mathbf{B}_t)$ for $t = 1, .., T$. These functions are determined by solving the model backward, starting at $t = T$. Let's define first the government's objective at t.

Proposition 4.1 *Given current states* $\mathbf{B}_t$ *and policy function* $\mathcal{B}_{t+1}(\mathbf{B}_{t+1})$ *for next period policies,*

[10] All the properties of the competitive equilibrium derived earlier apply to the model with stochastic survival. We only need to reinterpret the discount factor as $\beta = \hat{\beta}\omega$. The distributions of income and wealth, however, now converge to a steady state if public debt stays constant over time.

the problem solved by government j at time t is

$$\max_{B_{j,t+1}} \left\{ \Phi W_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) + V_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) \right\}, \tag{27}$$

where the functions $W_{j,t}$ and $V_{j,t}$ are defined recursively as

$$\begin{aligned} W_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) &= \ln\left(\bar{w} + \frac{\nu B_{j,t+1}}{R_{j,t}} - \nu B_{j,t}\right) + \beta W_{j,t+1}\Big(\mathbf{B}_{t+1}; \mathcal{B}_{t+1}(\mathbf{B}_{t+1})\Big), \\ V_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) &= \ln(1-\eta_t) + \left(\frac{1}{1-\eta_t}\right)\left[\mathbb{E}\ln\Big(A(z^i_{j,t}) + \nu B_{j,t} + p_{j,t}\Big) + \eta_t \ln\left(\frac{\eta_{j,t}\phi_{j,t}}{p_{j,t}}\right)\right] + \\ &\qquad \beta\mathbb{E}V_{j,t+1}\Big(\mathbf{B}_{t+1}, \mathcal{B}_{t+1}(\mathbf{B}_{t+1})\Big). \end{aligned}$$

Proof 4.1 *Appendix M.*

In solving problem (27),[11] the government of country j takes as given the debts chosen by all other countries, the vector $\mathbf{B}_{-j,t+1}$. The solution, denoted by $B_{j,t+1} = \varphi_{j,t}(\mathbf{B}_t, \mathbf{B}_{-j,t+1})$, represents the optimal response function to the policies chosen by the other governments.

Definition 4.1 (Nash policy game) *For given states $\mathbf{B}_t$, the solution to the Nash policy game at time t is the vector $\mathbf{B}^*_{t+1}$ that satisfies $B^*_{j,t+1} = \varphi_{j,t}(\mathbf{B}_t, \mathbf{B}^*_{-j,t+1})$, for all $j = 1, .., N$.*

The solution to the policy game at time t provides the policy function $\mathbf{B}_{t+1} = \mathcal{B}_t(\mathbf{B}_t)$. Starting from $t = T$ and taking into account the terminal condition $\mathcal{B}_T(\mathbf{B}_T) = 0$, we can then construct the whole sequence of policy functions backward.

The infinite horizon is obtained as $T \to \infty$. Assuming convergence, the politico-equilibrium is characterized by an invariant policy function $\mathcal{B}(\mathbf{B}) = \lim_{T\to\infty} \mathcal{B}_1(\mathbf{B}_1)$. Because of the complexity of the model, we are unable to find a closed-form solution. Thus, we will only provide a numerical characterization by solving the model for a large but finite number of periods T.

4.2 Quantitative analysis

To show the impact of financial liberalization, we start from a steady-state equilibrium without mobility of capital and compute the transition dynamics following financial integration. Similarly, to show the impact of rising income risk, we start from a steady-state with low income risk and compute the transition dynamics following the increase in the volatility of the idiosyncratic shock. We solve the model numerically using a global approach based on the discretization of the state space (the stocks of public debt in the N countries) and grid search optimization. The detailed description of the numerical procedure is in the online appendix. To find a steady state, we solve the model for a large number of periods T and simulate the model for $t = 1, ..., T/2$. The steady state is the equilibrium in period $t = T/2$. Provided that T is large, the stock of debt converges to the level reached at $t = T/2$ relatively quickly (see Figure 9).

[11] Newborn agents do not appear separately in the welfare functions because of the aggregation result and the assumption that assets left by existing entrepreneurs are distributed to newborn entrepreneurs.

Parameterization: We fix the number of countries to $N = 2$ and assume that they are symmetric. Although the numerical simulation is not meant to provide a rigorous quantitative exercise but to illustrate the qualitative dynamic features of the model, we try as much as possible to choose the parameters according to observed empirical targets. More specifically, we choose variables observed pre-1980s as the initial calibration targets. This is motivated by the view that the process of international financial liberalization started in the 1980s. The pre-1980s period can then be considered as closer to a regime of financial autarky. Also, as can be seen from Figure 1, the average income inequality in industrialized countries started to increase toward the end of the 1970s and early 1980s. This motivates our choice to calibrate the autarky version of the model to the early 1980s. In particular, we focus on two targets: a ratio of public debt over income of 30 percent and a share of income earned by the top 1 percent of the population equal to 6 percent. These are the approximate numbers reported in Figure 1 for the OECD countries in the 1970s. We now describe in detail how the initial calibration targets can be used to pin down the parameters of the model.

A period in the model is one year and the discount factor is set to $\beta = 0.9466$, which results from an intertemporal discount rate of 3 percent and a survival probability $\omega = 0.975$. The value of ω implies an average (active) life of 40 years.

For the production function we would like to use a Cobb-Douglas specification, that is, $F(z,k,l) = z^\theta k^\theta l^{1-\theta}$, with $\bar{z} = \mathbb{E}z$ normalized to 1. However, the amount of idiosyncratic risk generated by this specification is bounded by the nonnegativity of z. In order to have more flexibility, we assume that the shock z also affects the effective quantity of land after production. Thus, the total income generated by the entrepreneur is $z^\theta k^\theta l^{1-\theta} + (z - \bar{z})kp$. The first component is pure production while the second component can be interpreted as capital gains (or losses if negative). Notice that in aggregate the capital gains or losses are zero. Thus, the aggregate production is exactly the same in the two cases and θ represents the capital income share which we set to 0.2. This is lower than the typical number used in the literature because there is no depreciation in the model.

Productivity is uniformly distributed in the domain $1 \pm \Delta$, where Δ is chosen so that the share of income earned by the top 1 percent is equal to 6 percent in the autarky steady state.[12] However, this also depends on Φ, which in turn is chosen to have a steady state of public debt over income of 30 percent in the autarky steady state. These are the approximate numbers for income concentration and public debt in the OECD countries pre-1980s reported in Figure 1. To reach these two targets, the values of Δ and Φ are chosen simultaneously through an iterative procedure. The resulting values are $\Delta = 0.14$ and $\Phi = 3.902$. These values imply that the standard deviation of entrepreneurial income is about 15 percent the value of land used in production, pk, and the population share of workers is slightly below 80 percent.

Results: Figure 9 shows the evolution of public debt over time for different time-horizons under autarky and mobility. Debt is smaller when the horizon is short (third panel). This is because high borrowing in the initial periods will result in low workers' consumption in the terminal periods, when the debt needs to be repaid. Given the concavity of the utility function, this will reduce welfare. As the horizon expands, so does the number of periods over which the debt can be repaid, which

[12] Entrepreneurial income is equal to $A(z_t^i)k_t^i + (z_t^i - \bar{z})k_t^i p_t + b_t^i - b_t^i/R_t + \tau_t$, that is, profits, interests and transfers. The income of an individual worker is equal to $w_t/\Phi + \tau_t$, that is, labor income plus and transfers.

increases the desire to borrow initially. The infinite horizon economy can be well approximated with a large value of T: As the first panel indicates, B converges after about 40 periods. The figure also shows that financial integration has stronger effects when the horizon is longer.

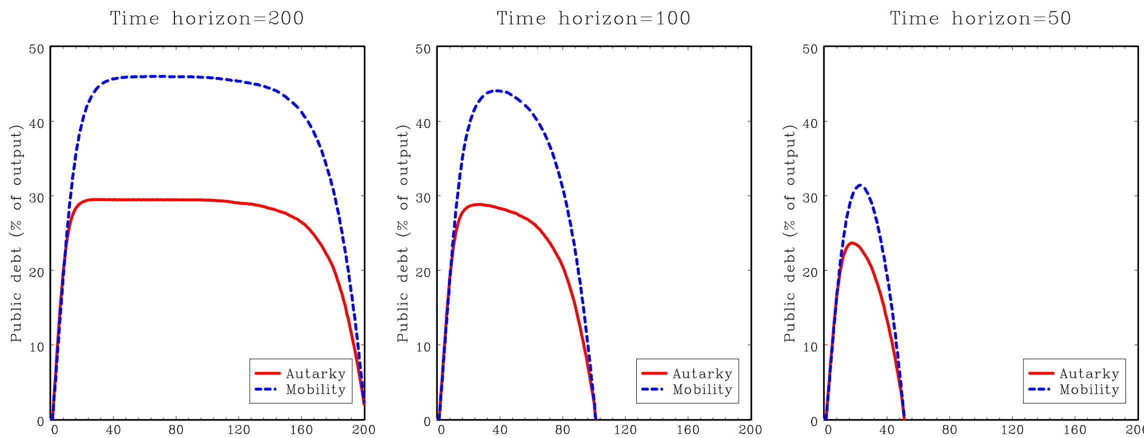

Figure 9: Evolution of government debt for different time horizons.

Figure 10 plots the transition dynamics for government debt induced by international capital market liberalization and increased income inequality. The initial equilibrium is the steady state under Autarky. This is the middle point of the first panel in Figure 9. The increase in income inequality is generated by a higher volatility of the idiosyncratic risk, which changes from $\Delta = 0.14$ to $\Delta = 0.1725$. As described above, $\Delta = 0.14$ was chosen to generate the 6 percent concentration of income at the top 1 percent in the autarky steady state. The new value is chosen to have a share of 7.5 percent for the top income earners in the steady state with capital mobility. As shown in Figure 1, this is about half the increase in concentration for the OECD countries during the sample period: the top 1 percent share is about 9 percent toward the end of the sample. Since we do not know which part of the increase in inequality is driven by income risk, as opposed to cross-sectional inequality that is predictable at the individual level, we have assumed that the increase in risk contributed only 50 percent. The targeted number has been replicated in the steady state with mobility because the 2000s are characterized by a high degree of financial integration among industrialized countries.

Before continuing, we would like to explain why we make the assumption that inequality increases in both countries even if in the data the increase is observed only in some countries (see Atkinson, Piketty, and Saez (2011)). Our choice is motivated by computational considerations. When countries have different Δ, Corollary 2.1 no longer holds. Thus, we cannot impose the condition that domestic and foreign entrepreneurs hold the same $\tilde{b}$. This implies that to compute the equilibrium we have to add another state variable: $\tilde{b}_1$ or $\tilde{b}_2$. With capital mobility this increases the computational complexity significantly. However, limiting the analysis to the symmetric case is not a major shortcoming because, as shown in Section 3 with the two-period model, the change in inequality in only one country also affects the debt chosen by the other country when financial markets are integrated. Thus, using the average change in inequality as the target for all countries

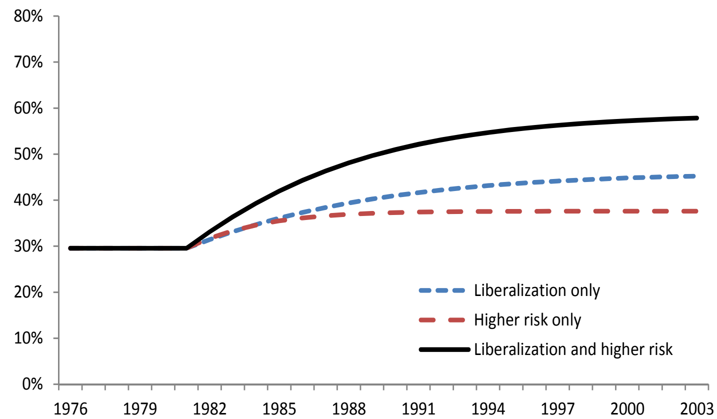

Figure 10: Responses of public debt financial liberalization and increased income risk.

provides a reasonable approximation to the response of public debt in all integrated economies when the change is asymmetric.

As can been seen in Figure 10, capital liberalization (ignoring higher risk) increases long-term debt from 30 percent of income to about 46 percent of income. If we focus instead on the change in risk alone (keeping the economies in autarky), long-term debt increases to 38 percent of income. When the two changes are considered together, long-term debt increases to 59 percent.

To compare the dynamics of the model to the empirical series, Figure 11 plots the data generated by the model (with both liberalization and increased risk) and the empirical data for the average of the OECD countries, Europe, and the United States. The response of the interest rate is also plotted. The dynamic path of public debt generated by the model (continuous line) resembles the dynamics observed in the data (dashed lines). The dynamics of the interest rates are also qualitatively similar. In particular, we see interest rate hikes at the beginning of the 1980s, with a subsequent decline later in the sample.

The initial jump in the interest rate generated by the model is necessary to make bonds attractive to entrepreneurs who are the buyers of the additional bonds. The increase in the holding of bonds requires entrepreneurs to reduce current consumption in compensation for higher future consumption, which in turn requires higher interest rates. Since the government continues to increase the debt after the first period, the interest rate remains high. However, since the increase in government debt slows down over time, the interest rate declines gradually after the initial jump. In the long run, R is higher than in the autarky steady state, but the difference is small.[13]

[13]The dynamic of the interest rate can be understood by looking at the Euler equation for bonds for a representative entrepreneur, $1 = \beta R_t \mathbb{E}_t\left(c_t/c_{t+1}\right)$. As long as the government borrows more in response to liberalization, the consumption growth increases (since current consumption must decline in order to buy the additional bonds). Then we can see from the Euler equation that the interest rate R_t increases. After the initial increase, the growth rate of consumption declines gradually and converges back to the steady state. This implies that the interest rate declines after the initial jump. In response to an increase in income risk, however, it is more difficult to illustrate the dynamics of the interest rate because there are two contrasting effects. The first is the effect already described. The second derives from the fact that, since entrepreneurs face higher risk, the term $\mathbb{E}_t(c_t/c_{t+1})$ increases (as a result of the

(a) Public debt (b) Interest rate

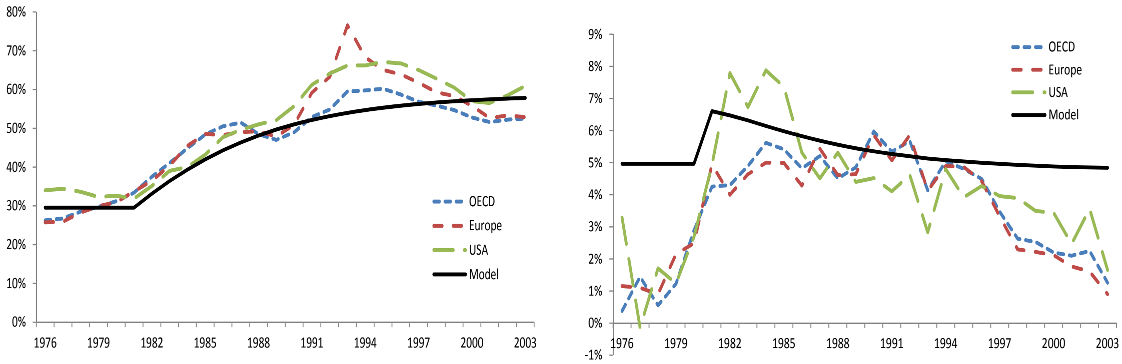

Figure 11: Responses of public debt and real interest rates to liberalization and increased income risk.

We would like to emphasize that the comparison of the dynamics of the interest rate generated by the model with the empirical series is not meant to show that the interest rate dynamics can be fully explained by capital markets liberalization and increased income risk. Of course, there are many other factors that contributed to the interest rate dynamics, especially the hike in the early 1980s. We only want to show that the pattern predicted by the model is not inconsistent with the pattern observed in the data. Also, if the changes in capital mobility and income risk were gradual, then the response of the interest rate generated by the model is likely to be more gradual.

5 Conclusion

The stock of public debt has increased in most advanced economies during the last 30 years, a period also characterized by extensive liberalization of international capital markets and a sustained increase in income inequality. In this paper we study a multi-country politico-economic model where the incentives of governments to borrow increase both when financial markets become internationally integrated and when inequality rises if this is associated with higher income risk. We propose this mechanism as one of the possible explanations for the growing stocks of government debt observed in most of the advanced economies since the early 1980s. We have also conducted a cross-country empirical analysis using OECD data and the results are consistent with the theoretical predictions.

The final remark relates to the relevance of the analysis conducted in this paper for understanding the recent difficulties in sovereign borrowing. If debt crises are more likely to arise when the stock of public debt is higher, then the growth in government borrowing induced by capital markets liberalization and increased income inequality may contribute to triggering a sovereign debt crisis. An extension that explicitly studies the possibility of default on sovereign debt is, however, left for future research.

higher volatility of c_{t+1}). This tends to reduce the interest rate as we can see from the Euler equation. Since there are two effects, we cannot say which one dominates. However, we have conducted some numerical sensitivity and found that the initial jump and the subsequent decline are quite robust.

Appendix

A Data description for Figures 1, 2 and 11

1) *Debt/GDP Ratio* is total (domestic plus external) gross central government debt over GDP, from Reinhart and Rogoff (2011). Sample period: 1973-2005.

2) *Financial Liberalization Index* is from Abiad, Detragiache, and Tressel (2008). Sample period: 1973-2005.

3) *Income Share of Top 1 percent* is from Alvaredo, Atkinson, Piketty, and Saez (2011).

4) *Deflator, p*, is the GDP deflator from Kaminsky, Reinhart, and Végh (2004). Sample period: 1970-2003.

5) *Inflation*, π, is computed as $\pi_t = p_t/p_{t-1} - 1$.

6) *Expected Inflation*, π^e, is computed as the fitted values from the regression

$$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + \alpha_4\pi_{t-4} + \epsilon_t.$$

7) *Nominal Interest Rate, i*, is the long-term (10 years) interest rates on government bonds from OECD Statistics. Generally the yield is calculated at the pre-tax level and before deductions for brokerage costs and commissions and is derived from the relationship between the present market value of the bond and at maturity, also taking into account interest payments paid through to maturity.

8) *Real Interest Rate, r*, is computed as $r_t = (1 + i_t)/(1 + \pi^e_{t+1}) - 1$, where i is the nominal interest rate and π^e is expected inflation.

9) *OECD*: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Japan, Korea, Mexico, the Netherlands, New Zealand, Norway, Poland, Portugal, Spain, Sweden, Switzerland, Turkey, the United Kingdom, and the United States.

10) *EUROPE*: Austria, Belgium, Bulgaria, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Netherlands, Norway, Poland, Portugal, Russia, Spain, Sweden, Switzerland, Turkey, and UK.

11) *Figure 2* plots the fitted values and confidence interval from the following linear regression

$$\frac{\Delta r_{j,t}}{\Delta \ln(B_{j,t})} = \alpha_0 + \alpha_1 \cdot t,$$

where $r_{j,t}$ is the real interest rate in country j in year t and $B_{j,t}$ is the real government debt in country j in year t. Since the the dependence variable is a ratio and could take very large values when the denominator is close to zero, we have eliminated observations for which the ratio is bigger than 5. Without this selection the graph has a similar shape but the elasticity values are bigger.

B Proof of Lemma 2.1

Terminal conditions imply $k^i_{j,T+1} = \tilde{b}^i_{j,T+1} = 0$. For $t < T$, we guess $k^i_{j,t+1} = \frac{\eta_t\phi_{j,t}}{p_{j,t}}a^i_{j,t}$ and $\tilde{b}^i_{j,t+1} = R_{j,t}\eta_t(1-\phi_{j,t})a^i_{j,t}$, where η_t is an unknown time-varying parameter. Then, $c^i_{j,t} = (1-\eta_t)a^i_{j,t}$ and $a^i_{j,t+1}$ satisfies

$$a^i_{j,t+1} = \eta_t\left[\left(\frac{A(z^i_{j,t+1}, w_{j,t+1}) + p_{j,t+1}}{p_{j,t}}\right)\phi_{j,t} + R_{j,t}(1-\phi_{j,t})\right]a^i_{j,t}.$$

The first-order conditions with respect to land and bond holdings for $t < T$ become

$$\frac{\eta_t}{1-\eta_t} = \beta\mathbb{E}\left\{\frac{\frac{A(z^i_{j,t+1},w_{j,t+1})+p_{j,t+1}}{p_{j,t}}}{(1-\eta_{t+1})\left[\left(\frac{A(z^i_{j,t+1},w_{j,t+1})+p_{j,t+1}}{p_{j,t}}\right)\phi_{j,t}+R_{j,t}(1-\phi_{j,t})\right]}\right\} \tag{28}$$

$$\frac{\eta_t}{1-\eta_t} = \beta\mathbb{E}\left\{\frac{R_{j,t}}{(1-\eta_{t+1})\left[\left(\frac{A(z^i_{j,t+1},w_{j,t+1})+p_{j,t+1}}{p_{j,t}}\right)\phi_{j,t}+R_{j,t}(1-\phi_{j,t})\right]}\right\} \tag{29}$$

Multiply the two conditions by $\phi_{j,t}$ and $1-\phi_{j,t}$, respectively, and add them to get

$$\frac{\eta_t}{1-\eta_t} = \beta\mathbb{E}\left(\frac{1}{1-\eta_{t+1}}\right).$$

Hence, $\eta_T = 0$ and

$$\eta_t = \beta\frac{1}{1+\beta^{T-t}\left(\sum_{s=1}^{T-t}\beta^{s-1}\right)^{-1}} \qquad \forall t < T$$

verify the guess, and the first optimality condition becomes

$$\mathbb{E}\left[\frac{R_{j,t}}{\left(\frac{A(z^i_{j,t+1},w_{j,t+1})+p_{j,t+1}}{p_{j,t}}\right)\phi_{j,t}+R_{j,t}(1-\phi_{j,t})}\right] = 1. \tag{30}$$

Q.E.D.

C Proof of Proposition 2.1

We first show that the wage rate does not depend on the distribution and it is constant. The optimality condition for the input of labor is $F_l(z^i_{j,t}, k^i_{j,t}, l^i_{j,t}) = w_{j,t}$. Because the production function is homogeneous of degree 1, the demand of labor is linear in land, that is, $l^i_{j,t} = l(z^i_{j,t}, w_{j,t})k^i_{j,t}$. If we integrate over all i and average over z, we obtain the aggregate demand of labor

$$\int_i \sum_\ell l(z_\ell, w_{j,t})k^i_{j,t}\mu_\ell = \sum_\ell l(z_\ell, w_{j,t})\mu_\ell \int_i k^i_{j,t},$$

where the expression on the right-hand-side uses the law of large numbers. Since in equilibrium the demand of labor must be equal to the supply, which is 1, and total land is also 1, the above condition can be rewritten as $1 = \sum_\ell l(z_\ell, w_{j,t})\mu_\ell$. This implicitly defines the wage, which does not depend on endogenous variables. Therefore, the wage is constant. Since the distribution of z is the same across countries, the wage rate must also be equal across countries, that is, $w_{j,t} = \bar{w}$.

Eq. (11) follows from replacing the government's budget constraint (5) into the worker's budget constraint (eq. (4)). Eq. (7) is obtained from eq. (28) after replacing $R_{j,t}(1-\phi_{j,t}) = \phi_{j,t}\tilde{b}_{j,t+1}/p_{j,t}$. This expression is derived from Lemma 2.1. To obtain eq. (8), combine aggregate asset holdings $\bar{a}_{j,t} = \sum_\ell A(z_\ell, \bar{w})\mu_\ell + p_{j,t} + \tilde{b}_{j,t}$ with the aggregated choice of land, $p_{j,t}\bar{k} = \eta_t\phi_{j,t}\bar{a}_{j,t}$. Taking into account that the wage is $\bar{w}$, $\bar{k} = 1$, and defining $\sum_\ell A(z_\ell, \bar{w})\mu_\ell = \bar{A}$, we obtain eq. (8).

To derive eq. (9), consider the aggregate entrepreneurs' budget constraint $c^e_{j,t} + \frac{\tilde{b}_{j,t+1}}{R_{j,t}} = \bar{A} + \tilde{b}_{j,t}$. We can now use the aggregate policy $c^e_{j,t} = (1-\eta_t)\bar{a}_{j,t}$ and aggregate asset holdings $\bar{a}_{j,t} = \bar{A} + p_{j,t} + \tilde{b}_{j,t}$ to eliminate consumption and use eq. (8) to eliminate $p_{j,t}$ and solve for $R_{j,t}$.

To derive eq. (10), aggregate individual entrepreneurs' budgets to obtain $c^e_{j,t} + \frac{\tilde{b}_{j,t+1}}{R_{j,t}} = \bar{A} + \tilde{b}_{j,t}$, and rearrange the terms to get eq. (10). *Q.E.D.*

D Proof of Corollary 2.1

Proof of Proposition 2.1 established that $\eta_T = 0$, which, together with eq. (8), implies that $p_{j,T} = 0\ \forall j$. Replacing this in eq. (7), we obtain

$$\phi_{j,T-1} = \mathbb{E}_{T-1}\left[\frac{A(z^i_{j,T})}{A(z^i_{j,T}) + \tilde{b}_{j,T}}\right]. \tag{31}$$

Rewriting eq. (9) at date $t = T-1$, and using $R_{j,t} = R_t$, $\forall j$ in an integrated equilibrium delivers

$$\tilde{b}_{j,T} = R_{T-1}(\bar{A} + \tilde{b}_{j,T-1})\frac{\eta_{T-1}(1-\phi_{j,T-1})}{(1-\eta_{T-1}\phi_{j,T-1})}. \tag{32}$$

Replacing eq. (32) into eq. (31) results in $\phi_{j,T-1}$ being a function of $\tilde{b}_{j,T-1}$. Notice that this holds because we have assumed a common shock structure across countries. Using the function $\phi_{j,T-1}(\tilde{b}_{j,T-1})$ in eq. (32) delivers $\tilde{b}_{j,T}(\tilde{b}_{j,T-1})$. Substituting these two functions in eq. (8), evaluated at $t = T-1$, yields $p_{T-1}(\tilde{b}_{j,T-1})$.

Evaluating eqs. (7) and (9) at $t = T-2$, and using the functions derived above, we can show that $\tilde{b}_{j,T-1}(\tilde{b}_{j,T-2})$. By repeated substitution over time (following the same steps) we can obtain an expression for $\tilde{b}_{j,2}$ which only depends on $\tilde{b}_{j,1}$. Since $\tilde{b}_{j,1} = \tilde{b}_1$, $\forall j$, then $\tilde{b}_{j,2} = \tilde{b}_2$, $\forall j$. Substituting forward, we can easily show that $\tilde{b}_{j,t+1} = \tilde{b}_{t+1}\,\forall j$. Adding up across countries, and using the bond-market equilibrium condition, $\sum_{j=1}^N \tilde{b}_{j,t+1} = N\tilde{b}_{t+1} = \nu\sum_{j=1}^N B_{j,t+1}$. *Q.E.D.*

E Proof of Lemma 3.1

Follow the steps in the proof of Proposition 2.1 (see section C) to derive

$$\frac{B}{R} = \frac{\beta\bar{A}(1-\phi(B))}{\nu[1+\beta(1-\phi(B))]} > 0, \quad \text{where} \tag{33}$$

$$\phi(B) = \mathbb{E}\left(\frac{A(z)}{A(z)+\nu B}\right) < 1. \tag{34}$$

i. Let B^A satisfy the FOC $\frac{\partial W(B)}{\partial B} = 0$, with $\frac{\partial W(B)}{\partial B} = \frac{\nu}{c^w_1}\frac{\partial(B/R)}{\partial B} - \beta\frac{\nu}{c^w_2}$, where $c^w_1 = \bar{w} + \nu B/R$ and $c^w_2 = \bar{w} - \nu B$ are aggregate workers' consumption. Since $\frac{\partial W(B)}{\partial B} > 0|_{B=0}$ and $\frac{\partial W(B)}{\partial B} \to -\infty$ as $B \to \frac{\bar{w}}{\nu}$, then $B^A \in \left[0, \frac{\bar{w}}{\nu}\right]$.

Uniqueness follows from the concavity of $W(B)$. Differentiating eq. (12) twice yields

$$\frac{\partial^2 W(B)}{\partial B^2} = -\frac{\nu^2}{(c^w_1)^2}\left[\frac{\partial(B/R)}{\partial B}\right]^2 + \frac{\nu}{c^w_1}\frac{\partial^2(B/R)}{\partial B^2} - \frac{\beta\nu^2}{(c^w_2)^2}. \tag{35}$$

Since

$$\frac{\partial^2 \phi(B)}{\partial B^2} = 2\mathbb{E}\left[\frac{A(z)\nu^2}{(A(z)+\nu B)^3}\right] > 0, \tag{36}$$

we have that

$$\frac{\partial^2 (B/R)}{\partial B^2} = -\frac{\beta \bar{A}}{\nu(1+\beta[1-\phi(B)])^3}\left[\frac{\partial^2 \phi(B)}{\partial B^2}(1+\beta[1-\phi(B)]) + 2\beta\left(\frac{\partial \phi(B)}{\partial B}\right)^2\right] < 0.$$

ii. Replace eq. (33) into the representative entrepreneur's consumption and obtain $c_1^e = \frac{\bar{A}}{1+\beta[1-\phi(B)]}$. Then, differentiate the resulting indirect utility

$$\frac{\partial V(B)}{\partial B} \quad = \quad \frac{\beta}{1+\beta(1-\phi(B))}\frac{\partial \phi(B)}{\partial B} + \beta\mathbb{E}\left(\frac{\nu}{A(z)+\nu B}\right).$$

Substitute

$$\frac{\partial \phi(B)}{\partial B} \quad = \quad -\mathbb{E}\left[\frac{\nu A(z)}{(A(z)+\nu B)^2}\right] \tag{37}$$

in the expression above and collect terms to show

$$\frac{\partial V(B)}{\partial B} = \beta\nu\mathbb{E}\left[\frac{\nu B + \beta[1-\phi(B)]\,(A(z)+\nu B)}{(A(z)+\nu B)^2\,(1+\beta[1-\phi(B)])}\right] > 0.$$

Q.E.D.

F Proof of Proposition 3.1

Suppose that $\Phi > \frac{(1+\beta)\bar{w}}{\bar{A}} + \beta$, and let the government's objective be defined by

$$G(B) \equiv \Phi W(B) + V(B)$$

where $W(B)$ and $V(B)$ are given by eqs. (12) and (13). To prove concavity, differentiate $G(B)$ twice, where $\frac{\partial^2 W(B)}{\partial B^2}$ is defined in eq. (35) and

$$\frac{\partial^2 V(B)}{\partial B^2} = -\frac{\nu^2}{(c_1^e)^2}\left[\frac{\partial (B/R)}{\partial B}\right]^2 - \frac{\nu}{c_1^e}\frac{\partial^2 (B/R)}{\partial B^2} - \beta\mathbb{E}\frac{\nu^2}{(c_2^e)^2}.$$

After some manipulations, we can show that

$$\begin{aligned}\frac{\partial^2 G(B)}{\partial B^2} \quad &= \quad -\left[\frac{\partial (B/R)}{\partial B}\right]^2 \nu^2\left[\frac{\Phi}{(c_1^w)^2} + \frac{1}{(c_1^e)^2}\right] - \beta\nu^2\left[\frac{\Phi}{(c_2^w)^2} + \mathbb{E}\frac{1}{(c_2^e)^2}\right] \\ &\quad + \quad \frac{\partial^2 (B/R)}{\partial B^2}\nu\left[\frac{\Phi}{c_1^w} - \frac{1}{c_1^e}\right].\end{aligned}$$

The first row is negative for all B. Hence, a sufficient condition for $\frac{\partial^2 G(B)}{\partial B^2} < 0$ is that the second row is non-positive. We established that $\frac{\partial^2 (B/R)}{\partial B^2} < 0$ in Section E (Part *i.*). In addition, we need that

$$\frac{\Phi}{c_1^w} - \frac{1}{c_1^e} \quad = \quad \frac{\Phi c_1^e - c_1^w}{c_1^w c_1^e} > 0,$$

since $c_1^e = \frac{\bar{A}}{1+\beta[1-\phi(B)]}$ and $c_1^w = \bar{w} + \nu B/R$. Substituting for R we get that

$$\begin{aligned} c_1^e - c_1^w/\Phi &= \frac{1}{1+\beta(1-\phi(B))}\left[\bar{A} - \frac{1}{\Phi}\left([1+\beta(1-\phi(B))]\bar{w} + \beta\bar{A}(1-\phi(B))\right)\right] \\ &\geq \frac{1}{\Phi}\frac{1}{1+\beta(1-\phi(B)}[\bar{A}(\Phi-\beta) - \bar{w}(1+\beta)]. \end{aligned}$$

Since $0 \leq \phi(B) \leq 1$, the denominator of the above equation is positive. Moreover, the assumption that $\Phi > \frac{(1+\beta)\bar{w}}{\bar{A}} + \beta$ is sufficient for the numerator to be positive. This establishes concavity.

Let B^A satisfy $\frac{\partial G(B)}{\partial B} = 0$. From Lemma 3.1, $V(B)$ is increasing in B $\forall B \in \left[0, \frac{\bar{w}}{\nu}\right]$ and $\frac{\partial W(B)}{\partial B}|_{B=0} > 0$ $\Rightarrow \frac{\partial G(B)}{\partial B}|_{B=0} > 0$. Additionally, $\frac{\partial V(B)}{\partial B}$ is finite at $\frac{\bar{w}}{\nu}$ and $\frac{\partial W(B)}{\partial B} \to -\infty$ as $B \to \frac{\bar{w}}{\nu}$, so $\frac{\partial G(B)}{\partial B} \to -\infty$. Hence $B^A \in \left[0, \frac{\bar{w}}{\nu}\right]$. Because $G(B)$ is strictly concave, B^A must be unique. *Q.E.D.*

G Proof of Proposition 3.2

Let the relative size of workers $\nu = 1$. To show that debt is increasing in N, replace $\Phi/(1+\Phi) = 1$ in eq. (21) to obtain

$$G(B,N) \equiv \Phi\left[\frac{\partial(B/R)}{\partial B}\left(\frac{1}{c_1^w}\right) - \beta\left(\frac{1}{c_2^w}\right)\right] = 0,$$

where

$$\frac{\partial(B/R)}{\partial B} = \frac{1}{R}\left(1 - \frac{B}{R}\frac{\partial R}{\partial b}\frac{1}{N}\right) \equiv \gamma \qquad \text{and} \qquad \frac{\partial R}{\partial b} = R\left[\frac{1}{b} + \frac{\partial\phi}{\partial b}\frac{1}{[1+\beta(1-\phi)](1-\phi)}\right]. \tag{38}$$

Recall that $b = \frac{\sum_{j=1}^N B_j}{N}$ denotes the demand of bonds.

Claim G.1: The interest rate is increasing in b, $\frac{\partial R}{\partial b} > 0$.

Proof: Rewrite eq. (38) as

$$\begin{aligned} \frac{\partial R}{\partial b} &= \frac{R}{b[1+\beta(1-\phi)](1-\phi)}\left[[1+\beta(1-\phi)](1-\phi) + b\frac{\partial\phi}{\partial b}\right] \\ &> \frac{R}{b[1+\beta(1-\phi)](1-\phi)}\left[1-\phi + b\frac{\partial\phi}{\partial b}\right] = 0 \end{aligned}$$

The inequality follows from $\beta(1-\phi) < 1$. Replace eqs. (34) and (37) in the bracketed term to show equality.

Claim G.2: *(i.)* $\partial G(B,N)/\partial B < 0$ and *(ii.)* $\partial G(B,N)/\partial N > 0$

Proof:

(i.) We can show that

$$\frac{\partial G(B,N)}{\partial B} = \Phi\left[\frac{\partial\gamma}{\partial b}\frac{1}{c_1^w} - \gamma^2\frac{\Phi}{(c_1^w)^2} - \beta\frac{\Phi}{(c_2^w)^2}\right]. \tag{39}$$

where

$$\frac{\partial\gamma}{\partial b} = -\left(1 - \frac{B}{Nb}\right)\frac{2}{NR^2}\frac{\partial R}{\partial b} - \frac{B}{Nb}\frac{\beta\bar{A}}{(1+\beta(1-\phi))^2}\left[\frac{\partial^2\phi}{\partial b^2} + \frac{2\beta\left(\frac{\partial\phi}{\partial b}\right)^2}{1+\beta(1-\phi)}\right].$$

Since $\frac{\partial^2\phi}{\partial b^2} > 0$ from eq. (36) and $\frac{\partial R}{\partial b} > 0$ from Claim G.1, then $\frac{\partial \gamma}{\partial b} < 0$. Because all terms in eq. (39) are negative, the result follows.

(ii.) We can show that

$$\frac{\partial G(B,N)}{\partial N} = \Phi\left[\frac{\partial\gamma}{\partial N}\frac{1}{c_1^w} - \gamma\frac{\Phi}{(c_1^w)^2}\frac{\partial(B/R)}{\partial N}\right].$$

The first term is positive. Noting that since $b = B$ then $\frac{\partial b}{\partial N} = \frac{b-B}{N^2} = 0$, and performing some algebraic manipulations, we obtain $\frac{\partial\gamma}{\partial N} = \frac{B}{R^2N^2}\frac{\partial R}{\partial b} > 0$ from Claim G.1. The second term is zero, since

$$\frac{\partial(B/R)}{\partial N} = -\left[\frac{1-\phi}{b} + \frac{1}{[1+\beta(1-\phi)]}\frac{\partial\phi}{\partial b}\right]\frac{B\beta\bar{A}}{[1-\beta(1-\phi)]b}\frac{\partial b}{\partial N}$$

and $\frac{\partial b}{\partial N} = 0$.

Using Claim G.2 and the implicit function theorem, we show that domestic debt B is increasing in N

$$\frac{\partial B}{\partial N} = -\frac{\partial G(B,N)/\partial N}{\partial G(B,N)/\partial B} > 0.$$

For the limiting case, let $N \to \infty$ in eq. (21). Substituting c_1^w and c_2^w and rearranging, we obtain

$$\beta R = 1 - \frac{1+\beta}{\bar{w}}B. \tag{40}$$

This equation determines country 1's supply of debt given R. In equilibrium, $B_1 = B_2 = ..B_N = b = b$ where the per capita demand for debt b satisfies eq. (18). The financially integrated equilibrium levels of b and R are thus determined by eqs. (18) and (40).

Existence and uniqueness follow from: (i) the LHS of eq. (40) is decreasing in b and equals 1 at the origin, and (ii) the RHS of eq. (40) is increasing in b (since $R_b > 0$) and has an intercept at $\left[\mathbb{E}\left(\frac{\bar{z}}{z}\right)\right]^{-1} < 1$. Denote the intersection point by B^{FI}. From (i) and (ii) it also follows that B^{FI} is bounded and $\beta R < 1$ when $b = B^{FI}$.

Under autarky, eq. (40) is instead

$$\beta R = 1 - \frac{1+\beta}{\bar{w}}b - \epsilon(b)\left(1 - \frac{b}{\bar{w}}\right). \tag{41}$$

The LHS is the same as before. The RHS is also equal to 1 at the origin because $\epsilon(0) = 0$. Since $\epsilon(b) > 0$ and $\bar{w} - b = c_2^w > 0$ when $b > 0$, the new term in the RHS is positive. Hence, the intersection of the two curves in eq. (41) occurs at $B^A < B^{FI}$, since the RHS is steeper.

Since debt is larger and V is increasing in b, $V(B^A) < V(B^{FI})$. Since W is concave in b and $W(b)$ is decreasing when $b > B^A$, then $W(B^A) > W(B^{FI})$.

What is left to prove is that the equilibrium must be symmetric. This can be shown starting from the first order condition of the government which must be satisfied for all countries,

$$\Phi\cdot\left[\frac{1-\frac{\epsilon(\overline{B})B}{N\overline{B}}}{c_1^w} - \frac{\beta R}{c_2^w}\right] = \left(\frac{1}{N}\right)\cdot\left[\frac{1-\epsilon(\overline{B})}{c_1^e} - \mathbb{E}_t\left(\frac{\beta R}{c_2^e}\right)\right]. \tag{42}$$

An equilibrium is characterized by a worldwide debt $\overline{B}$. Given $\overline{B}$, the elasticity ϵ and the interest rate R are determined. Also notice that the right-hand side of (42) is the same for all countries, since entrepreneurs choose to hold the same stock of bonds in all countries. The left-hand side could differ, since governments could choose different B. However, since the left-hand side is strictly decreasing in B (keeping $\overline{B}$ constant), the fact that the right-hand side is the same for all countries implies that B must be the same for all countries. Otherwise, the first-order condition (42) will not hold for all countries. Notice that this result applies for any value of Φ, not only for the limiting case $\Phi/(1+\Phi) = 1$. *Q.E.D.*

H Proof of Proposition 3.3

Setting $\Phi/(1+\Phi) = 1$, the first-order conditions for the domestic and foreign country become

$$1 - \frac{\alpha B_1}{\overline{B}}\epsilon(\overline{B}) = \beta R(\overline{B})\left(\frac{c_1^w(B_1)}{c_2^w(B_1)}\right) \tag{43}$$

$$1 - \frac{(1-\alpha) B_2}{\overline{B}}\epsilon(\overline{B}) = \beta R(\overline{B})\left(\frac{c_1^w(B_2)}{c_2^w(B_2)}\right), \tag{44}$$

where we have made it explicit that the interest rate elasticity, $\epsilon(\overline{B})$, and the interest rate, $R(\overline{B})$, are functions of the average worldwide debt $\overline{B} = \alpha B_1 + (1-\alpha)B_2$.

An equilibrium will be characterized by B_1 and B_2 (and $\overline{B}$) that satisfy conditions (43) and (44). We want to show that in an integrated economy $B_1 > B_2$ if $\alpha < 1/2$, that is, the per capita debt of the large country is lower than the per capita debt of the small country.

Subtracting (44) to (43) and substituting $(1-\alpha)B_2 = \overline{B} - \alpha B_1$ we get

$$\left(1 - \frac{2\alpha B_1}{\overline{B}}\right)\epsilon(\overline{B}) = \beta R(\overline{B})\left(\frac{c_1^w(B_1)}{c_2^w(B_1)} - \frac{c_1^w(B_2)}{c_2^w(B_2)}\right) \tag{45}$$

For a given $\overline{B}$ that characterizes the equilibrium, the left-hand-side term is decreasing in B_1. Since $\overline{B}$ is the equilibrium worldwide debt taken as given in this exercise, an increase in B_1 must be associated with a decline in B_2. Therefore, it is the ratio B_1/B_2 that matters. The right-hand-side term, instead, is increasing in B_1. To see this, we can define aggregate workers' consumption using the budget constraints as

$$c_1^w(B_1) = \bar{w} + \frac{B_1}{R(\overline{B})}, \qquad c_2^w(B_1) = \bar{w} - B_1 \tag{46}$$

$$c_1^w(B_2) = \bar{w} + \frac{B_2}{R(\overline{B})}, \qquad c_2^w(B_2) = \bar{w} - B_2 \tag{47}$$

From these equations it is clear that $c_1^w(B_1)/c_2^w(B_1)$ is increasing in B_1 and $c_1^w(B_2)/c_2^w(B_2)$ is increasing in B_2. Since an increase in B_1 must be associated with a decline in B_2, then $c_1^w(B_2)/c_2^w(B_2)$ is decreasing in B_1. Thus, the right-hand side of eq. (45) must be increasing in B_1.

So far, we have established that the LHS of eq. (45) is decreasing and the RHS is increasing in B_1. Next, we observe that, if $\alpha < 1/2$, then the LHS is positive when $B_1 = B_2$. The RHS, instead, is zero. Therefore, to equalize the LHS (which is decreasing in B_1) to the RHS (which is increasing in B_1) we have to increase B_1 (which must be associated with a decrease in B_2). Therefore, if $\alpha < 1/2$, $B_1 > B_2$.

Finally, since in the autarky equilibrium both countries had the same debt, the growth in debt following financial liberalization is bigger for the small country. *Q.E.D.*

I Proof of Proposition 3.4

Let $\Phi/(1+\Phi) = 1$, then the autarky equilibrium satisfies the government's first-order condition

$$\frac{1-\epsilon(B)}{R(B)c_1^w} = \frac{\beta}{c_2^w},$$

where we made it explicit that the interest elasticity ϵ and the interest rate R are functions of debt B. Since $c_1^w = \bar{w} + B/R(B)$ and $c_2^w = \bar{w} - B$, the first-order condition can be rewritten as

$$\frac{1-\epsilon(B)}{\bar{w}R(B) + B} = \frac{\beta}{\bar{w} - B}. \tag{48}$$

The right-hand side of (48) is clearly increasing in B. We now show that the left-hand side is decreasing in B. First let's rewrite the left-hand side as

$$\frac{1-\epsilon(B)}{\bar{w}R(B)+B} = \left(\frac{1-\epsilon(B)}{R(B)}\right) \cdot \left(\frac{1}{\bar{w}+B/R(B)}\right), \tag{49}$$

which is the product of two terms. We want to show that both terms are decreasing in B. Let's start with the first term which is equal to

$$\frac{1-\epsilon(B)}{R(B)} = -\frac{\beta\phi'(B)\bar{A}}{[1+\beta(1-\phi(B))]^2}.$$

Since $\phi(B) = \mathbb{E}[A(z)/(A(z)+B)]$ and $-\phi'(B) = \mathbb{E}[A(z)/(A(z)+B)^2]$ are both decreasing in B, then the first term in (49) is also decreasing in B. The second term in (49) depends negatively on $B/R(B) = \beta(1-\phi(B)\bar{A}/[1+\beta(1-\phi(B))]$. As we have already observed, $\phi(B) = \mathbb{E}A(z)/(A(z)+B)$ depends negatively on B and, therefore, $B/R(B)$ increases in B. Thus, the second term in (49) decreases with B. This proves that (49) is decreasing in B.

To summarize, we have shown that the left-hand side of first order condition (48) decreases with B, while the right-hand side increases with B. Therefore, if an increase in the mean preserving spread of z raises the term $(1-\epsilon(B))/[\bar{w}R(B)+B]$ on the left-hand side, to re-establish equality B has to rise. *Q.E.D.*

J Proof of Proposition 3.5

Let B^* denote the optimal level of debt in the benchmark case, where $\overline{D} = 0$. Denote by R^*, c_t^{w*}, and c_t^{e*} the associated interest rate and allocations.

Consider a case where $\overline{D} > 0$ and let $\tilde{d} = d + \frac{B}{1+\Phi}$ denote workers' excess supply of debt. Workers choose $\tilde{d}$ in order to maximize (1) subject to $c_1^w = \bar{w}\frac{1}{\Phi} + \frac{\tilde{d}}{R}$, $c_2^w = \bar{w}\frac{1}{\Phi} - \tilde{d}$, and the borrowing constraint $\tilde{d} \leq \overline{d} + \frac{B}{1+\Phi}$. The interest rate is determined by the entrepreneurs' optimality condition,

$$R = \frac{1}{A}\left(\frac{1}{\beta E\frac{1}{A(z)+\tilde{b}}} + \tilde{b}\right). \tag{50}$$

In equilibrium, $b = B + D$, where $D = \Phi d$, implying $\tilde{b} = \Phi e\tilde{d} = \nu B + D$. When workers are fully unconstrained ($\overline{D} \to \infty$), their FOC holds with equality: $c_2^w = \beta R c_1^w$. This equation, together with eq. (50), determines $\tilde{d}$ and R independently of B, so public debt is irrelevant (e.g., Ricardian equivalence holds). Since only $d - B/(1+\Phi)$ is determined, we can set $B = 0$ without loss of generality and denote the optimal aggregate amount of debt when workers are unconstrained by D^*. Clearly, $\nu B^* < D^*$.

Our conjecture is that, for $\overline{D} \in (0, \infty)$, the government will optimally set $B = B^* - \overline{D}/\nu$. To verify this, note that under the conjecture $\tilde{b} = \nu B^* + D - \overline{D} \leq \nu B^* < D^*$. But then, the FOC of workers is slack, so $D = \overline{D}$. Because $\tilde{b} = \nu B^*$, the interest rate and allocations will be R^*, c_t^{w*}, and c_t^{e*}, which satisfy the government optimality condition (15). *Q.E.D.*

K Proof of Proposition 3.6

Let τ_t denote a proportional tax on income and T_t a lump sum transfer. Entrepreneurs' consumption is $c_1 = \pi(\bar{z}, k_1)(1-\tau_1) + k_1 p_1 - p_1 k_2 - \frac{b}{R} + T_1$ and $c_2^i = \pi(z^i, k_2)(1-\tau_2) + b + T_2$, while workers' consumption becomes $c_t^w = (1-\tau_t)w_t n_t + T_t$. When $\Phi = \frac{1-\theta}{\theta}$, first period labor income $w_1 n_1 = \bar{z}^\theta(1-\theta)\frac{1}{\Phi}$ equals entrepreneurs' income $\pi(\bar{z}, k_1) = \bar{z}^\theta\theta$. In equilibrium, $c_1 = e - \nu\frac{B}{R}$ and $c_1^w = e + \nu\frac{B}{R}$. In the second period, $c_2^w = e - \nu B$ while $c_2^i = e\frac{z_i}{z}(1-\tau) + \tau e + \nu B$. Setting $\tau_2 = 1$ and $B = 0$ maximizes welfare, since the government can efficiently redistribute resources eliminating entrepreneurs' inequality in $t = 2$, without distorting the economy. *Q.E.D.*

L Proof of Proposition 3.7

Entrepreneurs maximize the expected value of lifetime utility, $\ln(c_1 - h_1) + \beta E \ln(c_2^i - h_2^i)$, where $c_1 = \pi(\bar{z}, k_1)(1-\tau_1) + k_1 p_1 - p_1 k_2 - \frac{b}{R} + T_1$ and $c_2^i = \pi(z^i, k_2)(1-\tau_2) + b + T_2$. Profits satisfy $\pi(z_t, k_t) = (z_t k_t)^{\theta\eta} l_t^{(1-\theta)\eta} h_t^{1-\eta} - w_t l_t$. As before, we assume $k_1 = 1$ and $z_1 = \bar{z}$.

The problem of a worker is identical to the benchmark model, but government constraints become

$$T_1 = \tilde{\tau}_1 + \frac{B}{R}\frac{1}{(1+\Phi)} \qquad \text{and} \qquad T_2 = \tilde{\tau}_2 - B\frac{1}{(1+\Phi)},$$

where $\tilde{\tau}_t = \tau_t \frac{[w_t l_t \Phi + \int \pi(z_t, k_t)]}{(1+\Psi)}$ denotes tax revenues per capita.

Lemma L.1 *Given B and $\{\tau_t\}_{t=1}^2$, the equilibrium wage is constant $\bar{w}$ and the remaining prices and allocations in an autarky equilibrium satisfy*

$$\begin{aligned}
h_t^i &= k_t \frac{z_t^i}{\bar{z}^{1-\theta}}[(1-\eta)(1-\tau_t)]^{1/\eta} && (51)\\
c_1 - h_1 &= A(\bar{z}, \tau_1) + \tilde{\tau}_1 - \frac{\nu B}{R} && (52)\\
c_2^i - h_2^i &= A(z^i, \tau_2) + \tilde{\tau}_2 + \nu B, && (53)\\
c_1^w &= A(\bar{z}, \tau_1) + \tilde{\tau}_1 + \frac{\nu B}{\Phi R}, && (54)\\
c_2^w &= A(\bar{z}, \tau_2) + \tilde{\tau}_2 - \frac{\nu B}{\Phi}, && (55)\\
R &= \left(\frac{1}{\beta E\left(\frac{1}{c_2^i - h_2^i}\right)} + \nu B\right)\frac{1}{\tilde{\tau}_1 + A(\bar{z}, \tau_1)} && (56)
\end{aligned}$$

where $A(z,\tau) = \frac{z\theta\eta(1-\eta)^{(1-\eta)/\eta}(1-\tau)^{1/\eta}}{\bar{z}^{1-\theta}}$ and $\tilde{\tau}_t = \tau_t \frac{\bar{z}^\theta[(1-\eta)(1-\tau_t)]^{(1-\eta)/\eta}}{1+\Psi}$.

Proof L.1 *The demands for labor l_t^i and h_t^i of entrepreneur i are,*

$$h_t^i = \left[(z_t^i k_t)^{\theta\eta} l_t^{i(1-\theta)\eta}(1-\eta)(1-\tau_t)\right]^{1/\eta},$$

$$l_t^i = z_t^i k_t \left(\frac{(1-\theta)\eta[(1-\eta)(1-\tau_t)]^{(1-\eta)/\eta}}{w_t}\right)^{1/\theta}.$$

In equilibrium, $\int_i l_t^i = 1$, implying $\bar{w} = \bar{z}^\theta(1-\theta)\eta[(1-\eta)(1-\tau_t)]^{(1-\eta)/\eta}$. Replacing into h_t^i delivers eq. (51). Eqs. (52) and (53) are obtained by replacing: (i) government transfers, (ii) the bond market and land market equilibrium conditions, $\tilde{b} = \nu B$ and $k_2 = 1$, and (iii) after-tax profits net of the disutility of effort in the entrepreneurs' budget constraints. Expressions (54) and (55) are obtained by replacing government transfers in the workers' budget constraints and noting that $w_1 l_1(1-\tau_1) = \pi(\bar{z}, k_1)(1-\tau_1) - h_1$ when $\Phi = \frac{1-\theta}{\theta}$. That *is, after-tax income of workers and entrepreneurs (net of the disutility of exerting effort) are identical in period 1. Finally, the interest rate, eq. (56), arises from rearranging the entrepreneur's FOC w.r.t. the excess demand for bonds $\tilde{b}$ and replacing in other equilibrium conditions.*

The lemma characterizes the autarky equilibrium given government policies. Taxes, transfers, and public debt are chosen in the same fashion as in the benchmark model. Because elections are held every period, we can solve the Markov perfect equilibrium of this finite-horizon economy by backward induction.

Lemma L.2 *The optimal tax in the second period is positive and bounded away from 1, $\tau_2 \in (0,1)$.*

Proof L.2 *The government's objective in the second period, given B, is*

$$\max_{\tau_2} \Phi \ln c_2^w + \int_i \ln(c_2^i - h_2^i)$$

s.t. eq. (53), (55) and

$$\tilde{\tau}_2 = \tau_2 \frac{\bar{z}^\theta [(1-\eta)(1-\tau_2)]^{(1-\eta)/\eta}}{1+\Phi}.$$

The necessary condition w.r.t. τ_2 simplifies to

$$\Phi \frac{1}{c_2^w}\left(-\frac{\tau_2(1-\eta)}{(1-\tau_2)\eta}\right) + \int_i \frac{1}{c_2^i - h_2^i}\left(\frac{-z^i}{\bar{z}} + 1 - \frac{\tau_2(1-\eta)}{(1-\tau_2)\eta}\right) = 0.$$

Clearly, the solution for τ_2 is interior, $\tau_2 \in (0,1)$.

Notice that this results from the fact that the government can redistribute resources across agents in the economy when entrepreneurs' effort is endogenous. This is not the case in period 1.

Lemma L.3 *The government does not tax in the first period, $\tau_1 = 0$.*

Proof L.3 *The government's objective in the first period is*

$$\max_{\tau_1, B} \Phi\left(\ln c_1^w + \beta \ln c_2^w\right) + \ln(c_1 - h_1) + \beta \int_i \ln(c_2^i - h_2^i),$$

s.t. $\tau_2 = \Psi(B)$ and eqs. (56), (52), (53), (54), and (55).

The optimality condition w.r.t τ_1 is

$$\Phi \frac{1}{c_1^w}\left(-\frac{\tau_1(1-\eta)}{(1-\tau_1)\eta} - \frac{\nu B}{R^2}\frac{\partial R}{\partial \tau_1}\frac{1}{\Phi}\right) + \frac{1}{c_1 - h_1}\left(-\frac{\tau_1(1-\eta)}{(1-\tau_1)\eta} + \frac{\nu B}{R^2}\frac{\partial R}{\partial \tau_1}\right) = 0,$$

Simplifying, this reduces to

$$\frac{-\tau_1}{(1-\tau_1)}\left[\frac{\Phi}{c_1^w}\left(1 - \xi(\nu B)\right) + \frac{1}{c_1}\left(1 + \xi(\nu B)\right)\right] = 0$$

with $\xi(\nu B) = \frac{\nu B}{\nu B + \frac{1}{\beta \mathbb{E}\frac{1}{c_2^i - h_2^i}}}$. Since the term in brackets is strictly positive, then $\tau_1 = 0$.

Because both agents have the same income in the first period, there is no redistribution from the affine tax system (that is, labor income and lump-sum transfers). The change in the interest rate caused by changes in τ_1 would result in some redistribution between workers and entrepreneurs. The government, however, can achieve this by simply changing B, a less distortionary instrument than τ_1.

Lemma L.4 *Debt is relevant, $B \neq 0$.*

Proof L.4 *By contradiction. The optimality condition w.r.t νB can be written as*

$$\left[\frac{1}{c_1^w}\left(1-\epsilon(\nu B)\right)-\beta R\frac{1}{c_2^w}\right]+\left[-\frac{1}{c_1-h_1}\left(1-\epsilon(\nu B)\right)+\beta R\mathbb{E}\frac{1}{c_2^i-h_2^i}\right]=0,$$

Suppose that $\tilde{B}=0$, then $c_1-h_1=c_1^w$, so the equation collapses to

$$-\frac{1}{c_2^w}+\mathbb{E}\frac{1}{c_2^i-h_2^i}=0,$$

re-arranging and substituting in consumption, this results in

$$\mathbb{E}\frac{A(\bar{z},\tau_2)+\tilde{\tau}_2}{A(z^i,\tau_2)+\tilde{\tau}_2}=1 \Rightarrow \mathbb{E}\frac{\eta+\frac{\tau_2(1-\eta)}{(1-\tau_2)\eta}}{\eta\frac{z^i}{\bar{z}}+\frac{\tau_2(1-\eta)}{(1-\tau_2)\eta}}=1,$$

a contradiction. *Q.E.D.*

M Proof of Proposition 4.1

Let's first derive the value for workers. Individual workers' consumption is equal to

$$c_{j,t}=\left(\frac{1}{\Phi}\right)\bar{w}+\tau_{j,t}.$$

Since the total government transfers are equal to $B_{j,t+1}/R_{j,t}-B_{j,t}$ and the population is $1+\Phi$, each worker gets $\tau_{j,t}=(B_{j,t+1}/R_{j,t}-B_{j,t})/(1+\Phi)$. Substituting and collecting $1/\Phi$ we get

$$c_{j,t}=\left(\frac{1}{\Phi}\right)\left[\bar{w}+\nu\left(\frac{B_{j,t+1}}{R_{j,t}}-B_{j,t}\right)\right]. \tag{57}$$

To derive the workers' value we start from the terminal period $t=T$ where

$$\widetilde{W}_{j,T}(\mathbf{B}_T,\mathbf{B}_{T+1})=\ln(c_{j,T}).$$

Substituting (57) we can rewrite it as

$$W_{j,T}(\mathbf{B}_T,\mathbf{B}_{T+1})=\ln\left(\bar{w}+\frac{\nu B_{j,T+1}}{R_{j,T}}-\nu B_{j,T}\right),$$

where $W_{j,T}(\mathbf{B}_T,\mathbf{B}_{T+1})=\widetilde{W}_{j,T}(\mathbf{B}_T,\mathbf{B}_{T+1})+\ln(\Phi)$.

We now consider the earlier period $t=T-1$ where the workers' value is

$$\widetilde{W}_{j,T-1}(\mathbf{B}_{T-1},\mathbf{B}_T)=\ln(c_{j,T-1})+\beta\widetilde{W}_{j,T}(\mathbf{B}_T,\mathbf{B}_{T+1}).$$

Substituting (57), the workers' value can be rearranged as

$$W_{j,T-1}(\mathbf{B}_{T-1},\mathbf{B}_T)=\ln\left(\bar{w}+\frac{\nu B_{j,T}}{R_{j,T-1}}-\nu B_{j,T-1}\right)+\beta W_{j,T}(\mathbf{B}_T,\mathbf{B}_{T+1}),$$

where $W_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) = \widetilde{W}_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) + (1+\beta)\ln(\Phi)$.

Continuing with $t = T-2, .., 1$ we can derive the general expression

$$W_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) = \ln\left(\bar{w} + \frac{\nu B_{j,t+1}}{R_{j,t}} - \nu B_{j,t}\right) + \beta W_{j,t+1}(\mathbf{B}_{t+1}, \mathbf{B}_{t+2}), \tag{58}$$

where

$$W_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) = \widetilde{W}_{j,t}(\mathbf{B}_t, \mathbf{B}_{t+1}) + \left(\frac{1}{1-\eta_t}\right)\ln(\Phi). \tag{59}$$

Replacing $\mathbf{B}_{t+2} = \mathcal{B}_{t+1}(\mathbf{B}_{t+1})$, we obtain the expression for $W_{j,t}$ reported in Proposition 4.1.

We now derive the value for entrepreneurs. By Lemma 2.1, entrepreneurs' consumption is equal to $c^i_{j,t} = (1-\eta_t)a^i_{j,t}$, where $a^i_{j,t} = [A(z^i_{j,t}) + p_{j,t} + \tilde{b}^i_{j,t}/k^i_{j,t}]k^i_{j,t}$. Since $\tilde{b}^i_{j,t}/k^i_{j,t}$ is the same across entrepreneurs and aggregate land is 1, we can write $a^i_{j,t} = [A(z^i_{j,t}) + p_{j,t} + \tilde{b}_{j,t}]k^i_{j,t} = \hat{a}^i_{j,t}k^i_{j,t}$, with consumption equal to

$$c^i_{j,t} = (1-\eta_t)\hat{a}^i_{j,t}k^i_{j,t}. \tag{60}$$

Using Lemma 2.1 we can also write the individual gross growth rate of land as

$$\frac{k^i_{j,t+1}}{k^i_{j,t}} = \frac{\eta_t \phi_{j,t}\hat{a}^i_{j,t}}{p_{j,t}}. \tag{61}$$

Since we have a finite number of periods, we start with the terminal period $t = T$. The indirect utility of an entrepreneur i at $t = T$ is equal to

$$\widetilde{V}^i_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}) = \ln(c^i_{j,T}).$$

Substituting (60) for $t = T$ we have

$$\widetilde{V}^i_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}) = \ln(1-\eta_T) + \ln(\hat{a}^i_{j,T}) + \ln(k^i_{j,T}).$$

Subtracting $\ln(k^i_{j,T})$ on both sides and integrating over $z^i_{j,t}$, we define the expected normalized value

$$V_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}) = \mathbb{E}\Big[\widetilde{V}^i_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}) - \ln(k^i_{j,T})\Big] = \ln(1-\eta_T) + \mathbb{E}\ln(\hat{a}^i_{j,T}).$$

Here the expectation operator $\mathbb{E}$ represents the integration over all individual entrepreneurs indexed by i. Once we integrate, the resulting value does not depend on individual characteristics nor the distribution of $k^i_{j,t}$. Thus, we have dropped the superscript i.

We move next to the earlier period $t = T-1$. The indirect utility of an entrepreneur i can be written as was derived above

$$\widetilde{V}^i_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) = \ln(c^i_{j,T-1}) + \beta\mathbb{E}\widetilde{V}^i_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}).$$

Substituting (60) for $t = T-1$ we have

$$\widetilde{V}^i_{j,T-1}(\mathbf{B}_{T-1}, \mathbf{B}_T) = \ln(1-\eta_{T-1}) + \ln(\hat{a}^i_{j,T-1}) + \ln(k^i_{j,T-1}) + \beta\mathbb{E}\widetilde{V}^i_{j,T}(\mathbf{B}_T, \mathbf{B}_{T+1}).$$

Subtracting $(1+\beta)\ln(k^i_{j,T-1})$ from both sides and adding and subtracting $\beta\ln(k^i_{j,T})$ on the right-hand side we obtain

$$\begin{aligned}\widetilde{V}^i_{j,T-1}(\mathbf{B}_{T-1},\mathbf{B}_T)-(1+\beta)\ln(k^i_{j,T-1}) &= \ln(1-\eta_{T-1})+\ln(\hat{a}^i_{j,T-1})+\beta\ln\left(\frac{k^i_{j,T}}{k^i_{j,T-1}}\right)\\ &\quad+\beta\mathbb{E}\Big[\widetilde{V}^i_{j,T}(\mathbf{B}_T,\mathbf{B}_{T+1})-\ln(k^i_{j,T})\Big].\end{aligned}$$

Using eq. (61) to eliminate $k^i_{j,T}/k^i_{j,T-1}$ and integrating over i we get,

$$V_{j,T-1}(\mathbf{B}_{T-1},\mathbf{B}_T)=\ln(1-\eta_{T-1})+(1+\beta)\mathbb{E}\ln(\hat{a}^i_{j,T-1})+\beta\ln\left(\frac{\eta_{T-1}\phi_{j,T-1}}{p_{j,T-1}}\right)+\beta V_{j,T}(\mathbf{B}_T,\mathbf{B}_{T+1}),$$

where we have defined

$$V_{j,T-1}(\mathbf{B}_{T-1},\mathbf{B}_T)=\mathbb{E}\Big[\widetilde{V}^i_{j,T-1}(\mathbf{B}_{T-1},\mathbf{B}_T)-(1+\beta)\ln(k^i_{j,T-1})\Big]$$

The next step is to consider $t=T-2$ and continue backward until $t=1$. For a generic t we have

$$V_{j,t}(\mathbf{B}_t,\mathbf{B}_{t+1})=\ln(1-\eta_t)+\left(\frac{1}{1-\eta_t}\right)\left[\eta_t\ln\left(\frac{\eta_t\phi_{j,t}}{p_{j,t}}\right)+\mathbb{E}_t\ln\hat{a}^i_{j,t}\right]+\beta V_{j,t+1}(\mathbf{B}_{t+1},\mathbf{B}_{t+2}), \quad (62)$$

where

$$V_{j,t}(\mathbf{B}_t,\mathbf{B}_{t+1})=\mathbb{E}\left[\widetilde{V}^i_{j,t}(\mathbf{B}_t,\mathbf{B}_{t+1})-\left(\frac{1}{1-\eta_t}\right)\ln(k^i_{j,t})\right] \quad (63)$$

Replacing $\mathbf{B}_{t+2}=\mathcal{B}_{t+1}(\mathbf{B}_{t+1})$, we obtain the expression for $V_{j,t}$ reported in Proposition 4.1.

The government objective is

$$\Phi\tilde{W}_{j,t}(\mathbf{B}_t,\mathbf{B}_{t+1})+\mathbb{E}\widetilde{V}^i_{j,t}(\mathbf{B}_t,\mathbf{B}_{t+1}).$$

Remember that the objective of the government is the integral of the 'non-normalized' values for workers and entrepreneurs. Using (59) and (63), the objective can be rewritten as

$$\Phi W_{j,t}(\mathbf{B}_t,\mathbf{B}_{t+1})+V_{j,t}(\mathbf{B}_t,\mathbf{B}_{t+1})+\left(\frac{1}{1-\eta_t}\right)\Big(\mathbb{E}\ln(k^i_{j,t})-\Phi\ln(\Phi)\Big)$$

The last term does not depend on $\mathbf{B}_{t+1}$. Thus, the optimal debt does not depend on this term and we can focus on the first two terms as reported in Proposition 4.1. *Q.E.D.*

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Online Appendix

A Numerical algorithm

To simplify the exposition we describe the numerical procedure when there are only two countries ($N = 2$). We first form a two-dimensional, equally spaced grid over the states B_1 and B_2, which we denote by $\mathbf{S}$. We then solve the model at any grid point for the states $(B_1, B_2) \in \mathbf{S}$ backward, starting from the terminal period $t = T$. Before describing the specific solution at any $t = T, T-1, ..., 1$, we state the following property based on Lemma 2.1, Proposition 2.1, Corollary 2.1 and Proposition 4.1.

Property 1 *Given $B_{j,t}$, $B_{j,t+1}$, $p_{j,t+1}$, $V_{j,t+1}$, $W_{j,t+1}$, $j \in \{1,2\}$, we can solve for all variables at time t using the equations*

$$\eta_t = \frac{\beta}{1 + \frac{\beta^{T-t}}{\sum_{s=1}^{T-t} \beta^{s-1}}} \tag{64}$$

$$\tilde{b}_{j,t} = \begin{cases} \nu B_{j,t}, & \textit{In autarky} \\ \nu \left(\frac{B_{1,t}+B_{2,t}}{2} \right), & \textit{With mobility} \end{cases} \tag{65}$$

$$\tilde{b}_{j,t+1} = \begin{cases} \nu B_{j,t+1}, & \textit{In autarky} \\ \nu \left(\frac{B_{1,t+1}+B_{2,t+1}}{2} \right), & \textit{With mobility} \end{cases} \tag{66}$$

$$\phi_{j,t} = \mathbb{E}_t \left[\frac{A(z^i_{j,t+1}) + p_{j,t+1}}{A(z^i_{j,t+1}) + p_{j,t+1} + \tilde{b}_{j,t+1}} \right], \tag{67}$$

$$p_{j,t} = \frac{\eta_t \phi_{j,t} (\bar{A} + \tilde{b}_{j,t})}{(1 - \eta_t \phi_{j,t})}, \tag{68}$$

$$R_{j,t} = \frac{(1 - \eta_t \phi_{j,t}) \tilde{b}_{j,t+1}}{\eta_t (1 - \phi_{j,t})(\bar{A} + \tilde{b}_{j,t})}, \tag{69}$$

$$\hat{a}^i_{j,t} = A(z^i_{j,t}) + p_{j,t} + \tilde{b}_{j,t}, \tag{70}$$

$$V_{j,t} = \ln(1 - \eta_t) + \left(\frac{1}{1 - \eta_t} \right) \left[\eta_t \ln \left(\frac{\eta_t \phi_{j,t}}{p_{j,t}} \right) + \mathbb{E}_t \ln \hat{a}^i_{j,t} \right] + \beta V_{j,t+1}, \tag{71}$$

$$W_{j,t} = \ln \left(\bar{w} + \nu \left(\frac{B_{j,t+1}}{R_{j,t}} - B_{j,t} \right) \right) + \beta W_{j,t+1}, \tag{72}$$

where $\nu = \frac{\Phi}{1+\Phi}$, $\bar{A} = \sum_\ell A(z_\ell)\mu_\ell$, $\bar{w} = (1-\theta)\bar{z}^\theta$.

We can solve exactly for the above variables sequentially, once we know $B_{j,t}$, $B_{j,t+1}$, $p_{j,t+1}$, $V_{j,t+1}$ and $W_{j,t+1}$. For the following description of the computational algorithm, it will be convenient to define the vectors $\mathbf{X}_t = \{B_{j,t}, B_{j,t+1}, p_{j,t+1}, V_{j,t+1}, W_{j,t+1}\}_{j=1}^2$ and $\mathbf{Y}_t = \{p_{j,t}, V_{j,t}, W_{j,t}\}_{j=1}^2$. Using Property 1, we can express $\mathbf{Y}_t$ as a function of $\mathbf{X}_t$,

$$\mathbf{Y}_t = \Upsilon(\mathbf{X}_t).$$

We describe next the solution at each time t, starting from the terminal period T.

Solution at $t = T$

In the terminal period governments fully repay their debts. Thus, $B_{j,T+1} = 0$. Furthermore we know that $p_{j,T+1} = V_{1,T+1} = W_{j,T+1} = 0$. Therefore, $\mathbf{X}_T = \{B_{j,T}, 0, 0, 0, 0\}_{j=1}^2$. We can then use the function $\Upsilon(\mathbf{X}_T)$ from property 1 to solve for $\mathbf{Y}_T = \{p_{j,T}, V_{j,T}, W_{j,T}\}_{j=1}^2$ at each grid point $(B_1, B_2) \in \mathbf{S}$.

The solution obtained for $\mathbf{Y}_T$ at each grid point $(B_1, B_2) \in \mathbf{S}$ is used to form the approximate function

$$\mathbf{Y}_T = \Gamma_T(B_{1,T}, B_{2,T})$$

The reason we need to create this approximate function is that, when we move to the next step $t = T-1$, we need to determine $\mathbf{Y}_T$ also for values of $B_{1,T}$ and $B_{2,T}$ that are not on the grid $\mathbf{S}$. The approximate function is created with bilinear interpolation of the solutions $\mathbf{Y}_T$ obtained at the grid points $(B_1, B_2) \in \mathbf{S}$. Armed with the approximate function $\Gamma_T(B_{1,T}, B_{2,T})$, we can move to period $t = T - 1$.

Solution at $t < T$

The main difference from the terminal period T is that now we need to solve for the optimal debts $B_{1,t+1}$ and $B_{2,t+1}$ chosen by governments. To find the optimal debt chosen by each government at each grid point $(B_1, B_2) \in \mathbf{S}$, we implement the following steps.

1. We first solve for the optimal response functions to the debt chosen by the other country. To find the optimal response function we need to find the government objective $O_{j,t}(B_{1,t+1}, B_{2,t+1}) = \Phi W_{j,t} + V_{j,t}$. The response function of country 1 and country 2 are defined, respectively, as

$$\begin{aligned} \varphi_{1,t}(B_{2,t+1}) &= \max_{B_{1,t+1}} O_{1,t}(B_{1,t+1}, B_{2,t+1}), \\ \varphi_{2,t}(B_{1,t+1}) &= \max_{B_{2,t+1}} O_{2,t}(B_{1,t+1}, B_{2,t+1}). \end{aligned}$$

The optimization is performed by searching over an equally spaced grid $\mathbf{B}$. This grid is finer than the two dimensional grid for the states $\mathbf{S}$ so that we obtain a more accurate approximation to the maximization problem. The detailed steps are as follows:

(a) Given $(B_{1,t+1}, B_{2,t+1}) \in \mathbf{B}\times\mathbf{B}$, we find $\mathbf{Y}_{t+1}$ using the approximate function $\Gamma_{t+1}(B_{1,t+1}, B_{2,t+1})$ found in the previous step $t + 1$.

(b) Given $\mathbf{Y}_{t+1}$ we have all the terms we need to construct the vector $\mathbf{X}_t$. Thus we can find $\mathbf{Y}_t$ using the function $\Upsilon_t(\mathbf{X}_t)$ from Property 1.

(c) The vector $\mathbf{Y}_t$ contains the necessary elements to compute the government objectives $O_{j,t}(B_{1,t+1}, B_{2,t+1})$ for $(B_{1,t+1}, B_{2,t+1}) \in \mathbf{B} \times \mathbf{B}$.

(d) Now that we know the government objectives at the grid points, we compute the optimal response functions as

$$\begin{aligned} \varphi_{1,t}(B_{2,t+1}) &= \max_{B_{1,t+1}\in\mathbf{B}} O_{1,t}(B_{1,t+1}, B_{2,t+1}), \\ \varphi_{2,t}(B_{1,t+1}) &= \max_{B_{2,t+1}\in\mathbf{B}} O_{2,t}(B_{1,t+1}, B_{2,t+1}), \end{aligned}$$

which are defined only over the grid $\mathbf{B}$. To make the response functions continuous, we join the grid values with piece-wide linear segments.

2. The optimal response functions allow us to compute the equilibrium policies chosen by the two governments. They are the fix point $(B^*_{1,t+1}, B^*_{2,t+1})$ to

$$\begin{aligned} B^*_{1,t+1} &= \varphi_{1,t}(B^*_{2,t+1}) \\ B^*_{2,t+1} &= \varphi_{2,t}(B^*_{1,t+1}) \end{aligned}$$

After making the response function continuous, we find the solution $(B^*_{1,t+1}, B^*_{2,t+1})$ using a nonlinear solver. Of course, the solution is not necessarily on the grid $\mathbf{B} \times \mathbf{B}$.

3. Given the solution for $(B^*_{1,t+1}, B^*_{2,t+1})$, we can compute $\mathbf{Y}_{t+1} = \Gamma_{t+1}(B^*_{1,t+1}, B^*_{2,t+1})$ and construct the vector $\mathbf{X}_t$ associated with the equilibrium policies. This allows us to compute $\mathbf{Y}_t = \Upsilon_t(\mathbf{X}_t)$.

Once we have completed the above steps and found the vector $\mathbf{Y}_t$ for each grid point $(B_1, B_2) \in \mathbf{S}$, we can then construct the approximate (bi-linearly interpolated) function

$$\mathbf{Y}_t = \Gamma_t(B_{1,t}, B_{2,t}).$$

We can then move to the earlier step until we reach the initial period $t = 1$.

A.1 Some properties of the equilibrium

Figure 12 plots the best response functions of the two countries in the autarky regime and in the regime with capital mobility. The parametrization of the model is described in Section 4.2 of the paper. The response functions are computed at steady state values—under autarky and mobility, respectively—of debt in each country. The best response function under autarky does not depend on the other country's choice of debt, as shown in the left panel of Figure 12. In the regime with capital mobility, instead, there is strategic interaction between the two countries and, therefore, the response functions depend on the debt chosen by the other country. The right panel of Figure 12 shows that the response functions intersect only once, suggesting that the Nash equilibrium is unique. The shape of the response functions when current debt differs from steady state is similar (graph available upon request).

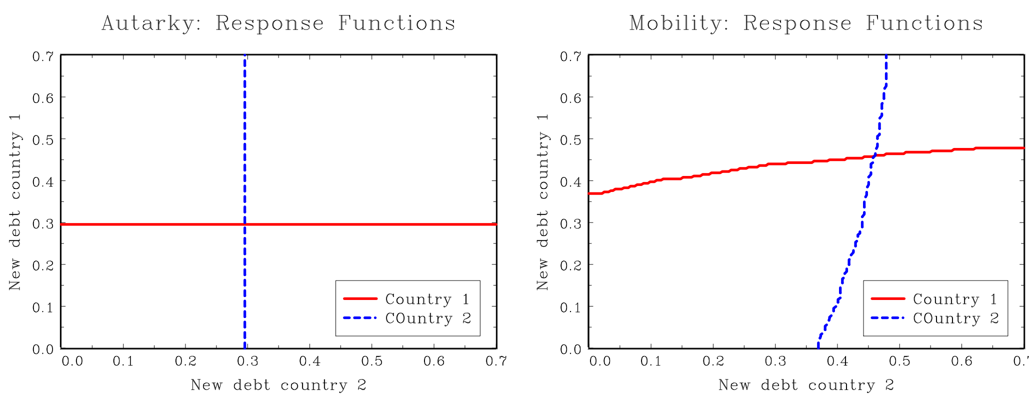

Figure 12: Optimal response functions at the steady state with autarky and with mobility.

Figure 13 plots the equilibrium policy function, $B_{j,t+1}$ as a function of $B_{j,t}$, under the two financial arrangements (autarky and mobility) and assuming that the two countries are symmetric (so they choose the same level of debt). As can be seen, the two functions cross the 45 degree line only once, implying that the steady state is unique. The fact that with mobility the crossing point is at a higher level of debt means that governments borrow more when financial markets are integrated, as shown in the two-period version of the model. The policy function in the regime with capital mobility also illustrates the dynamics of debt after liberalization. Starting from the autarkic steady state (where the 'autarky' policy function crosses the 45 degree line), capital liberalization induces immediate higher borrowing. This follows from the fact that the policy function with mobility is above the 45 degree line when current debt is at the autarkic steady state. This jump brings the two economies closer to the new steady state with mobility (where the 'mobility' policy function intersects the 45 degree line), but does not imply immediate convergence. Concavity of the policy function implies that debt increases, at a decreasing rate, until the new steady state is reached.

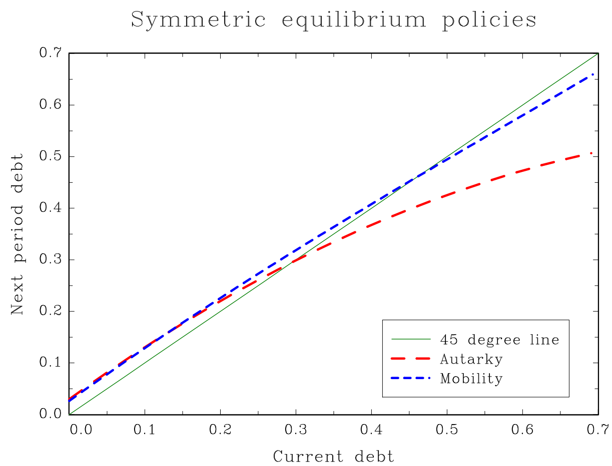

Figure 13: Equilibrium policy function at the steady state with autarky and with mobility.

B Empirical analysis

The theoretical analysis has shown that greater mobility of capital and higher income inequality raise government borrowing. In this section we conduct an empirical investigation of these theoretical predictions using cross-country data for the OECD countries. In subsection B.1 we test whether the issuance of government debt is positively associated to capital markets liberalization and income inequality. In subsection B.2 we test whether the elasticity of the interest rate to the issuance of public debt declines with capital markets liberalization.

B.1 Liberalization, inequality, and the supply of debt

To check whether there are statistically significant links between indices of capital market liberalization, income inequality, and government borrowing, we regress the growth rate of real government debt on two main variables: (i) an index that captures the change in capital mobility, dMOB, and (ii) changes in some measure of inequality, $dINEQ$. We estimate the following fixed effect regression equation:

$$\begin{aligned}\text{dDEBT}_{j,t} &= \alpha_D \cdot \text{DEBT}_{j,t-1} + \alpha_G \cdot \text{dGDP}_{j,t-1} + \alpha_M \cdot \text{dMOB}_t \\ &\qquad + \alpha_I \cdot dINEQ_t + \alpha_X \cdot X_{j,t} + u_{j,t}.\end{aligned}$$

- $\text{dDEBT}_{j,t}$: Log-change in real public debt of country j in year t.
- $\text{DEBT}_{j,t-1}$: Ratio of public debt to the GDP of country j in year $t-1$.
- $\text{dGDP}_{j,t}$: Log-change in the GDP of country j in year t.
- dMOB_t: Change in the index of capital mobility in year t or $t-1$.
- dINEQ_t: Change in a measure of inequality in year t or $t-1$.
- $X_{j,t}$: Set of control variables for country j.
- $u_{j,t}$: Residuals containing country and year fixed effects.

A few remarks are in order. First, we relate the *change* in public debt to the *change* in the liberalization index, instead of the *level* of the index. This better captures the dynamics predicted by the model. In fact, in the long run, there is no relation between the degree of capital mobility and the change in debt, since the stock of debt converges to the steady state.

The second remark pertains to the construction of the index of financial liberalization. This index is not country-specific as can be noticed from the absence of the country subscript j. Instead, we construct the index as the average of country-specific indices for all countries included in the sample, weighted by their size (measured by total GDP). The motivation for adopting this measure of capital liberalization can be explained as follows.

Indicators of financial liberalization refer to the private sector, not the public sector. Thus, the fact that one country has very strict international capital controls does not mean that the government is restrained from borrowing abroad. What is relevant for the government's ability to borrow abroad is the openness of other countries. Therefore, to determine the ease with which the government can sell its debt to foreign (private) investors, we have to look at the capital controls imposed by other countries. This is done by computing an average index for all countries included in the sample.[14]

A related issue is whether in computing the weighted average of the liberalization index we should exclude the country of reference. For example, to evaluate the importance of capital mobility for the U.S. public debt, we should perhaps average the indices of the OECD countries excluding the U.S. We have chosen not to do so for the following reason. Although the liberalization of other countries is what defines the foreign market for government bonds, the domestic liberalization can still affect domestic issuance through an indirect channel. However, we also tried the alternative index and the results (not reported) are robust.

Regarding the data for the liberalization variable, we use two indices, both based on de jure measures. The first is the liberalization index constructed by Abiad, Detragiache, and Tressel

[14] Another way of showing the irrelevance of the country's own indicator is with the following example. Suppose that country A liberalizes its capital markets, allowing free international mobility of capital. However, all other countries maintain strict controls. Obviously, the government of country A does not have access to the foreign market even if it had liberalized its own market.

(2008). The results based on this index are reported in Table 1. The second index uses the capital account openness indicator constructed by Chinn and Ito (2008),2008), with results reported in Table 2. Income inequality in Table 1 is proxied by the share of income earned by the top 1 percent of the population (specification (5)), compiled by Atkinson, Piketty, and Saez (2011), and by the averages of the gross Gini coefficients (specification (6)) obtained from the "Standardized World Income Inequality Database, version 3.0, July 2010 " compiled by Frederick Solt. The data sources are described in the tables.

We estimate the regression equation on a sample that includes 22 OECD countries. The selection of countries in the first set of regressions is based on data availability for government debt and the financial index, which restricts the sample to 26 countries. From this selected group, we exclude four countries: Hungary, Poland, Mexico, and Turkey. The first two countries are excluded, since the available data start in the 1990s, when they became market-oriented economies. Mexico and Turkey are excluded because they were at a lower stage of economic development compared to the other countries in the sample and they experienced various degrees of market turbulence during the sample period. For robustness, however, we also repeated the estimations for the whole sample with 26 countries, and the results are consistent with those obtained with the restricted sample, including 22 countries.

We start by analyzing the effects of financial integration on debt accumulation, but initially excluding inequality dINEQ_t. By doing so we can use a larger sample, since the inequality variable is unavailable for Austria, Belgium, Switzerland, Germany, Greece, and Korea. The sample size consists of 677 observations. In the simplest specification, we also abstract from any controls $X_{j,t}$. In the second specification we include a dummy for the countries that joined the European Monetary System. Since the membership was conditional on fulfilling certain requirements in terms of public debt (Maastricht Treaty), it is possible that the government debt of certain European countries has been affected by joining the EMU.

As can be seen in the first two columns of Tables 1 and 2, the coefficient on the financial index is positive and highly significant, meaning that the change in capital market integration is positively correlated with the change in public debt. Although we do not claim that this proves causation, there is a strong conditional correlation between these two variables. As far as the EMU dummy is concerned, the coefficient is negative, consistent with the view that EMU countries were forced to adjust their public finances before becoming full members.

Next, we add the interaction term between the financial index and the size of the country, measured by real GDP. The motivation to include this term is dictated by the theory. We have seen in Section 3 that the effect of capital liberalization is stronger for smaller countries. Since small countries have a lower ability to affect the world interest rate, their governments have more incentive to borrow once they have access to the world financial market. The third column of Tables 1 and 2 shows that the coefficient of the interaction term is negative, as expected from the theory, and statistically significant in some cases.

The fourth specification adds a demographic variable. This is the *old dependency ratio* between the population in the age group 65 and over and the population in the age group 15-64. Although our model abstracts from demographic considerations, there is a widespread belief that aging in industrialized countries is an important force for the rising public debt. This is because the political

Table 1: Country fixed-effect regression. The dependent variable is real public debt growth. The financial index is based on Abiad, Detragiache, and Tressel (2008).

	(1)	(2)	(3)	(4)	(5)	(6)
Lag debt to GDP ratio	-0.149^{***}	-0.146^{***}	-0.149^{***}	-0.170^{***}	-0.162^{***}	-0.156^{***}
	(0.0374)	(0.0375)	(0.0378)	(0.0383)	(0.0253)	(0.0387)
Lag real GDP growth	-1.235^{***}	-1.210^{**}	-1.216^{***}	-1.159^{**}	-1.381^{**}	-0.887^{*}
	(0.433)	(0.430)	(0.429)	(0.413)	(0.571)	(0.473)
Lag change in financial index	0.688^{**}	0.697^{**}	0.966^{***}	1.180^{***}	1.555^{***}	1.314^{***}
	(0.269)	(0.270)	(0.281)	(0.278)	(0.331)	(0.226)
Lag EMU dummy		-0.0478^{**}	-0.0474^{**}	-0.0521^{**}	-0.084^{***}	-0.0545^{***}
		(0.0189)	(0.0190)	(0.0185)	(0.0259)	(0.0179)
Size × Lag change in FI			-6.136	-6.602^{*}	-7.883^{*}	-6.418^{*}
			(3.818)	(3.554)	(3.932)	(3.522)
Change in dependency ratio				0.0695^{**}	0.0636^{**}	0.0650^{**}
				(0.0256)	(0.0223)	(0.0273)
Change in inequality					0.128^{**}	0.0126^{*}
					(0.0536)	(0.00673)
Observations	677	677	677	677	435	630
R-squared	0.130	0.132	0.137	0.150	0.199	0.160
Number of countries	22	22	22	22	16	21

Notes: The variable *Financial Index* (FI) is constructed using the liberalization index of Abiad, Detragiache, and Tressel (2008). We compute the financial index for a year as a weighted average of all the country indexes where weights are given by their relative GDP shares. The ratio of debt to GDP is from Reinhart and Rogoff (2011), and real GDP and population data are from the World Development Indicators (World Bank). Real debt is constructed by multiplying the ratio of debt to GDP by real GDP. Size is the lagged logarithm of real GDP weights. The EMU dummy is equal to 1 in the year the country joined the European Monetary Union and 0 otherwise. The old dependency ratio is the population 65 and above divided by the population in the age group 15-64. In specification (5), the inequality index is measured by the logarithm of the top 1 percent income share calculated by Atkinson, Piketty, and Saez (2011), and we include its change at t.In specification (6), inequality is measured by the gross Gini coefficient obtained from the "Standardized World Income Inequality Database, version 3.0, July 2010 ", and we include its change at $t-1$ .The sample period is 1973-2005 and includes the following countries for specifications (1) to (4): Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Korea, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States. Due to data availability, Austria, Belgium, Germany, Greece, and Korea are excluded in specification (5), and Korea is excluded from specification (6). Robust standard errors are in parenthesis.

* Significant at 10 percent. ** Significant at 5 percent. *** Significant at 1 percent.

weight shifts toward older generations that may prefer higher debt. As can be seen from the fourth column of Tables 1 and 2, the coefficient associated with the change in this variable is positive. However, the inclusion of the old dependency ratio does not affect the sign and significance of the financial index, confirming the importance of capital market liberalization for government borrowing.

The final specification introduces income inequality. With the inclusion of the inequality index we lose some observations, since the index is not available for all countries. As a result, the sample shrinks to 435 observations. The coefficient is positive and statistically significant, indicating that rising income inequality is associated with higher borrowing.

As far as the other variables are concerned, we find that the lagged stock of debt is negatively correlated with its change. This is what we expect if the debt tends to converge to a long-term level. The change in GDP is meant to capture business cycle effects, and it has the expected negative sign: when the economy does well, government revenues increase and automatic expenditures decline so that government debt increases less.

Table 2: Country fixed-effect regression. The dependent variable is real public debt growth. The financial index is based on Chinn and Ito (2008).

	(1)	(2)	(3)	(4)	(5)
Lag debt to GDP ratio	-0.150^{***}	-0.147^{***}	-0.148^{***}	-0.166^{***}	-0.157^{***}
	(0.0366)	(0.0366)	(0.0368)	(0.0380)	(0.0267)
Lag real GDP growth	-1.262^{***}	-1.235^{***}	-1.230^{***}	-1.189^{***}	-1.400^{**}
	(0.428)	(0.425)	(0.423)	(0.410)	(0.585)
Change in financial index	0.113^{**}	0.116^{**}	0.177^{***}	0.205^{***}	0.253^{***}
	(0.0539)	(0.0539)	(0.0575)	(0.0630)	(0.0606)
Lag EMU dummy		-0.0485^{**}	-0.0487^{**}	-0.0528^{**}	-0.0854^{***}
		(0.0189)	(0.0192)	(0.0187)	(0.0264)
Size × Change in fin index			-1.375^{**}	-1.428^{**}	-1.437^{**}
			(0.728)	(0.680)	(0.617)
Change in dependency ratio				0.0594^{**}	0.0535^{**}
				(0.0259)	(0.0250)
Change in top 1 percent share					0.106^{*}
					(0.0599)
Observations	677	677	677	677	435
R-squared	0.130	0.132	0.137	0.150	0.199
Number of countries	22	22	22	22	16

Notes: The variable *Financial Index* is constructed using the capital account openness index of Chinn and Ito (2008). For the other variables, see notes in Table 1

B.2 Liberalization and interest rate elasticity

We have shown in the introduction of the paper that the interest rate elasticity has declined over time (see Figure 2 in the paper). In this section we investigate whether the declining elasticity is associated with increased financial globalization as predicted by the theory.

To check the importance of financial globalization, we regress the percentage change in a country's interest rate on the percentage change in the country's public debt and its interaction with measures of financial markets integration. More specifically, we estimate the fixed-effect regression

$$\mathrm{dINT}_{j,t} = \alpha_1 \mathrm{dDEBT}_{j,t} + \alpha_2 \mathrm{dDEBT}_{j,t}\mathrm{MOB}_{j,t} + \alpha_3 \mathrm{dDEBT}_{j,t}\mathrm{DEBT}_{-j,t} + \alpha_4 X_{j,t} + u_{j,t},$$

where

- $\text{dINT}_{j,t}$: Percentage change in interest rate of country j in year t.
- $\text{dDEBT}_{j,t}$: Percenatge change in public real debt of country j in year t.
- $\text{MOB}_{j,t}$: Financial integration index in year t from Abiad, Detragiache, and Tressel (2008).
- $\text{DEBT}_{-j,t}$: Average global public real debt in year t excluding country j.
- $X_{j,t}$: Set of control variables for country j in year t.
- $u_{j,t}$: Residuals containing country and year fixed effects.

The most important variable is the interaction term between the growth of public debt and the financial index. According to the theory, we expect the coefficient of this interaction term to be negative. The regression is estimated using the same sample used in the previous subsection.

By looking at the time series for each country, it is clear that in 1999 and 2000 the interest rate for the "satellite" European countries that joined the Euro zone (Spain, Ireland, Italy, and Belgium) converged to the prevalent interest rates of core European countries such as France and Germany. This justifies the inclusion of the EMU dummy. We have also included a dummy variable for Germany for the years 1989, 1990 and 1991, to account for the consolidation of the public debt of East and West Germany.

The results support our main hypothesis: the interest rate elasticity decreases with financial liberalization or global market size. This is true regardless of whether we use a country-specific or the average financial liberalization index used in the previous section, or whether we use an actual measure of the size of the global market. In all cases, the coefficients on the interaction terms are negative and statistically significant. The goodness of fit, however, is quite small.

These findings complement the observation of higher cross-country convergence in interest rates during the last three decades. See Obstfeld and Taylor (2005).

Table 3: Country fixed-effect regression. The dependent variable is percentage change of real interest rates. The financial index measures are based on Abiad, Detragiache, and Tressel (2008).

	(1)	(2)	(3)	(4)
percent change in B_t	0.0120	0.112*	0.0504*	0.0541**
	(0.0095)	(0.0588)	(0.0245)	(0.0253)
percent change in $B_t \times MOB_{j,t}$	−0.116*	−0.129*		−0.1486**
	(0.0586)	(0.0683)		(0.0631)
percent change in $B_t \times B_{-j,t}$			$-7.8e^{-16}$**	$-7.9e^{-16}$**
			($3.7e^{-16}$)	($3.8e^{-16}$)
Lag EMU dummy	0.0006	0.0008	0.0008	0.0009
	(0.0020)	(0.0020)	(0.0020)	(0.0020)
German reunification dummy	0.0088***	0.0084***	0.0086***	0.0083***
	(0.0005)	(0.0005)	(0.0004)	(0.0004)
Constant	−0.0009**	−0.0013***	−0.0014***	−0.0013**
	(0.0003)	(0.0004)	(0.0004)	(0.0004)
Observations	459	459	459	459
R-squared	0.004	0.013	0.012	0.012
Number of countries	22	22	22	22

Notes: The variable $MOB_{j,t}$ in specifications (1) and (2) uses the liberalization index of Abiad, Detragiache, and Tressel (2008). In specification (1), $MOB_{j,t}$ is the financial index weighted by size: the interaction between the country-specific index and the country size (described in the previous section). In specification (2), $MOB_{j,t}$ is the weighted average index of financial liberalization used in Tables 1 and 2 where weights are given by their relative GDP shares. And finally in specification (3), $MOB_{j,t}$ is the average of the real debt at time t of the countries in the sample excluding country j. Specification (4) includes both the interaction terms of specifications (1) and (3). The sample period is 1974-2003. For the other variables, see notes in Table 1. Robust standard errors are in parenthesis.

* Significant at 10 percent. ** Significant at 5 percent. *** Significant at 1 percent.