Optimal Policy and (the lack of) Time Inconsistency: Insights from Simple Models[*]

Marina Azzimonti[†]   Pierre-Daniel Sarte[‡]   Jorge Soares[§]

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Abstract

In the standard neoclassical model with a representative agent, a benevolent planner who can commit to future policies will, if feasible, levy a single confiscatory tax on capital in the initial period and commit never to set positive taxes thereafter. We show that this policy, which allows for the disposal of distortional taxes entirely, can arise even when sequential governments are unable to credibly promise future tax rates, regardless of how public expenditures are determined. We suggest that Markov-perfect distortional tax rates emerge more naturally in an overlapping generations setting. In that setting, an intergenerational distribution concern arises that limits the degree to which the initial old generation is taxed, in effect creating an endogenous upper limit on initial capital income taxes. Furthermore, this intergenerational objective reintroduces time inconsistency as a policy issue since, in a Markov-perfect equilibrium, taxes are set to equate marginal utilities of consumption across generations regardless of the implications for capital income tax rates. Unlike a Ramsey planner, at each date, sequential governments in a Markov-perfect equilibrium consider the past as sunk and, therefore, treat capital income taxes as non-distortionary.

JEL: H3, H6, H21, E62

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1   Introduction

Following the work of Judd (1985), and Chamley (1986), much of the literature on optimal taxation has argued that with full commitment, it is best to leave capital income untaxed in the long run while imposing a confiscatory capital levy in the short run, either at some initial date or, when the tax on capital income is bounded above, over some adjoining time interval.[1] As policy prescriptions, these findings are evidently stark. A central aspect of Ramsey taxation, however, is the government's ability to credibly commit to a sequence of future tax rates. For instance, the derivation of a single capital levy as optimal assumes that, at some initial date, the government can credibly promise to set taxes to zero forever after. More specifically, Sargent and Lungqvist (2000) write that *"taxing the capital stock at time 0 (...) disposes of distortionary taxation. It follows that a government without a commitment technology would be tempted in future periods to renege on its promises. (...) An interesting question arises: can there exist a reputational mechanism that replaces the assumption of a commitment technology?"*[2] We first show that such a reputational mechanism is not necessary in this case, and that disposing of distortionary taxes is possible even when one takes away the government's ability to commit to future taxes.

In contrast to Kydland and Prescott (1980), who first established the time inconsistency of optimal taxes in a neoclassical setting without debt, the presence of sovereign lending and borrowing is a key feature that helps make Ramsey taxes time consistent. In the standard growth model with an infinitely-lived representative agent, a finitely-lived government can manipulate its successor's behavior by varying the level of debt it receives upon taking office. Thus, we show that disposing of distortionary taxes can occur even when policy is chosen sequentially by different governments (i.e. without commitment), and whether or not the sequence of government expenditures is determined endogenously within the model. When an upper limit is exogenously placed on the capital income tax, the Ramsey plan is not strictly time consistent. However, once this constraint ceases to bind, the time consistent policy continues to admit a solution where taxes on labor and capital income are set to zero after that date.

Time consistent policies throughout our analysis are Markov perfect and, therefore, do not rely on reputational mechanisms. The literature on optimal time consistent policy to date has mainly followed two approaches. One line of work has used the loss of good reputation as a way of com-

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[1] These findings are extended to a stochastic environment by Chari, Christiano, and Kehoe (1994). But Correia (1996), and Jones, Manuelli, Rossi (1997), show that the optimal long-run tax on capital can differ from zero when other factors of production are either untaxed or not taxed optimally.

[2] See end of Chapter 12.

mitting the government to desirable policies.[3] Another approach, as exemplified by Klein, Krusell, Rios-Rull (2008), has relied on Markov perfect equilibria whereby policies are constrained to be functions of the state the economy only. Without exception, the literature on optimal time consistent fiscal policy has abstracted from either government debt or capital, and a working assumption at the outset is the solutions with and without commitment differ.[4] In fact, what matters in our analysis is a government's ability to bequeath revenue-generating assets to its successor that, potentially, render the use of future taxes unnecessary. Thus, optimal confiscatory short-run capital taxes might also emerge as time consistent in environments where the government can lend directly to firms or, alternatively, where it has no disadvantage in operating production directly. Such features are all missing from the seminal work of Kydland and Prescott (1977, 1980), and Fisher (1980), underlining the time inconsistency of optimal taxes under commitment.

Ultimately, the lessons learned in the standard neoclassical setting suggest that an alternative framework might more naturally generate distortionary Markov-perfect taxes at all dates. While the presence of sovereign debt constitutes a central aspect of our analysis, the assumptions of a benevolent government and that of a single representative agent also matter crucially. In a world with overlapping generations, for instance, the current (possibly retired) old have a capital position that is mainly inelastic since their investment decisions are essentially behind them. However, unlike an infinitely-lived agent construct, should their capital be confiscated today, they will not be alive in future periods to make up for this levy by borrowing from the government. Taking this feature into account, a benevolent planner who weighs all generations equally will not place the entire tax burden associated with the present value of government expenditures on the initial old generation. This fact immediately ensures that, even without imposing an upper limit on the initial capital income tax, other distortionary taxes will not be zero either in a Ramsey or time consistent equilibrium.[5]

In addition, we show that the intergenerational distribution concern that arises with overlapping

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[3]See Chari and Kehoe (1990). For more recent advances on this literature, see Dominguez (2007) and Reis (2013).

[4]See also Klein and Rios-Rull (2003), Martin (2010, 2015), Ortigueira (2006), and Quadrini (2005), for papers that impose a balanced budget, as well as Martin (2009), Debortoli and Nunes (2010), and Occhino (2006), for recent studies of time consistent fiscal policy absent capital. Gervais and Mennuni (2015) analyze quantitatively optimal taxation under physical capital and debt, but assume government commitment.

[5]A capital levy in an overlapping generations environment involves redistribution among agents. The fact that optimal steady state capital income taxes are not zero in such a framework, assuming full commitment, is shown in the early work of Atkinson and Sandmo (1980), and later Garriga (2003), and Erosa and Gervais (2002) with age-dependent taxes. See also Mateos-Planas (2010), as well as Hassler, Rodríguez Mora, Storesletten, and Zilibotti (2004), for overlapping generations settings in which policy is chosen through majority voting.

generations, one that is necessarily absent in a representative agent construct, makes the Ramsey plan time inconsistent. At each date, a Markov government chooses taxes on labor so as to equate marginal utilities of consumption across generations, even when this implies taxing capital income. Changing the tax on labor generally implies a change in the tax on capital via the government budget constraint but, from the standpoint of a Markov government which takes the states it inherits as given, capital income taxes are non-distortionary. In contrast, from the standpoint of a date 0 Ramsey planner, the capital income tax is obviously distortionary and, in taking this fact into account, its policies will necessarily be time inconsistent.

This paper is organized as follows. Section 2 describes the problem of optimal Markov-perfect taxation in a finite horizon representative agent economy. Section 3 extends the analysis to an infinite horizon. Section 4 suggests a deviation from the representative agent framework by considering optimal taxation with and without commitment in a simple overlapping generations environment. Section 5 concludes.

2 Optimal time consistent taxation in a finite horizon economy

This section shows that the well-known optimal initial capital levy, which obtains when capital income taxes are not constrained exogenously and the government can credibly promise not to levy future distortionary taxes, also emerges without commitment. In other words, credible promises are redundant in this case. A two-period version of the economy studied by Chamley (1986) is perhaps the environment in which one can most easily establish this particular coincidence of full commitment and time consistent optimal fiscal policies. This result holds irrespective of whether the sequence of government spending is determined endogenously within the model, and easily generalizes to the case with an arbitrarily finite horizon. More generally, the fact that full commitment and discretionary policies coincide in this case does not rely on any peculiarity associated with an infinite horizon. Thus, we are able to frame our main finding as a recursive Markov-perfect equilibrium in which date 0 plays no pivotal role. When the capital income tax is subject to an upper limit, the resulting Ramsey policy is not strictly time consistent, but a Markov solution continues to set labor and capital income taxes to zero forever once this limit stops binding.

2.1 The environment

There exists a representative household who has finite life indexed by $t = 0, 1$. This household lives in a single good economy and values consumption and leisure streams, $\{c_t, l_t\}_{t=0}^1$, according

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to preferences given by

$$u(c_0, l_0) + \beta u(c_1, l_1), \tag{1}$$

where $l_t = 1 - n_t$, with n_t denoting labor input. The function u is increasing, strictly concave, and twice continuously differentiable in c and l. The unique good is produced by combining labor and capital, k_t, using the production technology

$$F(k_t, n_t), \tag{2}$$

where F is constant returns to scale with respect to its inputs. Production can be used for either private or government consumption, or to increase the capital stock,

$$c_t + k_{t+1} + g_t = F(k_t, n_t) + (1 - \delta)k_t, \tag{3}$$

where $0 < \delta < 1$ denotes the capital depreciation rate, and $\{g_t\}_{t=0}^1$ represents an exogenously given sequence of public expenditures.

As in Chamley (1986), the government finances its purchases using time-varying linear taxes on labor, τ_t^n, and capital, τ_t^k. The government can also make up for any imbalance between its revenues and expenditures by issuing one-period bonds that are perfectly substitutable with capital. We denote the level of government debt by b_t, where $b_t < 0$ when the government lends to the public. At each date, the government's budget constraint is given by

$$\tau_t^k r_t k_t + \tau_t^n w_t n_t + b_{t+1} = g_t + R_t b_t, \tag{4}$$

where r_t and w_t are the market rates of return to capital and labor, and R_t denotes the gross rate of return on government bonds from $t - 1$ to t. The left and right-hand side of equation (4) represent sources and uses of government revenue respectively.

The representative household maximizes (1) subject to the sequence of budget constraints,

$$c_t + k_{t+1} - (1 - \delta)k_t + b_{t+1} = (1 - \tau_t^n)w_t n_t + (1 - \tau_t^k)r_t k_t + R_t b_t, \tag{5}$$

from which we obtain the static equation describing households' optimal labor-leisure choice,

$$u_l(c_t, 1 - n_t) = u_c(c_t, 1 - n_t)(1 - \tau_t^n)w_t, \tag{6}$$

the conventional Euler equation,

$$u_c(c_t, 1 - n_t) = \beta u_c(c_{t+1}, 1 - n_{t+1}) \left[(1 - \tau_{t+1}^k)r_{t+1} + 1 - \delta \right], \tag{7}$$

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and the asset arbitrage condition,

$$R_{t+1} = (1 - \tau_{t+1}^k)r_{t+1} + 1 - \delta. \tag{8}$$

A representative firm in this economy takes as given the sequence of prices, $\{r_t, w_t\}_{t=0}^1$, and maximizes profits. It follows that

$$r_t = F_k(k_t, n_t) \text{ and } w_t = F_n(k_t, n_t). \tag{9}$$

Given the environment we have just laid out, second-best Ramsey tax rates are found by maximizing welfare in (1) subject to public and private sector budget constraints (4) and (5), optimal household behavior summarized by (6), (7), and (8), as well as firms' decision rules, (9). In solving the Ramsey problem, we imagine that the government can commit to its chosen course of actions through time, and that no limit is placed on τ_t^k. Under these assumptions, it is well known that Ramsey tax rates imply a capital levy in the initial period and zero distortionary taxes otherwise. Because the capital stock at time 0 is fixed, this policy prescription amounts to a single initial lump sum tax. The literature on optimal taxation, therefore, typically conjectures that without a commitment technology, a government would be tempted at future dates to renege on its promises of zero labor and capital taxes, and revert back to a confiscatory tax on capital. We show that this is in not the case, and that a time consistent Markov-perfect equilibrium yields the same solution.

2.2 Markov-perfect optimal tax rates

We now assume that the government no longer has access to a commitment technology. Rational households recognize that the public sector may have an incentive to deviate from the sequence of taxes they promise. Hence, policy announcements made at time 0 are not always credible. In fact, a general feature of Ramsey policies without public debt is that if a government were given the chance to re-optimize at some date $t > 0$, it would choose to deviate from the policy sequence prescribed at $t = 0$.

The question then is whether alternative optimal policies exist which, when implemented, no subsequent government would ever have an incentive to abandon. In this section we define a maximization problem that is associated with a time consistent tax structure in equilibrium. Optimal policy rules are functions of the current states of the economy only. In particular, time consistent tax rates at any date depend on the level of debt and the stock of capital available at the beginning

of that date. Policy is history independent and reputation plays no role.[6] We define a stationary Subgame Perfect Markov Equilibrium along the lines of Klein, Krusell, and Rios Rull (2008). We consider a sequence of successive governments, each choosing labor and capital tax rates based on the state it inherits when taking office. In choosing this policy rule, each government takes as given the following government's optimal choice of taxes, given the relevant states at that date. If a policymaker can correctly infer the rule optimally used by her successor, she may then be able to manipulate the decisions of whatever government follows by creating the appropriate state through her choice of policy. For this scenario to be an equilibrium, it must be the case that all policymakers actually choose the rule anticipated by their predecessor.

In the simple two-period economy considered in this section, time consistent optimal tax rates of the kind we have just described can most easily be found by focusing on the last period and proceeding backwards.

Formally, the problem of the government at date 1 is

$$\max \ u(c_1, 1 - n_1) \tag{10}$$

subject to the government budget constraint (4), the household budget constraint (5), and the condition describing optimal labor-leisure choice (6), all evaluated at date 1. The Euler equation constraint (7) that determines savings is not relevant in the last period for two reasons. First, since the world ends with date 1, households have no incentive to save for future consumption. In particular, $k_2 = b_2 = 0$. Second, from the perspective of this government, the capital stock, k_1, is given and past investment decisions are sunk. One important implication is that the optimal choices of taxes, τ_1^k and τ_1^n, depend only on the relevant states for that date, $\{k_1, b_1\}$, and we denote these choices by $\tau_1^k(k_1, b_1)$ and $\tau_1^n(k_1, b_1)$. Furthermore, we designate optimal allocations for consumption and labor that emerge from this maximization problem as $c_1(k_1, b_1)$ and $n_1(k_1, b_1)$ respectively.

Knowing how its successor behaves, a government at date 0 can then optimally choose how to set taxes given $\{k_0, b_0\}$. In doing so, this government takes fully into account the optimal behavior followed by the date 1 government. Formally, the maximization problem is,

$$\max \ u(c_0, 1 - n_0) + \beta V(k_1, b_1), \tag{11}$$

where $V(k_1, b_1) = u(c_1(k_1, b_1), 1 - n_1(k_1, b_1))$, subject to the government budget constraint (4), the household budget constraint (5), and the condition describing optimal labor-leisure choice (6), all

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[6]See Chari and Kehoe (1990) for this alternative approach towards optimal time consistent policy.

evaluated at date 0, as well as the Euler equation constraint,

$$u_c(c_0, 1 - n_0) = \beta \left[F_k(k_1, n_1(k_1, b_1))(1 - \tau_1^k(k_1, b_1)) + 1 - \delta \right] u_c(c_1(k_1, b_1), n_1(k_1, b_1)). \qquad (12)$$

In this last equation, tomorrow's consumption, leisure, and taxes, are explicitly written in terms of the date 1 decision rules. By solving the backwards induction problem we have just described, we are ensuring that each government at each date responds in the best possible way to the states inherited from previous governments. Therefore, the solution obtained in this way will necessarily be time consistent, and is summarized in the following proposition.

Proposition 1 *Optimal time consistent taxes on labor and capital are such that $\tau_0^k \geqslant 0$, $\tau_1^k = 0$, and $\tau_0^n = \tau_1^n = 0$.*

Proof.   See Appendix A.   ∎

One can easily show that the policy described in proposition 1 is in fact identical to the one that emerges under the Ramsey plan. Under commitment, if the initial tax on capital income is unconstrained, the policy that maximizes lifetime utility involves setting a positive capital tax rate at date 0 (which finances the discounted sum of future public expenditures), and no distortions to the labor decision in either period or capital taxes at date one. In principle, once date 0 has passed, the planner in power in period one might well have incentives to deviate from this path and set positive taxes. However, in choosing taxes today, the date 0 government is able to anticipate the incentives facing the date 1 government, summarized by $\{\tau_1^n(k_1, b_1), \tau_1^k(k_1, b_1)\}$, and manipulate its successor's policy decisions. Hence, the initial government chooses taxes, in particular, τ_0^k, so as to leave the period 1 government with equilibrium stocks of debt and capital that exactly induce it to set taxes on capital and labor to zero.[7] Put another way, when one allows for the presence of both capital and bonds, strategic manipulation by the initial government helps reproduce Ramsey outcomes despite the fact that choices are sequential and that the past cannot be undone.

The solution for the initial optimal tax on capital is given by[8]

$$\tau_0^k = \frac{g_0 + (F_k(k_0, n_0(k_0, b_0)) + 1 - \delta)b_0 + \frac{g_1}{F_k(k_1(k_0, b_0), n_1(k_0, b_0)) + 1 - \delta}}{F_k(k_0, n_0(k_0, b_0))(k_0 + b_0)}, \qquad (13)$$

where $n_0(k_0, b_0)$, $n_1(k_0, b_0)$ and $k_1(k_0, b_0)$ denote allocations as a function of the initial states. As with the standard Ramsey solution, it is possible for taxes to be zero even in the initial period.

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[7] Recall that as τ_0^k changes, so does disposable income at that date and, therefore, c_0 which in turns affects the optimality conditions associated with k_1 and b_1.

[8] See Appendix A for the derivation.
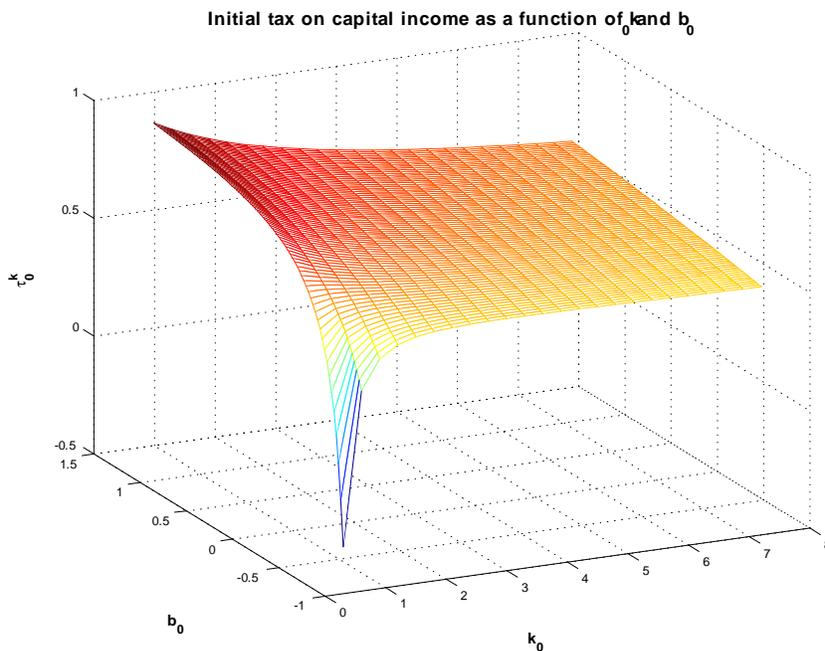
Specifically, there exist combinations of initial capital stocks and debt, k_0 and b_0, that deliver $\tau_0^k = 0$.

Figure 1 above plots an example of equation (13) as a function of initial asset holdings. As one might expect, when initial public debt increases (given a fixed value of k_0), so does the tax on capital required in order to finance a given stream of expenditures, $\{g_0, g_1\}$. The relationship linking τ_0^k to private capital, however, is non-monotonic for different levels of debt.[9]

2.3 Endogenous public expenditures

The fact that Markov-perfect taxes replicate Ramsey policies in the case we have just described does not depend on the assumption of exogenous public expenditures. Consider, for instance, an economy where the government provides pure public goods – such as national parks – from which

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[9]To see this, note first that given a return to savings, $F_k(k_0, n_0)$, a large stock of capital generates high tax revenues, $\tau_0^k F_k(k_0, n_0) k_0$, which suggests that a lower value of τ_0^k may be required to meet a given sequence of public spending. However, since the return to savings itself decreases with k_0, it helps erode the tax base as the level of capital increases and raises the equilibrium choice of τ_0^k to maintain feasibility. In addition, since tomorrow's return to savings falls with k_0 – because $k_1(k_0, b_0)$ increases with initial capital – the present value of tomorrow's expenditures also rises, thus requiring a larger levy in period 0. In the end, the net effect of changes in k_0 on the equilibrium value of τ_0^k required to finance expenditures depends on both these forces acting in opposite direction. Moreover, these forces are themselves affected by the level of initial debt, which help determine both date 0 and date 1 labor choices, $n_0(k_0, b_0)$ and $n_1(k_0, b_0)$, as well as savings in the current period, $k_1(k_0, b_0)$.

households derive utility directly. Assuming that these goods can be financed with both bonds and taxes on factor income, the environment is then identical to the one analyzed above, except that government expenditures are now determined endogenously. It is still the case that the optimal policy involves confiscatory capital taxes initially with no distortions thereafter, and that this policy is time consistent. Consequently, the justification for a given sequence of public spending and the means of financing it can be studied separately.

Proposition 2 *Consider the economy described in section 2.1, but let preferences be defined over both private and public consumption, $u(c, 1 - n, g)$, where u is increasing, strictly concave, and twice continuously differentiable in its arguments. Then, the optimal sequence of taxes on labor and capital under commitment are such that $\tau_0^k \geqslant 0$, $\tau_1^k = 0$, and $\tau_0^n = \tau_1^n = 0$. Furthermore, this solution is time consistent.*

The proof of this proposition is entirely straightforward and follows the exact reasoning described in Appendix A, with two additional conditions that determine the optimal size of government spending at date 0 and 1. These conditions equate the marginal utility derived from the consumption of private and public goods at each date.

2.4 Extensions to an arbitrary finite horizon

The intuition we have just laid out in a two-period model easily generalizes to the case of an arbitrary finite horizon, $T > 0$. Under government commitment, what matters is the notion that irrespective of the time horizon, a planner at date 0 never wishes to distort either the labor-leisure decision or the savings decision. The same reasoning turns out to apply when different governments, but with the same objective, make sequential decisions over time. Given that a policymaker in office in the last period chooses policy based on the states it receives, k_T and b_T, the government in period $T - 1$ chooses taxes so as to leave its successor with exactly those states that induce it to set $\tau_T^k = \tau_T^n = 0$. This process repeats itself backwards until the initial period, independently of how public expenditures are determined. In effect, each government acts in such a way as to provide the exact incentives needed for its successor not to introduce distortions in the economy.

2.5 An upper limit on the capital income tax

In the original framework studied by Chamley (1986), an upper limit is exogenously placed on the capital income tax, $\tau_t^k \leq 1$, in order to precisely prevent the date 0 capital levy that otherwise emerges as optimal. In a discrete time version of that economy, Atkeson, Chari, and Kehoe (1999)

show that this restriction binds at most for a finite number of periods, say $J^R > 0$, after which capital income taxes take on an intermediate value for one period and are zero thereafter. In that constrained neoclassical setting, therefore, a capital levy of sorts is not ruled out by assumption but nevertheless emerges over the first few periods.[10] In this case, taxes on labor will generally never be zero.

While the Ramsey solution is not strictly time consistent in this case, we now show that if the upper limit on capital income taxes ceases to bind at some date J, the problem admits a very similar Markov-perfect solution, at least in finite horizon. Under this solution, capital income taxes are always zero after date J. Unlike in the commitment case, labor taxes are also zero when $t \geq J$ so that in general, J and J^R above do not coincide.

Relaxing the full commitment assumption does not necessarily yield distortionary taxes at all dates, even with an upper limit on capital income. The proposition below formalizes this result.

Proposition 3 *Consider the economy described in section 2.1 with $t = 0, 1, ..., T$, and $\tau_t^k \leq 1$. Then, if $0 < \tau_J^k < 1$ for some $J \leq T$, in a Markov-perfect equilibrium, $\tau_{J+t}^k = \tau_{J+t}^n = 0$ for $t = 1, ..., T - J$.*

Proof. See Appendix B. ∎

The intuition underlying this last proposition is as follows: under commitment, a Ramsey planner wishes to set, in the short run, the highest feasible capital taxes that will finance enough of an asset base to eliminate distortions in the long run. Hence, she sets τ_t^k to its upper bound for J^R periods, after which capital taxes take on an intermediate value for one period. One of the trade offs involved with commitment is that the planner could chose to set $\tau_{J^R-1}^k$ lower than its upper bound – (and therefore reduce the associated intertemporal distortion) – at the expense of increasing labor taxes. This margin, however, is largely inaccessible to sequential Markov planners since, at each date, a planner chooses only current labor and capital income taxes given previous governments decisions, and can only influence future taxes through the states she leaves her successor government. As a result, Ramsey taxes will generally not be time consistent in this case. Ignoring the latter margin for the moment, it remains that once the upper limit on capital income taxes ceases to bind, enough assets are held by the public sector that after that date, governments no longer have to levy any taxes. In effect, the problem from that date on reduces to the benchmark

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[10]In fact, if utility is separable in consumption and leisure, the capital income tax is zero from period 2 onwards. See Renstrom (1999).

case introduced in section 2.2. In fact, even when an exogenous upper limit is imposed on capital income taxes, the type of policy described in Atkeson et al. (1999) can be time consistent for some specification of preferences. This is shown in Xie (1997) for the utility specification $u(c, l) = \ln(c - l)$, in which case labor income taxes are also always zero. Ultimately, these results suggest that removing commitment does not necessarily preclude the front-loading of taxes, and only the exogenous upper limit on τ_t^k prevents a complete initial-period confiscation. It is worth noting that the Markov path for capital income taxes, from period J onwards, continues to hold even when labor is supplied inelastically and untaxed. In contrast, it is well known that in such a framework, the optimal capital income tax under full commitment is generally not zero in the final period (Correia [1996]).

3 Optimal time consistent taxation in an infinite horizon economy

At this stage, we are in a position to show that the intuition laid out above extends to the standard infinite horizon environment. In other words, the time horizon is not an important determinant of outcomes. However, there does exist one notable difference with the previous section in that our emphasis now lies on a *stationary recursive* Markov-prefect equilibrium. Therefore, date 0 no longer plays a pivotal role. We revert back to a problem without an exogenous constraint on capital income taxes since, over an infinite horizon, one presumes that a date is eventually reached where consolidated assets are such that this constraint becomes irrelevant.[11]

Consider the economy described in section 2 but let time range over an infinite horizon, $t = 0, 1, ..., \infty$. Define the stationary Markov-perfect equilibrium policy rules,

$$\tau^n = \theta(\bar{k}, \bar{b}) \text{ and } \tau^k = \psi(\bar{k}, \bar{b}), \tag{14}$$

where $\bar{k}$ and $\bar{b}$ denote the aggregate stocks of capital and debt respectively so that, at any point in time, taxes on labor and capital depend only on the payoff relevant state variables.[12] Given the policy rules in (14), it is useful to write down the individual maximization problem recursively.

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[11]To be clear, the solution described in this section holds over a region of the asset space that rules out large amounts of public debt or small capital stocks.

[12]Krusell and Kuruscu (2002) show that Markov-perfect solutions potentially allow for an infinite number of discontinuous equilibria (i.e. where the policy rules are discontinuous in the state variables). When they arise, however, these equilibria typically result from assuming an infinite horizon. For the purpose of our analysis, we shall narrow our definition of a stationary Markov-perfect equilibrium and focus only on the limit of the finite horizon solution described in the previous section. Assuming that the policy functions in (14) are differentiable, we show that such an equilibrium can indeed be found.

In what follows, we denote next period's value of any variable x by x'. Let $k' = \mathcal{H}(k, b, \bar{k}, \bar{b})$ and $b' = \mathcal{B}(k, b, \bar{k}, \bar{b})$ represent optimal capital and debt accumulation decision rules under the Markov policy functions θ and ψ. In other words, given aggregate and individual states for capital and debt respectively, $\mathcal{H}$ and $\mathcal{B}$ solve the following dynamic program,

$$W(k, b, \bar{k}, \bar{b}) = \max_{c, n, k', b'} \left\{ u(c, 1 - n) + \beta W(k', b', \bar{k}', \bar{b}') \right\} \tag{P1}$$

subject to

$$c + b' + k' = \left[(1 - \psi(\bar{k}, \bar{b}))r + 1 - \delta \right] k + R^b b + (1 - \theta(\bar{k}, \bar{b}))wn, \tag{15}$$

where $r = F_k(\bar{k}, \bar{n})$, $R^{b'} = (1 - \psi(\bar{k}', \bar{b}'))F_k(\bar{k}', \bar{n}') + 1 - \delta$, $w = F_n(\bar{k}, \bar{n})$. Aggregate variables evolve according to $\bar{k}' = \bar{\mathcal{H}}(\bar{k}, \bar{b})$ and $\bar{b}' = \bar{\mathcal{B}}(\bar{k}, \bar{b})$. In equilibrium, it must be the case that aggregate outcomes are consistent with individual optimization, $\bar{k} = k$ and $\bar{b} = b$. Consistency then implies that $\mathcal{H}(\bar{k}, \bar{b}, \bar{k}, \bar{b}) = \bar{\mathcal{H}}(\bar{k}, \bar{b})$ and $\mathcal{B}(\bar{k}, \bar{b}, \bar{k}, \bar{b}) = \bar{\mathcal{B}}(\bar{k}, \bar{b})$.

In order that the Markov policies stated in (14) constitute a subgame perfect equilibrium, no government must ever have an incentive to deviate from $\theta(\bar{k}, \bar{b})$ and $\psi(\bar{k}, \bar{b})$ at any time. Joint deviations are not feasible since, in a Markov-perfect equilibrium, a new government chooses policy every period and cannot enter binding contracts with future governments. It is sufficient, therefore, to analyze the problem of a government that is allowed to "cheat" in the current period by setting tax rates $\tau^n \neq \theta(\bar{k}, \bar{b})$ and $\tau^k \neq \psi(\bar{k}, \bar{b})$, under the assumption that $\theta(\bar{k}, \bar{b})$ and $\psi(\bar{k}, \bar{b})$ are forever followed in the future. Since the representative agent assumption implies that $\bar{k} = k$ and $\bar{b} = b$ in equilibrium, we simplify the notation below and only refer to k and b.

Taking as given household optimal behavior summarized by (5) through (7), a government at any date that is free to set current taxes solves

$$\max_{c, n, \tau^k, \tau^n, k', b'} u(c, 1 - n) + \beta W(k', b') \tag{P2}$$

subject to

$$c + k' + b' = \left[(1 - \tau^k)F_k + 1 - \delta \right] (k + b) + (1 - \tau^n)F_n n \tag{16}$$

$$u_l(c, 1 - n) = u_c(c, 1 - n)(1 - \tau^n)F_n$$

$$u_c(c, 1 - n) = \beta \left\{ \left[(1 - \psi(k', b'))F_k' + 1 - \delta \right] u_c(\tilde{c}(k', b', \mathcal{H}'), 1 - n') \right\}$$

$$\tau^k F_k(k + b) + \tau^n F_n n = g + (F_k + 1 - \delta)b + b',$$

where

$$\tilde{c}(k', b', \mathcal{H}') = \left(F_k' + 1 - \delta \right) k' + F_n' n' - \mathcal{H}(k', b') - g'$$

obtains from the consolidation of household and government budget constraints, $F_i' = F_i(k', n')$ with $i = k, n$, and $W(k', b')$ denotes households' continuation value when they behave optimally under the Markov policy rules θ and ψ. Then, in order for θ and ψ to constitute time consistent equilibrium policy functions, it must be the case that the optimal choices of τ^n and τ^k that solve (P2) yield precisely θ and ψ respectively. In other words, for θ and ψ to be equilibrium time consistent policies, it must be the case that a government that is allowed to deviate at any date finds it in its best interest not to do so and to actually follow the rules prescribed by θ and ψ in (14).

Proposition 4 *In a stationary Markov perfect equilibrium, θ and ψ are such that $\forall k$ and b, $\theta(k, b) = 0$ and $\psi(\mathcal{H}(k, b), \mathcal{B}(k, b)) = 0$.*

Proof.   See Appendix C. ∎

Relative to the previous sections, the key insight of proposition 4 is that calendar dates no longer matter. In particular, in problems (P1) and (P2), the past is sunk and, no matter how one has arrived to the states k and b, taxes on labor, $\theta(k, b)$, are always zero. Moreover, while time consistent taxes on capital, $\tau^k = \psi(k, b)$, are generally not zero for arbitrary values of k and b, it is also the case that once we move forward in the dynamic program (P1) using the decision rules $k' = \mathcal{H}(k, b)$ and $b' = \mathcal{B}(k, b)$, the Markov-perfect tax on capital is always zero. Observe, for instance, that two periods hence in program (P1), the values of capital, k'', and debt, b'', are simply obtained by applying the functions $\mathcal{H}$ and $\mathcal{B}$ to states $\{k', b'\}$ respectively. Therefore, since proposition 4 holds for all values of the states k and b, it follows that $\psi(k'', b'') = 0$. The argument, of course, holds at any stage of the household dynamic program under the policy rules θ and ψ. The fact that $\psi(k, b)$ is generally not zero for arbitrary values of k and b simply reflects the fact that taxes still have to be levied to finance government expenditures. However, given states k and b, the functions $\mathcal{H}$ and $\mathcal{B}$ map these states, as well as all following states, into a region of the asset space where ψ is always zero. Analogously to the finite horizon problem in the previous section, the planner at any date chooses policy such that the resulting equilibrium asset holdings place her successor on the surface depicted in Figure 1 where zero capital taxes are optimal.

4   A simple model of optimal taxation in overlapping generations

In the environment studied by Chamley (1986), one of the fundamental reasons driving the time consistency of the optimal initial capital levy relates to households' ability to undo this burden

in future periods by borrowing from the government. In essence, the presence of bonds lets the government front-load all taxes in the initial period in a lump sum fashion while still allowing households to meet their desired consumption path through sovereign lending. And indeed, we showed above that when optimal policy is chosen sequentially by different governments, all with the objective of maximizing household utility, the same solution arises.

In practice, however, the mechanism that disposes of distortionary taxes breaks down simply because individuals have finite lives. Specifically, consider a world with overlapping generations. The current old, whose capital position is relatively inelastic (i.e. their investment decisions are largely behind them and sunk), may simply not be alive in subsequent periods to enjoy the benefits of lower distortions that could be made possible by a large initial capital levy. Taking this fact into account, a planner who weighs all generations equally will not place the entire burden of public expenditures on the initial old generation, which in effect translates into an endogenous upper limit on capital income taxes. In addition, a role for distortionary taxes immediately emerges as the planner optimally apportions the cost of public spending across different generations. This differs sharply from the representative agent construct where, without a binding constraint on the initial capital income tax, all distortionary taxes are zero.

It is also noteworthy that when individuals die in finite time, optimal taxes under commitment will be time inconsistent, thus restoring the government's ability to commit to future actions as a policy concern. In particular, because the cost of financing the present discounted value of government expenditures cannot be raised all at once, the government can no longer credibly promise to set future taxes to zero.

We can illustrate these ideas most simply by addressing the problem of optimal taxation in a two-period economy analogous to that in section 2, but with overlapping generations. The objective here is to strip down the model to its essential features, thus highlighting the intuition underlying the time-inconsistency problem, and underscoring the importance of endogenous barriers to initial-period government confiscation of private assets.

Time is indexed by $t = 0, 1$, and individuals live for two periods. The economic environment, therefore, is populated by three types of households: those who are already old at date 0, those who are born at date 0 and old and date 1, and those who are born at date 1. Let c_{yt} and c_{ot} denote the consumption of the young and old at time t respectively. Individuals are endowed with one unit of time at birth, and use the first period of life to work and save for old age. In the second period of life, individuals use the return on their savings to finance their consumption expenditures and pass away. For transparency, this section assumes Constant Relative Risk Aversion (CRRA)

preferences and Cobb-Douglas technology.

Under these assumptions, consumption by the old generation at date 0 is simply given by

$$c_{o0} = \left[(1 - \tau_0^k)r_0 + 1 - \delta\right] a_0, \tag{17}$$

where $a_0 = b_0 + k_0$. From the standpoint of a Ramsey planner (i.e. one who can commit to a sequence of taxes), this generation's investment decisions are sunk, in the sense that a_0 is given. However, contrary to the representative agent construct of section 2, this generation will not be alive in the following period to make up through borrowing for whatever taxes they face.

Taking policy and prices as given, the generation born at date 0 solves

$$\max_{c_{y0},n_0,a_1,c_{o1}} \frac{c_{y0}^{1-\sigma}}{1-\sigma} - \chi \frac{n_0^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} + \beta \frac{c_{o1}^{1-\sigma}}{1-\sigma},$$

subject to its budget constraints,

$$c_{y0} = (1 - \tau_0^n)w_0 n_0 - a_1, \tag{18}$$

and

$$c_{o1} = \left[(1 - \tau_1^k)r_1 + 1 - \delta\right] a_1. \tag{19}$$

The conditions $\sigma > 0$, $\chi > 0$, and either $\nu > 0$ or $\nu < -\frac{1}{\chi\sigma}$ guarantee a concave problem. Thus, optimal labor and savings decisions for this generation are summarized by

$$\chi n_0^{\frac{1}{\nu}} = (1 - \tau_0^n)w_0 c_{y0}^{-\sigma}, \tag{20}$$

and

$$c_{y0}^{-\sigma} = \beta \left[(1 - \tau_1^k)r_1 + 1 - \delta\right] c_{o1}^{-\sigma}, \tag{21}$$

respectively.

The generation born at date 1 sets $a_2 = 0$ since the world ends on that date. Its budget constraint is given by

$$c_{y1} = (1 - \tau_1^n)w_1 n_1, \tag{22}$$

and its only relevant decision relates to optimal labor supply,

$$\chi n_1^{\frac{1}{\nu}} = (1 - \tau_1^n)w_1 c_{y1}^{-\sigma}. \tag{23}$$

4.1 Optimal taxation with commitment revisited

Given a planner who weighs every generation equally, second-best Ramsey tax rates and corresponding allocations are found by solving

$$\max W = \underbrace{\frac{c_{o0}^{1-\sigma}}{1-\sigma}}_{\text{initial old generation}} + \underbrace{\frac{c_{y0}^{1-\sigma}}{1-\sigma} - \chi\frac{n_0^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} + \beta\frac{c_{o1}^{1-\sigma}}{1-\sigma}}_{\text{initial young generation}} + \beta\underbrace{\left[\frac{c_{y1}^{1-\sigma}}{1-\sigma} - \chi\frac{n_1^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}\right]}_{\text{generation born at date 1}} \tag{P3}$$

subject to the sequence of government budget constraints,

$$g_0 + [(1-\tau_0^k)r_0 + 1 - \delta]b_0 = \tau_0^n w_0 n_0 + \tau_0^k r_0 k_0 + b_1, \tag{24}$$

and

$$g_1 + [(1-\tau_1^k)r_1 + 1 - \delta]b_1 = \tau_1^n w_1 n_1 + \tau_1^k r_1 k_1, \tag{25}$$

agents' budget constraints (17), (18), (19), and (22), as well as agents' optimal labor and savings decisions (20), (21), and (23).

Although analytical solutions for optimal taxes and corresponding allocations are not available in this context, the numerical example depicted in Table 1 is enough to illustrate two important points.[13] First, in contrast to the representative agent setting, τ_0^n, $\tau_1^n > 0$ despite the fact that no upper limit is exogenously imposed on capital income tax rates, τ_t^k. Because the planner weighs all generations equally, she does not impose the entire burden of taxes on the initial old generation. Second, while it is still the case that capital income taxes are zero in the final period, as in the representative agent economy, we shall see that without government commitment, this finding no longer holds. The problem of time inconsistency, therefore, plays a role in generating distortionary taxes.

In an overlapping generations economy, a benevolent government strives towards two objectives, namely to find the optimal allocation of resources both along an intratemporal and an intertemporal dimension. As with actual economies, the typical life-cycle model is such that old agents own a greater part of aggregate wealth while younger agents mostly work. A direct implication in the setting we have just laid out is that the tax on capital income affects the old generation while the tax on labor income affects only the young. It follows that increases in the initial capital income tax are associated with greater intergenerational inequity. In other words, avoiding distortionary taxes by relying more intensely on an initial capital levy creates greater inequality across generations. Given

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[13]This numerical example is computed using the following specification: $\beta = 0.96$, $\delta = 0.10$, $\sigma = 2.5$, $\nu = -0.3$, $\chi = 10$, and $\alpha = 1/3$, $z = 10$, $k_0 = 0.75$, $b_0 = 0.30$, $g_0 = g_1 = 3.5$.

this feature of the environment, a benevolent government will limit the degree to which the initial old generation is taxed. How then are the remainder of public expenditures to be financed? Since a tax on tomorrow's capital income is highly distortionary, a benevolent planner will generally prefer to tax current and future young generations through labor income taxes. In fact, in the example above, the tax on capital income in period 1 is exactly zero just as in the representative agent construct. The latter result is a direct application of the uniform commodity taxation principle as well as the fact that, in our framework, taxes are age-specific: young agents contribute labor income taxes while older agents only contribute capital income taxes (see Erosa and Gervais [2002] for a discussion of the uniform commodity taxation principle in overlapping generations models).

Table 1

Ramsey and Markov Tax Rates (percent)		
	Ramsey Policy	Markov Policy
Initial tax on capital income, τ_0^k:	12.67	12.56
Initial tax on labor income, τ_0^n:	14.92	14.75
Final tax on capital income, τ_1^k:	0.00	6.77
Final tax on labor income, τ_1^n:	20.84	17.55

Finally, observe that in period 0, aggregate economic wealth is given by k_0. The level of debt, b_0, then tells us how this wealth is initially distributed between the government and the old generation. Specifically, the initial old have wealth given by $a_0 = k_0 + b_0$ while government assets are $-b_0$. Since the government can use a non-distortionary tax on capital, τ_0^k, to redistribute initial wealth in any way it wishes, for any given initial level of wealth, k_0, the initial level of debt is irrelevant for the remaining policy decisions.

Having described the basics of the Ramsey plan in this simple overlapping generations context, we now turn our attention to time consistent tax policies. Interestingly, when different generations overlap, Ramsey policies will be time inconsistent owing to an intergenerational distribution concern that is necessarily absent in a representative agent framework.

4.2 Time consistent optimal tax rates

As in section 2, time consistent optimal tax rates can be found in two stages starting from the last period and proceeding backwards. Thus, the government at date 1 solves

$$\max \frac{c_{o1}^{1-\sigma}}{1-\sigma} + \frac{c_{y1}^{1-\sigma}}{1-\sigma} - \chi \frac{n_1^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}, \tag{P4a}$$

subject to agents' resource constraints (19) and (22), equation (23) which describes the optimal labor-leisure decision of the generation born at date 1, and the period 1 government budget constraint, (25). The set of first-order necessary conditions associated with this problem reduce to a single key equation (shown in Appendix D),

$$\left\{ 1 - \frac{\tau_1^n}{1 - \tau_1^n} \frac{1 - \sigma}{\frac{1}{\nu} + \sigma} \right\} c_{o1}^{-\sigma} - c_{y1}^{-\sigma} = 0 \tag{26}$$

This expression describes the trade-off involved in transferring one unit of consumption from the young to the old through labor income taxes. Given a fixed level of expenditures, g_1, an increase in labor income taxes allows for a reduction in capital income taxes while still ensuring that the government budget constraint holds. This policy change, therefore, translates into a transfer of resources from the young to the old generation. If labor taxes were not distortionary, the decline in marginal utility of the young resulting from one less unit of consumption, $c_{y1}^{-\sigma}$, would be exactly offset by the gain in marginal utility of the old, $c_{o1}^{-\sigma}$. However, taxes on labor income are distortionary and an increase in τ_1^n leads to a proportional change in the labor supply of the young given by

$$\frac{\partial n_1 / \partial \tau_1^n}{n_1} = \frac{-(1 - \sigma)}{(1 - \tau_1^n)(\frac{1}{\nu} + \sigma)}. \tag{27}$$

where $\frac{\partial n_1 / \partial \tau_1^n}{n_1} \gtrless 0$ depending on whether the substitution or income effect dominates.[14] Given that $\tau^n < 1$, the substitution effect dominates, $\frac{-(1-\sigma)}{(1-\tau_1^n)(\frac{1}{\nu}+\sigma)} < 0$, when $\sigma < 1$. Because labor taxes are distortionary, when higher labor taxes reduce consumption of the young by one unit, the corresponding decrease in capital taxes that keeps the government budget constraint balanced yields less than one additional unit of consumption to the old, as reflected by the second term in parenthesis in equation (26). The reverse is true when $\sigma > 1$ as the income effect dominates the substitution effect. Taking as given the states $\{k_1, b_1\}$, the planner then chooses taxes on labor income so as to equate consumption transfers across generations, measured in utility terms and taking into account the distortionary effect of the tax. In contrast to the Ramsey problem, she ignores any intertemporal feedback this might have on period 0 decision making. Furthermore, this intratemporal distribution motive is entirely absent in the representative agent setting of section 2.

Mechanically, the four constraints in this problem along with equation (26) form a set of five equations in five unknowns, c_{o1}, c_{y1}, n_1, τ_1^n and τ_1^k, and thus implicitly determine $c_{o1}(k_1, b_1)$, $c_{y1}(k_1, b_1)$, $n_1(k_1, b_1)$, $\tau_1^n(k_1, b_1)$ and $\tau_1^k(k_1, b_1)$.

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[14]See Appendix D.

At date 0, the government solves

$$\max \frac{c_{o0}^{1-\sigma}}{1-\sigma} + \frac{c_{y0}^{1-\sigma}}{1-\sigma} - \chi \frac{n_0^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} + \beta V(k_1, b_1), \tag{P4b}$$

where

$$V(k_1, b_1) = \frac{c_{o1}(k_1, b_1)^{1-\sigma}}{1-\sigma} + \frac{c_{y1}(k_1, b_1)^{1-\sigma}}{1-\sigma} - \chi \frac{n_1(k_1, b_1)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}},$$

subject to agents' budget constraints (17) and (18), optimal behavior summarized by (20) and (21), the period 0 government budget constraint (24), as well as all period 1 constraints used in solving (P4a) along with equation (26).

It should be clear that the resulting optimization problem exactly replicates the problem with full commitment, but with one additional constraint, equation (26). This equation captures the behavior of future governments and, in particular, the fact that today's policy choices have an effect on tomorrow's policies. Therefore, when equation (26) holds independently of the policy chosen in the Ramsey problem, as is the case when preferences are logarithmic in consumption for instance, the Ramsey plan is time consistent.

When utility is logarithmic in consumption, $\sigma = 1$ and $\frac{1-\sigma}{\frac{1}{\nu}+\sigma} = 0$. Hence, equation (26) implies that $c_{o1} = c_{y1}$ without any references to $\tau_1^n(k_1, b_1)$. To see why such a case is associated with time consistent Ramsey policies, consider the problem of the planner with full commitment. With $\sigma = 1$, it is well known that the young generation's savings at time 0, $k_1 + b_1$, depend only on current income and are invariant to the after-tax rate of return at time 1. As a consequence, the tax on capital in period 1 is non-distortionary and, at date 0, the Ramsey planner only considers its implications for intergenerational redistribution and labor taxes at date 1. However, from equations (22) and (23), when $\sigma = 1$, equilibrium labor supply is inelastic and given by

$$n_1 = \chi^{\frac{\nu}{1+\nu}}, \tag{28}$$

so that taxes on labor income are *also* non-distortionary. In this case, therefore, the Ramsey government will not have to minimize any distortionary effects of either labor or capital taxes in period 1, and can focus solely on attaining its intergenerational distribution objective in that period. In such a setting, a Ramsey planner that is given the chance to re-optimize in period 1 will have no incentive to deviate from its chosen policies at date 0 and the Ramsey plan will be time consistent.

In general (i.e. when $\sigma \neq 1$), taxes on capital in period 1 will not be zero unlike in the commitment problem and despite preferences being separable in consumption and leisure. Put another way, relative to the Ramsey problem, a Markov government at date 1 will use labor and

capital income taxes to resolve its intergenerational distribution concerns (i.e. make equation 26 hold), even if this means setting non-zero taxes on capital income. Recall that the government budget constraint still has to hold but that, from the standpoint of a Markov government, capital taxes are non distortionary. That is, sequential governments at each date take the states they inherit as given. It is precisely in this sense that the intergenerational concern which arises with overlapping generations creates a time inconsistency problem.

Finally, we should note that a future government may find it optimal to subsidize capital and increase taxes on labor income. When $\frac{1-\sigma}{\frac{1}{\nu}+\sigma} < 0$, the expression in parenthesis in equation (26) is greater than one and more weight is given to the marginal utility of old agents. The intuition here is relatively straightforward. When $\frac{-(1-\sigma)}{(1-\tau_1^n)(\frac{1}{\nu}+\sigma)} > 0$, $\frac{\partial n_1}{\partial \tau_1^n} > 0$ so that the income effect dominates and labor supply increases in response to a higher tax on labor income. Therefore, for the same cost to the young, more transfers can be made to the old. The date 1 government transfers more resources from the young to the old by decreasing capital taxes and increasing labor taxes.

5   Concluding remarks

In the standard neoclassical model with a representative agent, the literature on optimal taxation has argued that a benevolent planner with the ability to commit to a sequence of future tax rates will, if feasible, choose to levy a single confiscatory tax on capital in the initial period. Since the initial capital stock is fixed, this policy in effect disposes of distortionary taxation entirely. This paper has shown that disposing of all distortionary taxes remains possible even if one takes away the ability to credibly commit to future taxes. In particular, in a Markov-perfect equilibrium, a single capital levy can emerge as optimal even when policy is chosen sequentially by different governments, and whether or not government expenditures are determined endogenously within the model. When an upper limit is exogenously imposed on capital income taxes, the Ramsey plan is not strictly time consistent, but once this limit ceases to bind, the problem continues to admit a Markov solution where taxes on capital and labor income are set to zero after that date. Therefore, removing commitment can yield rather stark policies even in the latter case.

The lessons learned in the neoclassical framework suggest that distortionary time consistent tax rates more naturally emerge in an overlapping generations context. In such a world, the initial old, whose capital position is inelastic, may not be alive in future periods to borrow from the government to make up for a potentially large tax. Taking this feature into account, a benevolent planner will limit the extent to which it taxes the initial old, which in effect creates an endogenous upper limit

on capital income taxes. It follows that other distortionary taxes will generally not be zero either in the Ramsey or time consistent equilibrium, although the long-run tax on capital income may still be zero with full commitment.

Finally, in a world with overlapping generations, the intergenerational distribution concern that arises, one that is necessarily absent in a representative agent framework, makes the Ramsey plan time inconsistent. In that world, Markov governments choose labor taxes so as to equate marginal utilities of consumption across generations, regardless of the implications for capital income taxes. From their vantage point, capital income taxes are non-distortionary as each government treats the past as sunk, but this is obviously not the case for a Ramsey planner who, at some initial date, can set the entire sequence of future tax rates.

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APPENDIX A - Markov perfect optimal tax rates

As stated in the text, we prove proposition 1 using backwards induction, starting with the last period.

Optimal policy in the final period

A benevolent government in the last period is faced with states $\{k_1, b_1\}$, and maximizes household utility subject to the constraints (4) and (5), as well as the private sector decision rules (6), (7), (8), and (9), as they apply to date 1. To simplify notation, we shall use at times $u_c(t)$, $u_l(t)$, $F_k(t)$, etc... to denote time$-t$ values of the indicated functions, to be evaluated at their appropriate arguments. Thus, the Lagrangian corresponding to the date 1 problem is

$$\max \ u(c_1, 1 - n_1) \tag{PA1}$$

$$+\mu \left[u_c(1)(1 - \tau_1^n)F_n(1) - u_l(1) \right]$$

$$+\lambda \left[\left[F_k(1)(1 - \tau_1^k) + 1 - \delta \right] (k_1 + b_1) + (1 - \tau_1^n)F_n(1)n_1 - c_1 \right]$$

$$+\gamma \left[\tau_1^k F_k(1)(k_1 + b_1) + \tau_1^n F_n(1)n_1 - g_1 - (F_k(1) + 1 - \delta) b_1 \right].$$

The first order conditions associated with this problem are

$$c_1 : u_c(1) + \mu \left[u_{cc}(1)(1 - \tau_1^n)F_n(1) - u_{lc}(1) \right] - \lambda = 0, \tag{A1}$$

$$\tau_1^k : -\lambda + \gamma = 0, \tag{A2}$$

$$\tau_1^n : -\mu u_c(1) - \lambda n_1 + \gamma n_1 = 0, \tag{A3}$$

and

$$n_1 \quad : \quad -u_l(1) + \mu \left\{ (1 - \tau_1^n) \left[F_{nn}(1)u_c(1) - F_n(1)u_{cl}(1) \right] + u_{ll}(1) \right\} \tag{A4}$$

$$+\lambda \left\{ F_{kn}(1)(1 - \tau_1^k)(k_1 + b_1) + (1 - \tau_1^n) \left[F_{nn}(1)n_1 + F_n(1) \right] \right\}$$

$$+\gamma \left\{ \tau_1^k F_{kn}(1)(b_1 + k_1) + \tau_1^n \left[F_{nn}(1)n_1 + F_n(1) \right] - F_{kn}(1)b_1 \right\}$$

$$= \quad 0$$

Given these conditions, we now solve for the decision rules, τ_1^n, τ_1^k, n_1, and c_1 as functions of the states, k_1 and b_1. It is easiest to first establish the following result in proposition 1.

Result 1: $\tau_1^n = 0$

From equations (A2) and (A3), we have that $\mu = 0$. This implies that

$$u_c(1) = \lambda$$

in (A1). Furthermore, given that $\mu = 0$, that $\lambda = \gamma$, and that $F_{kn}(1)k_1 + F_{nn}(1)n_1 = 0$ under the assumption that F is constant returns to scale, equation (A4) reduces to

$$u_l(1) = u_c(1)F_n(1). \tag{A5}$$

Since equation (6) describes households' optimal labor-leisure decision at all dates, it follows that

$$\tau_1^n = 0, \tag{A6}$$

so that labor is not taxed in the final period.■

We now turn our attention to the remaining decision rules. Using the fact that $\tau_1^n = 0$, we obtain from the government budget constraint that

$$\tau_1^k = \frac{g_1 + (F_k(1) + 1 - \delta)b_1}{F_k(1)(k_1 + b_1)}, \tag{A7}$$

which allows us to re-write the household resource constraint as

$$c_1 = \left[\left(1 - \frac{g_1 + (F_k(1) + 1 - \delta)b_1}{F_k(1)(k_1 + b_1)}\right) F_k(1) + 1 - \delta\right](k_1 + b_1) + F_n(1)n_1.$$

This last expression yields, after some basic manipulations,

$$c_1 = F(1) - g_1 + (1 - \delta)k_1. \tag{A8}$$

Therefore, we can write (A5) as

$$u_l(F(k_1, n_1) - g_1 + (1 - \delta)k_1, 1 - n_1) = u_c(F(k_1, n_1) - g_1 + (1 - \delta)k_1, 1 - n_1) F_n(k_1, n_1)$$

which defines the solution for labor in the last period as a function of k_1, and the exogenous variable, g_1,

$$n_1 \equiv n_1(k_1). \tag{A9}$$

Observe that n_1 is independent of the level of debt inherited in period 1. Moreover, we can then substitute (A9) in equations (A7) and (A8) to obtain policy functions for the capital tax rate and consumption at date 1 respectively,

$$\tau_1^k \equiv \tau_1^k(k_1, b_1) \text{ and} \tag{A10}$$

$$c_1 \equiv c_1(k_1), \tag{A11}$$

where the level of consumption, c_1, in (A8) is independent of b_1. Equations (A6), (A9), (A10), and (A11) are the optimal decision rules, as functions of the state, associated with the period 1 problem.

We now turn to the optimal policy problem from the standpoint of the initial period and address the remaining results in proposition 1.

Optimal policy in period zero

Knowing how a benevolent government behaves in the last period, so that one can anticipate what tax rates emerge given the relevant states for that date, $\{k_1, b_1\}$, one can address the problem of optimal fiscal policy from the standpoint of date 0. The relevant Lagrangian for period 0 is

$$\max \ u(c_0, 1 - n_0) + \beta \overbrace{u(c_1(k_1,), 1 - n_1(k_1))}^{V(k_1)} \tag{PA2}$$

$$+ \mu \left[u_c(0)(1 - \tau_0^n) F_n(0) - u_l(0) \right]$$

$$+ \lambda \left\{ u_c(0) - \beta \left[F_k(k_1, n_1(k_1))(1 - \tau_1^k(k_1, b_1)) + 1 - \delta \right] u_c(c_1(k_1), n_1(k_1)) \right\}$$

$$+ \gamma \left[\left(F_k(0)(1 - \tau_0^k) + 1 - \delta \right)(k_0 + b_0) + (1 - \tau_0^n) F_n(0) n_0 - c_0 - k_1 - b_1 \right]$$

$$+ \omega \left[\tau_0^k F_k(0)(k_0 + b_0) + \tau_0^n F_n(0) n_0 - g_0 - (F_k(0) + 1 - \delta) b_0 + b_1 \right],$$

where, for transparency, we have written solutions for date 1 variables explicitly in terms of the date 1 states as worked out above.

The first order conditions associated with (PA2) yield

$$c_0 : u_c(0) + \mu \left[u_{cc}(0)(1 - \tau_0^n) F_n(0) - u_{lc}(0) \right] + \lambda u_{cc}(0) - \gamma = 0,$$

$$\tau_0^k : -\gamma + \omega = 0, \tag{A12}$$

$$\tau_0^n : -\mu u_c(0) - \gamma n_0 + \omega n_0 = 0. \tag{A13}$$

The optimal choice of bonds to be carried into date 1 yields

$$b_1 : \lambda \beta F_k(1) \tau_b^k(1) u_c(1) - \gamma + \omega = 0, \tag{A14}$$

Observe that, since $\gamma = \omega$ in (A12), (A13) implies $\mu = 0$ and (A14) implies $\lambda = 0$.

The optimality condition with respect to k_1 is simply

$$k_1 : \beta \left[u_c(1) c_k(1) - u_l(1) n_k(1) \right] - \gamma = 0. \tag{A15}$$

28

The optimal allocation of labor is given by

$$n_0 \quad : \quad -u_l(0) + \tag{A16}$$

$$+\gamma \left\{ F_{kn}(0)(1-\tau_0^k)(k_0+b_0) + (1-\tau_0^n)\left[F_{nn}(0)n_0 + F_n(0)\right] \right\}$$

$$+\omega \left\{ \tau_0^k F_{kn}(0)(b_0+k_0) + \tau_0^n \left[F_{nn}(0)n_0 + F_n(0)\right] - F_{kn}(0)b_0 \right\}$$

$$= \quad 0.$$

We are now in a position to establish the following results regarding the optimal labor tax at date 0 and the optimal capital tax rate in the final period.

Result 2: $\tau_0^n = 0$ and $\tau_1^k \equiv \tau_1^k(k_1, b_1) = 0$.

Observe that (A13) implies $\mu = 0$ since $\gamma = \omega$ in (A12), and that the first order condition with respect to initial consumption reduces to

$$u_c(0) = \gamma.$$

Hence, the optimal allocation of labor in (A16) simplifies to

$$u_l(0) = u_c(0)F_n(0) \tag{A17}$$

As before, since equation (6) holds in every period, it follows that

$$\tau_0^n = 0,$$

which confirms the first part of result 2. Moreover, note that so far, results 1 and 2 combine to give us $\tau_0^n = \tau_1^n = 0$. To prove the second part of result 2, take the derivative of (A8) with respect to k_1 to obtain

$$c_k(1) = F_k(1) + F_n(1)n_k(1) + (1-\delta).$$

Consequently, using (A5), equation (A15) simplifies to

$$u_c(0) = \beta \left[u_c(1)F_k(1) + 1 - \delta\right]. \tag{A18}$$

Therefore, by equation (7), it follows that

$$\tau_1^k = 0, \tag{A19}$$

which establishes the second part of Result 2.∎

As in the steady state associated with the Ramsey problem, time consistent tax rates on labor and capital are zero in the last period. It remains to find the policy functions for τ_0^k, n_0, c_0, k_1 and b_1, given the initial levels of private capital and public debt, k_0 and b_0.

Given (A19), equation (A7) implies that

$$b_1 = \frac{-g_1}{F_k(1) + 1 - \delta},$$ (A20)

which is to say that government purchases at date 1 are financed solely by interest payments on loans made to the private sector in the initial period. Using the fact that $\tau_0^n = 0$ and the expression for b_1 above, we can re-write the government budget constraint at date 0 as

$$\tau_0^k = \frac{g_0 + (F_k(0) + 1 - \delta)b_0 + \frac{g_1}{F_k(1) + 1 - \delta}}{F_k(0)(k_0 + b_0)}.$$ (A21)

Therefore, the size of the initial tax on capital is equal to that of the present discounted value of public expenditures less the return on any initial assets owned by the government. Note that *ceteris paribus*, τ_0^k can be negative if the government starts off with a large enough assets.

From the household's resource constraint, we have that

$$c_0 = F(0) - g_0 + (1 - \delta)k_0 - k_1.$$ (A22)

Therefore, equation (A18) can be re-written as

$$u_c(F(k_0, n_0) - g_0 + (1 - \delta)k_0 - k_1, 1 - n_0)$$ (A23)
$$= \beta u_c(c_1(k_1), 1 - n_1(k_1))\left[F_k(k_1, n_1(k_1)) + 1 - \delta\right]$$

Observe also that equation (A17) can be expressed as

$$u_l(F(k_0, n_0) - g_0 + (1 - \delta)k_0 - k_1, 1 - n_0)$$ (A24)
$$= u_c(F(k_0, n_0) - g_0 + (1 - \delta)k_0 - k_1, 1 - n_0)F_n(k_0, n_0)$$

Given the initial capital stock, k_0 (and exogenous government spending g_0), equations (A23) and (A24) make up a system of two equations in two unknowns, n_0 and k_1, which define the policy functions

$$n_0 \equiv n_0(k_0)$$

and

$$k_1 \equiv k_1(k_0).$$

Finally, we can now use the expressions above in equations (A20), (A21), and (A22) to formally define the remaining policy functions $b_1(k_0)$, $\tau_0^k(k_0, b_0)$, and $c_0(k_0)$ respectively.∎

APPENDIX B - An upper limit on the capital income tax

Optimal policy in period J

Assume that the upper limit on capital income taxes ceases to bind at some date J. A benevolent government in the period is faced with states $\{k_J, b_J\}$, and maximizes household utility subject to the constraints (4) and (5), as well as the private sector decision rules (6), (7), (8), and (9), as they apply to date J. The Lagrangian corresponding to the date J problem is

$$\max \ u(J) + \beta.V_{J+1}(J+1)$$

$$+\mu_J\left[u_c(J)(1-\tau_J^n)F_n(J) - u_l(J)\right]$$

$$+\lambda_J.\left[\begin{array}{c} \beta\left[F_k(J+1).(1-\tau^k(J+1)) + 1 - \delta\right].u_c(J+1) \\ - u_c(J) \end{array}\right]$$

$$+\gamma_J.\left[F(k_J) - g_J - c_J + (1-\delta)k_J - k_{J+1}\right]$$

$$+\omega_J.\left[\tau_J^k.F_k(J).(k_J + b_J) + \tau_J^n F_n(J)n_J - g_J - (F_k(J) + 1 - \delta).b_J + b_{J+1}\right]$$

$$+\xi_J.(1-\tau_J^k)$$

The first order conditions associated with this problem are

$c_J \quad : \quad u_c(J) + \mu_J.\left[u_{cc}(J).(1-\tau_J^n).F_n(J) - u_{lc}(J)\right] - \lambda_J.u_{cc}(J) - \gamma_J = 0$ \hfill (B1)

$n_J \quad : \quad -u_l(J)$ \hfill (B2)

$\qquad \qquad +\mu_J\left\{(1-\tau_J^n)\left[u_c(J)F_{nn}(k_J) - u_{cl}(J)F_n(J)\right] + u_{ll}(J)\right\}$

$\qquad \qquad -\lambda_J u_{cl}(J) + \gamma_J.F_n(J)$

$\qquad \qquad +\omega_J.\left[\tau_J^k.F_{kn}(k_J).(k_J + b_J) + \tau_J^n\left[F_{nn}(k_J)n_J + F_n(J)\right] - F_{kn}(k_J).b_J\right] = 0$

$\tau_J^n \quad : \quad -\mu_J u_c(J)F_n(J) + \omega_J.F_n(J)n_J = 0$ \hfill (B3)

$\tau_J^k \quad : \quad \omega_J.F_k(J).(k_J + b_J) = \xi_J$ \hfill (B4)

$b_{J+1} \quad : \quad \beta.V_b(J+1) + \omega_J - \lambda_J.\beta.\left\{\begin{array}{c} F_k(J+1).\tau_b^k(J+1).u_c(J+1) \\ + \left[F_k(k_{J+1}).(1-\Upsilon^k(J+1)) + 1 - \delta\right].u_{cc}(J+1)c_b(J+1) \end{array}\right\} = 0$ \hfill (B5)

$k_{J+1} \quad : \quad \beta.V_k(J+1) - \gamma_J$ \hfill (B6)

$\qquad \qquad +\lambda_J.\beta.\left\{\begin{array}{c} \left[F_{kk}(k_{J+1}) + F_{kn}(k_{J+1}).n_k(J+1)\right](1-\Upsilon_{J+1}(J+1)).u_c(J+1) \\ -F_k(J+1).\Upsilon_k^k(J+1).u_c(J+1) \\ + \left[F_k(J+1).(1-\Upsilon_{J+1}^k(J+1)) + 1 - \delta\right].\left[u_{cc}(J+1).c_k(J+1) + u_{cl}(J+1).n_k(J+1)\right] \end{array}\right\}$

$\qquad = \quad 0$

Given these conditions, we assume that the constraint on capital income taxes is not binding at date J and find a solution for the Markov problem from date J on using a guess and verify method. We guess some characteristics of the solution path and then verify that the derived solution has those characteristics.

If the constraint on the capital income tax is not binding at date J:

$$\xi_J = 0$$

From equation (B4) we have that $\omega_J = 0$, and then equation (B3) implies that $\mu_J = 0$.

Guess 1: $V_b(J+1) = 0$

If we assume that next period's value function is not affected by marginal changes to the level of debt, equation (B5) implies that:

$$\lambda_J = 0$$

Equations (B1) and (B2) become

$$u_c(J) = \gamma_J$$

and

$$u_c(J).F_n(J) = u_l(J)$$

Since equation (6) describes households' optimal labor-leisure decision at all dates, it follows that

$$\tau_J^n = 0$$

so that labor is not taxed at date J.

Guess 2: $\tau_{J+l}^n = 0$ and $\tau_{J+l+1}^k = 0$ for any $l > 0$

If we assume that from next period on the tax on labor income is zero, $\tau_{J+l}^n = 0$ for any $l > 0$,and from the following period on the tax on capital income is zero, $\tau_{J+l+1}^k = 0$ for any $l > 0$, we get:

$$V_k(k_{J+1}) = u_c(c_{J+1}(J+1)). \left[F_k(J+1) + (1-\delta)\right].$$

Then (B6) implies

$$k_{J+1} : \beta.u_c(c_{J+1}(J+1)). \left[F_k(J+1) + (1-\delta)\right] = u_c(J)$$

Therefore, by equation (7), it follows that

$$\tau^k_{J+1} = 0$$

Hence, if the constraint on the tax rate on capital is not binding this period ($\xi_J = 0$) and if $V_b(J+1) = 0$, and $\tau^n_{J+l+1} = 0$, $\tau^k_{J+l} = 0$ for any $l > 0$, then $\tau^n_J = 0$ and $\tau^k_{J+1} = 0$.

Moving one period ahead we know that $\tau^k_{J+1} = 0$ and therefore the constraint on the capital income tax is not binding, $\xi_{J+1} = 0$. We can then use the same procedure to show that if $V_b(J+2) = 0$, and $\tau^n_{J+l+2} = 0$, $\tau^k_{J+l+1} = 0$ for any $l > 0$ then $\tau^n_{J+1} = 0$ and $\tau^k_{J+2} = 0$. We can apply the same procedure until before the last period.

Optimal policy in period T

So now we arrive at period T with a sequence of zero taxes on capital and labor: $\tau^n_l = 0$, $\tau^k_{l+1} = 0$ for any $J \leq l \leq T - 1$. The Lagrangian corresponding to the date T problem is

$$\max \ u(c_T, 1 - n_T)$$
$$+\mu_T \left[u_c(c_T, 1 - n_T)(1 - \tau^n_T)F_n(T) - u_l(c_T, 1 - n_T) \right]$$
$$+\gamma_T. \left[F(T) - g_T - c_T + (1 - \delta)k_T \right]$$
$$+\omega_T. \left[\tau^k_T.F_k(T).(k_T + b_T) + \tau^n_T F_n(T)n_T - g_T - (F_k(T) + 1 - \delta).b_T \right]$$
$$+\xi_T.(1 - \tau^k_T)$$

$$c_T \ : \quad u_c(T) + \mu_T. \left[u_{cc}(T).(1 - \tau^n_T).F_n(T) - u_{lc}(T) \right] - \lambda_T.u_{cc}(T) - \gamma_T = 0$$

$$n_T \ : \quad -u_l(c_T, 1 - n_T)$$
$$+\mu_T \left\{ (1 - \tau^n_T) \left[u_c(T)F_{nn}(T) - u_{cl}(T)F_n(T) \right] + u_{ll}(T) \right\}$$
$$+\gamma_T.F_n(T)$$
$$+\omega_T. \left[\tau^k_T.F_{kn}(T).(k_T + b_T) + \tau^n_T \left[F_{nn}(T)n_T + F_n(T) \right] - F_{kn}(T).b_T \right] = 0$$

$$\tau^n_T \ : \quad -\mu_T u_c(c_T, 1 - n_T)F_n(T) + \omega_T.F_n(T)n_T = 0$$

$$\tau^k_T \ : \quad \omega_T.F_k(T).(k_T + b_T) = \xi_T$$

As $\tau^k_T = 0$ then $\xi_T = 0$ which implies that

$$c_T \quad : \quad u_c(c_T, 1 - n_T) = \gamma_T$$

$$n_T \quad : \quad u_l(c_T, 1 - n_T) = u_c(c_T, 1 - n_T).F_n(T)$$

$$\tau_T^n \quad : \quad \mu_T = 0$$

$$\tau_T^k \quad : \quad \omega_T. = 0$$

Therefore, by equation (6), it follows that

$$\tau_T^n = 0.$$

So, $\tau_l^n = 0$, $\tau_{l+1}^k = 0$ for any $l \geq J$.

Verifying our guesses

We should now confirm that our remaining guesses are correct. That is $V_b(J + l) = 0 \;\; \forall \, l > 0$.

The value function at time $J + l$ is a function of consumption and leisure in period $J + l$ and in all future periods.

k_{J+l+1}, and c_{J+l}, and n_{J+l} are determined by solving the following system:

$$u_c(J + l)(1 - \tau_{J+l}^n)F_n(k_{J+l}) = u_l(J + l)$$

$$\beta \left[F_k(J + l + 1).(1 - \tau_{J+l+1}^k) + 1 - \delta \right].u_c(J + l + 1) = u_c(J + l)$$

$$c_J = F(k_J) - g_J + (1 - \delta)k_J - k_{J+1}$$

As $\tau_{J+l}^n = 0$ and $\tau_{J+l+1}^k = 0 \; \forall \, l > 0$, we have.

$$u_c(J + l)F_n(k_{J+l}) = u_l(J + l)$$

$$\beta \left[F_k(J + l + 1) + 1 - \delta \right].u_c(J + l + 1) = u_c(J + l)$$

$$c_J = F(k_J) - g_J + (1 - \delta)k_J - k_{J+1}$$

And it is clear that the solutions to this system are independent of b_{J+l}.Hence, the value function at time $J + l$, for any $l > 0$, is independent of the level of debt.

APPENDIX C - Optimal time consistent taxation in an infinite horizon economy

The Lagrangian corresponding to problem (P2) is

$$L = \max_{c,n,\tau^k,\tau^n,k',b'} u(c, 1-n) + \beta W(k', b') \tag{PA3}$$

$$+\mu \left\{ \left[(1-\tau^k)F_k + 1 - \delta \right](k+b) + (1-\tau^n)F_n n - k' - b' - c \right\}$$

$$+\lambda \left\{ u_l(c, 1-n) - u_c(c, 1-n)(1-\tau^n)F_n \right\}$$

$$+\gamma \left(u_c(c, 1-n) - \beta \left\{ \left[(1-\psi(k', b'))F_k' + 1 - \delta \right] u_c(\widetilde{c}(k', b', H'), 1 - n') \right\} \right)$$

$$+\omega \left\{ \tau^k F_k(k+b) + \tau^n F_n n - g - (F_k + 1 - \delta)b + b' \right\},$$

where

$$\widetilde{c}(k', b', H') = \left(F_k' + 1 - \delta \right) k' + F_n' n' - H(k', b') - g',$$

denotes next period's consumption under the assumption that tomorrow's government abides by the Markov policy rules θ and ψ.

Since the one period deviation considered in (PA3) is such that households behave optimally thereafter under the Markov policy rules θ and ψ, observe from the dynamic program in (P2) that

$$W(k, b) = u(c(k, b), 1 - n(k, b)) + \beta W(k', b')$$

where

$$c(k, b) = \left[(1 - \psi(k, b))F_k + 1 - \delta \right](k+b) + (1 - \theta(k, b)F_n n - \mathcal{H}(k, b) - \mathcal{B}(k, b)$$

and $n(k, b)$ solves

$$u_l(c(k, b), 1 - n) = u_c(c(k, b), 1 - n)F_n(1 - \theta(k, b)).$$

The first order conditions associated with problem (PA3) are as follows (for notational convenience, when the context is clear, we do not explicitly write u_c, F_k, F_n, etc... as a function of their arguments):

$$\tau^k \quad : \quad -\mu F_k(k+b) + \omega F_k(k+b) = 0$$

$$\Rightarrow \quad \mu = \omega.$$

$$\tau^n \quad : \quad -\mu F_n n + \lambda F_n u_c + \omega F_n n$$

$$\Rightarrow \quad \lambda = 0.$$

$$b' \quad : \quad \beta W_b(k', b') - \mu$$

$$-\gamma\beta \left\{ -\psi_b' F_k' u_c(\tilde{c}', 1 - n') + \left[(1 - \psi') F_k' + 1 - \delta \right] \left[u_{cc}' \tilde{c}_b' - u_{cl} n_b' \right] \right\}$$

$$+\omega$$

$$= \quad 0.$$

Since $\mu = \omega$, this last expression reduces to

$$W_b(k', b') = \gamma \left\{ -\psi_b' F_k' u_c(\tilde{c}', 1 - n') + \left[(1 - \psi') F_k' + 1 - \delta \right] \left[u_{cc}' \tilde{c}_b' - u_l n_b' \right] \right\}. \tag{C1}$$

$$c \quad : \quad u_c - \mu + \lambda \left[u_{lc} - u_{cc}(1 - \tau^n) F_n \right] + \gamma u_{cc} = 0 \tag{C2}$$

$$\Rightarrow \quad u_c + \gamma u_{cc} = \mu \text{ since } \lambda = 0.$$

After some manipulations, the first-order conditions with respect to n yields

$$n : u_l = \mu F_n - u_{cl}\gamma. \tag{C3}$$

Finally, the optimality condition with respect to tomorrow's capital stock reads as

$$\beta W_k'(k', b') - \mu -$$

$$\gamma\beta \left\{ \left(-\psi_k' F_k' + (1 - \psi') \left[F_{kk}' + F_{kn}' n_k' \right] \right) u_c' + \left[(1 - \psi') F_k' + 1 - \delta \right] \left[u_{cc}' \tilde{c}_k' - u_{cl} n_k' \right] \right\}$$

$$= \quad 0.$$

To arrive at the results stated in proposition 2, as well as to highlight some interesting properties of the problem above, we begin by focusing on equation (C1). In particular, let us conjecture for now that $\mathcal{H}(k, b) = \mathcal{H}(k)$ and $W_b(k, b) = 0$. In other words, we conjecture that the value function associated with problem (PA3) is independent of the level of debt along the equilibrium path. Then, since the expression in brackets is strictly non-zero, we have that

$$\gamma = 0.$$

It immediately follows that

$$u_c = \mu$$

and that

$$u_l = u_c F_n \tag{C4}$$

36

from equations (C2) and (C3) respectively. In particular, this last equation implies that $\tau^n = 0$ so that, in a Markov perfect equilibrium, $\theta(k, b) = 0 \ \forall k$ and b. This establishes the first part of proposition 2.

From the first-order condition with respect to k', we have that

$$u_c = \beta W_k', \tag{C5}$$

where

$$W_k' = u_c' \left[F_k' + F_n' n_k' + 1 - \delta - \mathcal{H}_k' \right] - u_l' n_k' + \beta (W_k'' \mathcal{H}_k' + \underbrace{W_b'' \mathcal{B}_k'}_{=0}).$$

Since $u_l' = u_c' F_n'$ and $W_k'' = u_c'/\beta$ from equations (C4) and (C5) respectively, this last equation simplifies to

$$u_c = \beta \left[F_k' + 1 - \delta \right] u_c',$$

from which it follows that $\psi(k', b') = 0$ when k' and b' are chosen optimally. Put another way, $\psi(\mathcal{H}(k, b), \mathcal{B}(k, b)) = 0$, which establishes the second part of proposition 2.

It now remains to verify that our conjectures $\mathcal{H}(k, b) = \mathcal{H}(k)$ and $W_b(k, b) = 0$ is actually correct in a Markov-perfect equilibrium.

Evaluating the agents' first order condition for leisure in the political equilibrium, we see that labor is a function of capital holdings and savings (recall that g is exogenously given): $n(k, k')$.

$$u_l(F(k, n) + (1 - \delta)k - k' - g, 1 - n) = u_c(F(k, n) + (1 - \delta)k - k' - g, 1 - n)F_n(k, n).$$

Replacing $n(k, k')$ into the optimality condition for savings, we find that k' is indeed independent of bond holdings,

$$u_c(F(k, n(k, k')) + (1 - \delta)k - k' - g, 1 - n(k, k')) = \beta \left[F_k(k', n(k, k')) + 1 - \delta \right]$$
$$u_c(F(k', n(k', k'')) + (1 - \delta)k - k'' - g, 1 - n(k', k'')),$$

since $k' = \mathcal{H}(k)$ and $k'' = \mathcal{H}(\mathcal{H}(k))$ satisfy this equation.

The optimal labor choice is then, $n(k, \mathcal{H}(k)) \equiv \eta(k)$ and consumption equals $\mathfrak{c}(k) = F(k, \eta(k)) + (1 - \delta)k - \mathcal{H}(k) - g$. Finally, the value function becomes, $W(k, b) = u(\mathfrak{c}(k), 1 - \eta(k)) + \beta W(\mathcal{H}(k)) = W(k)$, which is indeed independent of b.

APPENDIX D - Optimal time consistent tax rates in an overlapping generations economy

This appendix shows the derivations of equations (26) and (27) in the main text. In period 1, a benevolent government solves

$$W = \max \frac{c_{o1}^{1-\sigma}}{1-\sigma} + \frac{c_{y1}^{1-\sigma}}{1-\sigma} - \chi \frac{n_1^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \tag{PA4}$$

subject to agents' resource constraints (19) and (22), the optimal labor-leisure decision equation, (23), and the government budget constraint, (25).

From (23) and (22), we have that

$$n_1 = \chi^{\frac{-\nu}{1+\sigma\nu}}[(1-\tau_1^n)w_1]^{\frac{\nu\sigma-\nu}{1+\sigma\nu}}. \tag{D1}$$

Using the latter expression, equation (22) gives

$$c_{y1} = \chi^{\frac{-\nu}{1+\sigma\nu}}[(1-\tau_1^n)w_1]^{\frac{1+\nu}{1+\sigma\nu}}. \tag{D2}$$

From the period 1 government budget constraint, we have

$$\tau_1^k = \frac{g_1 + [r_1 + 1 - \delta]b_1 - \tau_1^n w_1 n_1}{r_1(k_1 + b_1)}.$$

Substituting the latter expression into the old generation's budget constraint, we have

$$c_{01} = [r_1 + 1 - \delta]k_1 - g_1 + \tau_1^n w_1 n_1. \tag{D3}$$

Having solved for n_1, c_{y1}, and c_{01}, in equations (D1) through (D3) respectively, it follows from (PA4) that

$$\frac{\partial W}{\partial \tau_1^n} = c_{o1}^{-\sigma}\frac{\partial c_{o1}}{\partial \tau_1^n} + c_{y1}^{-\sigma}\frac{\partial c_{y1}}{\partial \tau_1^n} - \chi n_1^{\frac{1}{\nu}}\frac{\partial n_1}{\partial \tau_1^n} = 0. \tag{D4}$$

In taking the derivatives above, the effects on factor prices offset each other (a proof of this result is available upon request), so that we can ignore these effects without loss of generality to simplify the expressions below. Thus,

$$\frac{\partial n_1}{\partial \tau_1^n} = \frac{-(1-\sigma)n_1}{(1-\tau_1^n)(\frac{1}{\nu}+\sigma)},$$

or equation (27) in the text,

$$\frac{\partial c_{y1}}{\partial \tau_1^n} = \frac{-(1+\nu)c_{y1}}{(1-\tau_1^n)(1+\sigma\nu)},$$

and

$$\frac{\partial c_{o1}}{\partial \tau_1^n} = w_1\left(n_1 + \tau_1^n\frac{\partial n_1}{\partial \tau_1^n}\right).$$

Substituting these last three equations into (D4), we obtain after some manipulations equation (26),

$$\left(1 - \frac{\tau_1^n}{1 - \tau_1^n}\frac{1 - \sigma}{\frac{1}{\nu} + \sigma}\right).c_{o1}^{-\sigma} - c_{y1}^{-\sigma} = 0.$$